The Pomeron in QCD

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September 19, 2013

Topics to be Covered

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- Introduction to Regge Theory- the “classical” Pomeron

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- Soft and hard Pomerons

Strong Interactions Before QCD

Extract information from unitarity and analyticity properties of S-matrix.

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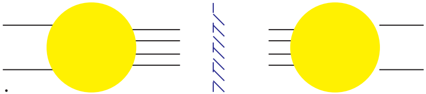
Unitarity - Optical Theorem



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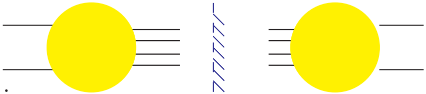

$$\Im m \mathcal{A}_{\alpha\alpha} = \sum_n \mathcal{A}_{\alpha n} \mathcal{A}^*_{n\alpha}$$

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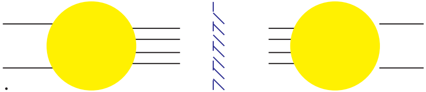

$$\Im m \mathcal{A}_{\alpha\alpha} = \sum_n \mathcal{A}_{\alpha n} \mathcal{A}^*_{n\alpha}$$

$$\Im m \mathcal{A}(s, t=0) = \frac{1}{2s} \sigma_{TOT}$$

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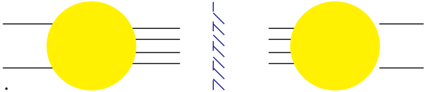

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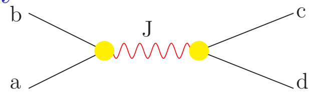
$$\Im m \mathcal{A}(s, t=0) = \frac{1}{2s} \sigma_{TOT}$$

Can be extended (Cutkosky Rules)

$$\Delta_s \mathcal{A}_{\alpha\beta} = \sum_{\text{cuts}} \mathcal{A}_{\alpha n} \mathcal{A}^*_{n\beta}$$

Leads to self-consistency relations for scattering amplitudes (Bootstrap)

Partial Wave Analysis



$$\mathcal{A}^{ab\to cd}(s,t) \ = \ \sum_J a_J(s) P_J(1-2t/s), \quad [\cos\theta = (1-2t/s), \ m_i \to 0]$$

Crossing:

$$\mathcal{A}^{a\bar{c}\to\bar{b}d}(s,t) \ = \ \mathcal{A}^{ab\to cd}(t,s) \ = \ \sum_J a_J(t) P(J, 1-2s/t)$$

In the limit $s \gg t$ (diffractive scattering)

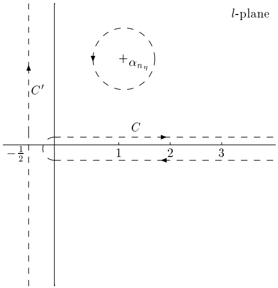
$$P(J, 1-2s/t) \sim s^J$$

so that

$$\mathcal{A}^{a\bar{c}\to\bar{b}d}(s,t) \ \stackrel{s\to\infty}{\to} \ \sum_J b_J(t) s^J$$

Sommerfeld-Watson Transformation

$$\mathcal{A}^{a\bar{c}\to\bar{b}d}(s,t) \;=\; \oint_C \sum_{\eta=\pm1} \frac{(2J+1)}{\sin\pi J}\frac{\left(\eta+e^{i\pi J}\right)}{2} a^{\eta}(J,t)P(J,2s/t)$$



Poles at integer J

Deform contour to C'

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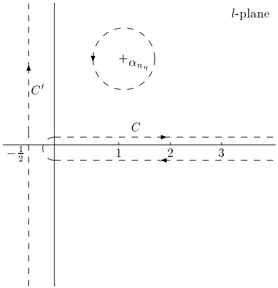

Poles at integer J

Deform contour to C'

For large s, integral along C' is zero.

Pick up only contributions from poles in J-plane.

N.B. Important (and possibly incorrect) assumption is that all singularities in J-plane are poles.

Sommerfeld-Watson Transformation

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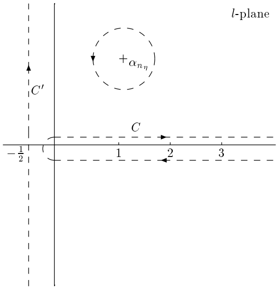


Poles at integer J

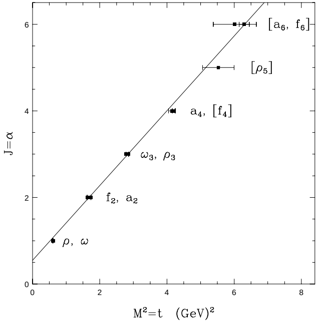
Deform contour to C'

$$\mathcal{A}(s,t) \overset{s\to\infty}{\to} \sum_i \frac{\left(\eta+e^{i\pi\alpha_i(t)}\right)}{2} \beta_i(t) s^{\alpha_i(t)}$$

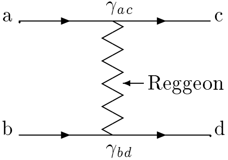
α_i are the poles in the J-plane

For $s\to\infty$ we only need the leading pole.

Chew-Frautschi Plot



Continue to negative t



$$\mathcal{A} \sim \gamma_{ac}(t)\gamma_{bd}(t)s^{\alpha_R(t)}$$

(Factorisation)
We can think of Regge exchange as the superposition of the exchange of many particles.

The Pomeron

Leading trajectory

Using Optical Theorem

$$\mathcal{A}(s,0) \stackrel{s\to\infty}{\to} \sim s^{\alpha(0)}$$

implies

$$\sigma_{TOT} \sim s^{\alpha(0)-1}$$

Okun-Pomeranchuk Theorem

If exchanged Regge trajectory does NOT have the quantum numbers of the vacuum, $\alpha(0) < 1$.

Foldy-Peierls Theorem

If $\alpha(0) \geq 1$, the Regge trajectory exchanged MUST have the quantum numbers of the vacuum.

THIS IS THE POMERON

N.B. $\alpha(0) > 1$ is NOT allowed by unitarity.

Froissart-Martin bound (derived from unitarity)

$$\sigma_{TOT} < A \ln^2 s$$

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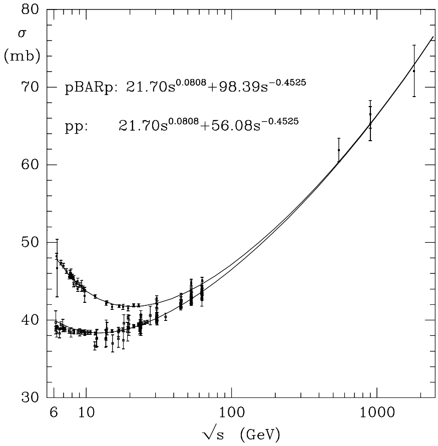

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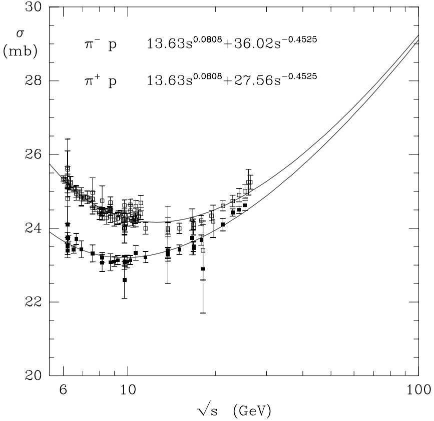

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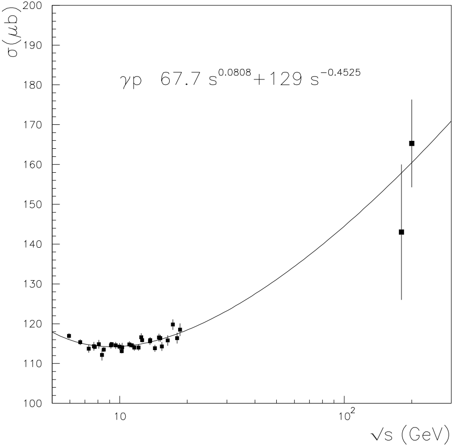

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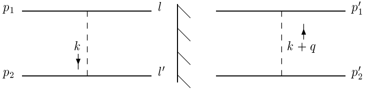
- How is the Pomeron explained in QCD?
- Data fitted by Landshoff-Donnachie cannot be calculated in purely perturbative QCD - non perturbative effects must be included
- Expect a hint of the Pomeron from perturbative QCD - but we don't really find one
- Phenomenological models have to be used to explain data.
- Nevertheless perturbative QCD in the kinematic regime where Pomerons are expected to dominate give some interesting results (solutions to the BFKL equation) which can be compared with data in certain cases.

Toy Model

$$\mathcal{L}_I = \lambda\phi^3$$

Use Optical theorem to calculate $\Im m\mathcal{A}(\sqrt{s},\mathbf{q})$



Leading order
Neglecting masses.

$$k = (k^+, k^-, \mathbf{k})$$

$$q = (0, 0, \mathbf{q})$$

$$dLIPS = \frac{1}{2} dk^+ dk^- d^2\mathbf{k}\,\delta(k^+(\sqrt{s} - k^-) - \mathbf{k}^2)$$

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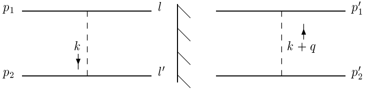

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$$\Im m \mathcal{A} = (2\pi)\lambda^4 \int dk^+ dk^- d^2\mathbf{k} \frac{\delta(k^+(\sqrt{s} - k^-) - \mathbf{k}^2)\delta(k^-(\sqrt{s} - k^+) - \mathbf{k}^2)}{(k^+k^- - \mathbf{k}^2)(k^+k^- - (\mathbf{k} - \mathbf{q})^2)}$$

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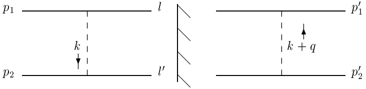

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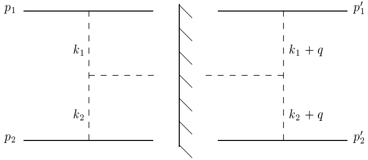
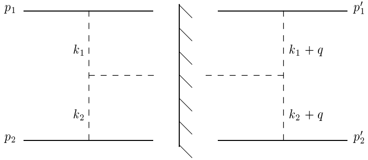
Higher Orders

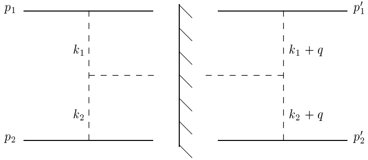
In higher order we want to sum all terms $\sim \lambda^{2n} \ln^n(s)$.
Is this approximation valid?
Maybe not - renormalon studies suggest that leading logarithm sums are not reliable.


p_1
p'_1
k_1
$k_1 + q$
k_2
$k_2 + q$
p_2
p'_2



$$\frac{\lambda^6}{8\pi}\int dk_1^+ dk_1^- d^2\mathbf{k_1} dk_2^+ dk_2^- d^2\mathbf{k_2}\delta(k_1^+(\sqrt{s}-k_1^-)-\mathbf{k_1}^2)$$

$$\frac{\delta((k_2-k_1)^+(k_2-k_1)^- - (\mathbf{k_1}-\mathbf{k_2})^2)\delta(k_2^- - (\sqrt{s}-k_2^+)-\mathbf{k_2}^2)}{(k_1^+k_1^- - \mathbf{k_1}^2)(k_1^+k_1^- - (\mathbf{k_1}-\mathbf{q}^2)(k_2^+k_2^- - \mathbf{k_2}^2)(k_2^+k_2^- - (\mathbf{k_2}-\mathbf{q})^2)}$$



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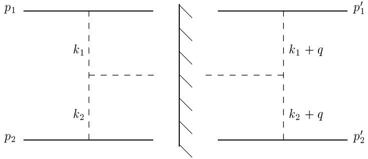
$$= \int_{k_1^+}^{\sqrt{s}} \frac{dk_2^+}{(k_2^+-k_1^+)}\frac{d^2\mathbf{k_1}d^2\mathbf{k_2}}{\mathbf{k_1}^2(\mathbf{k_1}-\mathbf{q})^2\mathbf{k_2}^2(\mathbf{k_2}-\mathbf{q})^2}$$



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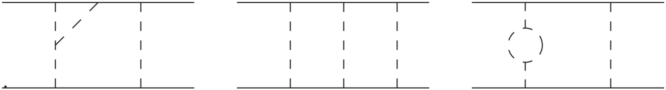
$$= \int_{k_1^+}^{\sqrt{s}} \frac{dk_2^+}{(k_2^+ - k_1^+)} \frac{d^2\mathbf{k_1} d^2\mathbf{k_2}}{\mathbf{k_1}^2(\mathbf{k_1}-\mathbf{q})^2\mathbf{k_2}^2(\mathbf{k_2}-\mathbf{q})^2}$$

Integral over k_2^+ gives $\ln(s)$
Dominated by the region $\sqrt{s} \gg k_2^+ \gg k_1^+$.

On the other hand

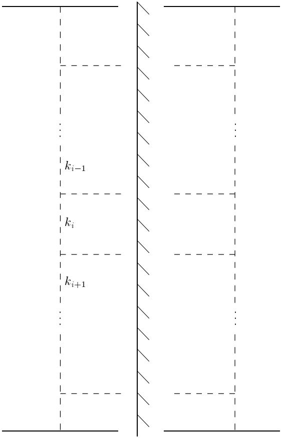


etc .

give an extra power of λ^2, but NO extra $ln(s)$.

Such graphs may be dropped in the leading log approximation.

Multi-rung Ladder Graph



$$\Im m \mathcal{A}^n \sim \int_{k_{n-1}^+}^{\sqrt{s}/2} \frac{dk_n^+}{(k_n^+ - k_{n-1}^+)} \cdots \int_{k_1^+}^{\sqrt{s}/2} \frac{dk_2^+}{(k_2^+ - k_2^+)}$$

$$\ln^n s$$

Dominant region of k^+ integration

$$\sqrt{s} \gg k_n^+ \gg k_{n-1}^+ \cdots \gg k_2^+ \gg k_1^+$$

"Multi-Regge regime"

In this model only uncrossed ladder graphs contribute to the leading log approximation.

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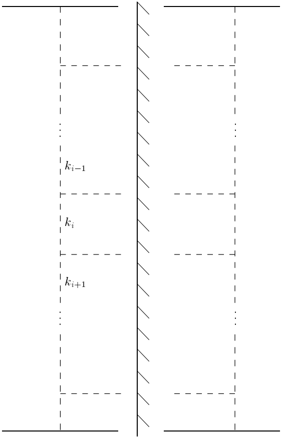

$$\sqrt{s} \gg k_n^+ \gg k_{n-1}^+ \cdots \gg k_2^+ \gg k_1^+$$

Similarly

$$\sqrt{s} \ll k_n^- \ll k_{n-1}^- \cdots \gg k_2^- \gg k_1^-$$

The on-shell delta functions for cut lines

$$\delta(k_{i+1}^- k_i^+ - (\mathbf{k_i} - \mathbf{k_{i-1}})^2)$$

Propagators of the vertical lines

$$\frac{1}{k_i^+ k_i^- - \mathbf{k_i}^2} \approx -\frac{1}{\mathbf{k_i}^2} \quad (\text{since } k_i^+ \ll \mathbf{k_i}^2 / k_i^-)$$

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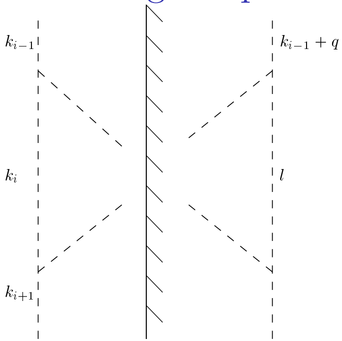
Crossed-rung Graphs



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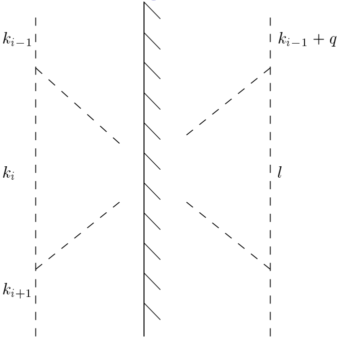

$$l^2 = (k_i + k_{i+1} - k_{i-1})^2 = (k_i^+ + k_{i+1}^+ - k_{i-1}^+)(k_i^- + k_{i+1}^- - k_{i-1}^-) - \sim \mathbf{k_i}^2$$

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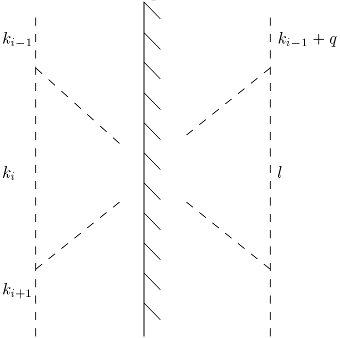

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$$\approx k_{i+1}^+ k_{i-1}^- \gg k_{i+1}^+ k_i^- \gg \mathbf{k}^2$$

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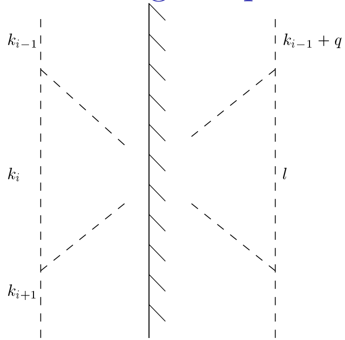

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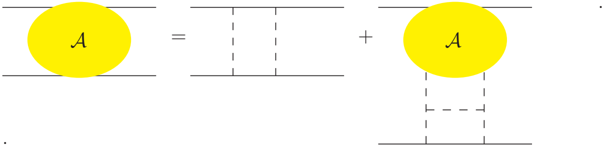
$$\approx k_{i+1}^+ k_{i-1}^- \gg k_{i+1}^+ k_i^- \gg \mathbf{k}^2$$

In the multi-Regge region crossed-ladder graphs are suppressed because denominators of propagators are larger.

Integral Equation



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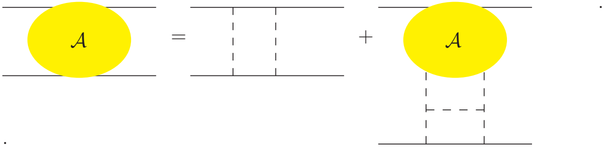
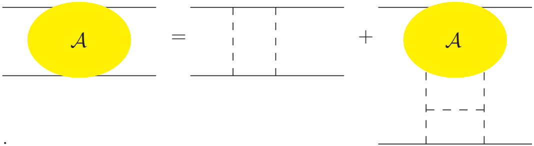


$$\Im m \mathcal{A}(\sqrt{s}, \mathbf{q}) \;=\; \Im m \mathcal{A}_0 + \frac{\lambda^2}{16\pi^3} \int^{\sqrt{s}/2} \frac{dk^+}{k^+} \frac{d^2\mathbf{k}}{\mathbf{k}^2 (\mathbf{k}-\mathbf{q})^2} \Im m \mathcal{A}(k^+, \mathbf{q})$$

Integral Equation



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↑
effect of adding rung.

Mellin Transform

(Equivalent to Sommerfeld-Watson transformation)

$$\tilde{\mathcal{A}}(\omega) \; = \; \int_{s_0}^{\infty} \left(\frac{s}{s_0}\right)^{-\omega-1} \mathcal{A}(s) ds$$

If $\mathcal{A}(s) \sim s^{\omega_0}$

$$\tilde{\mathcal{A}}(\omega) \sim \frac{1}{(\omega - \omega_0)}$$

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Furthermore

$$\begin{aligned}
\tilde{\mathcal{A}}(\omega)\tilde{\mathcal{B}}(\omega) &= \int_{s_0}^{\infty} \left(\frac{s}{s_0}\right)^{-\omega-1} \mathcal{A}(s) ds \int_{s_0}^{\infty} \left(\frac{s'}{s_0}\right)^{-\omega-1} \mathcal{B}(s') ds' \\
&= \int_{s_0}^{\infty} \left(\frac{s}{s_0}\right)^{-\omega-1} \int \frac{dk^+}{k^+} \mathcal{A}(k^+) \mathcal{B}\left(\frac{\sqrt{s}}{k^+}\right)
\end{aligned}$$

Integral Equation in Mellin Space

$$\Im m\tilde{\mathcal{A}}(\omega) = \Im m\tilde{\mathcal{A}}_0 + \frac{\lambda^2}{16\pi^3}\frac{1}{\omega}\int \frac{d^2\mathbf{k}}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}\Im m\tilde{\mathcal{A}}(\omega)$$

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QCD

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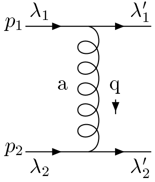
- Need to account for colour factors - distinguish between colour singlet and colour octet exchange

Colour Octet Exchange



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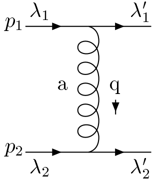
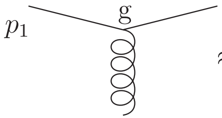


For $|\mathbf{q}|^2 \ll s$ use eikonal approximation



$$\approx 2gp_1^\mu$$

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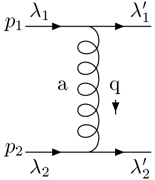

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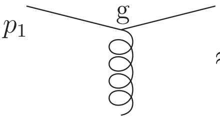

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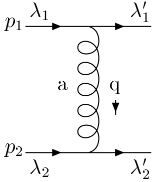

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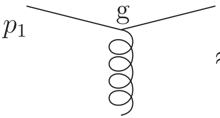

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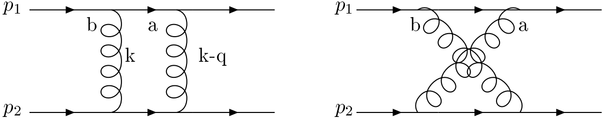
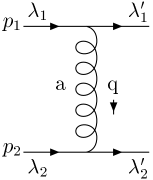
NLO



(a) (b)

Colour Octet Exchange



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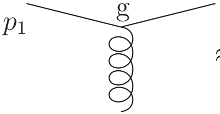

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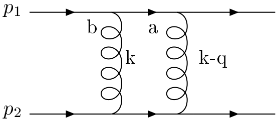

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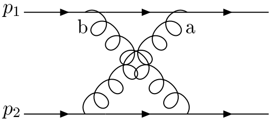

(b)

$$\Im m \mathcal{A}^{(8)} = \frac{\alpha_s C_A}{4\pi^2} \int d^2\mathbf{k} \frac{-\mathbf{q}^2}{\mathbf{k}^2 (\mathbf{k}-\mathbf{q})^2} g^2 \frac{2s}{\mathbf{q}^2} \tau^a \otimes \tau^a$$

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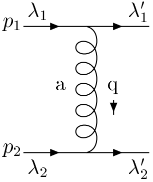

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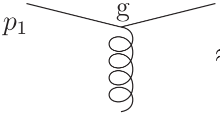

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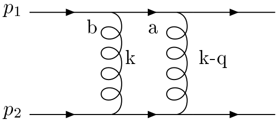

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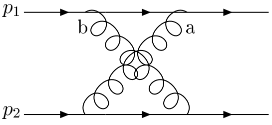

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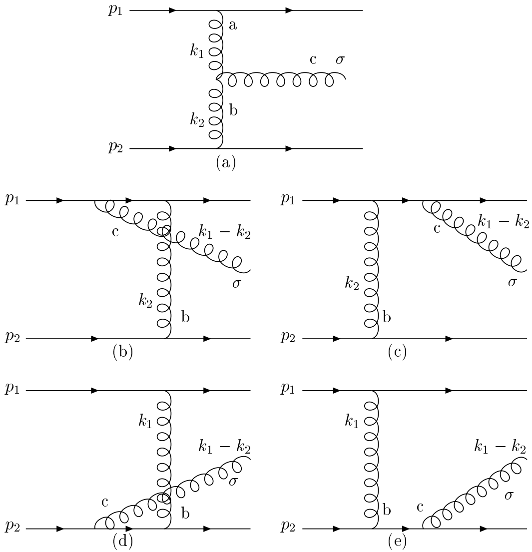
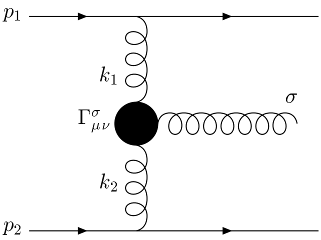
N.B. Infrared divergent - regularise by any means

NNLO



Effective vertex

$$k_1^+ \gg k_2^+, \; k_2^- \gg k_1^-$$



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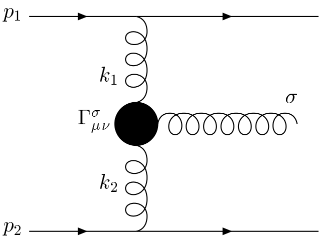
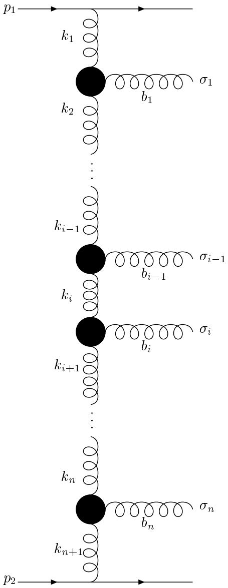


$$\Gamma^{\sigma}_{+-}(k_1,k_2) \;=\; 2gf^{abc}\left(k_1^+ + \frac{2\mathbf{k_1}^2}{k_2^-}, k_2^- + \frac{2\mathbf{k_2}^2}{k_1^+}, -(\mathbf{k_1}+\mathbf{k_2})\right)$$

(other components negligible)

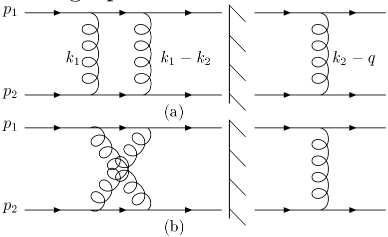


Generalises to all orders at tree-level in multi-Regge region (leading log approximation)

Each vertex replaced by $\Gamma_{+-}^{\sigma_i}(k_i, k_{i+1})$

Virtual Corrections

Need graphs such as



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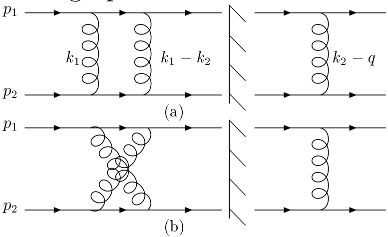

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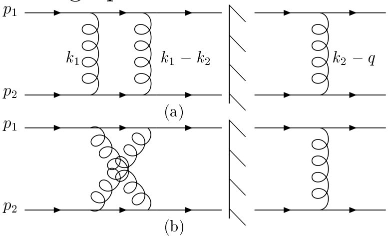

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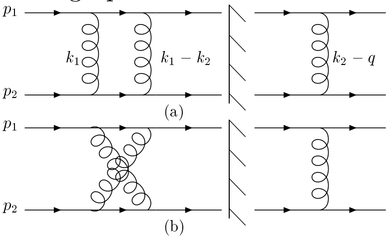

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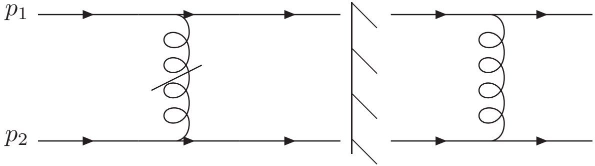
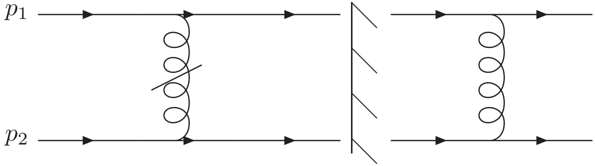
$$\varepsilon_G(\mathbf{k_2}^2) = -\frac{C_A \alpha_s}{4\pi^2} \int d^2\mathbf{k_1} \frac{\mathbf{k_2}^2}{\mathbf{k_1}^2(\mathbf{k_1}-\mathbf{k_2})^2}$$



LHS part indicates that exchanged gluon has been corrected.
(also need graph with correction of RHS of cut)



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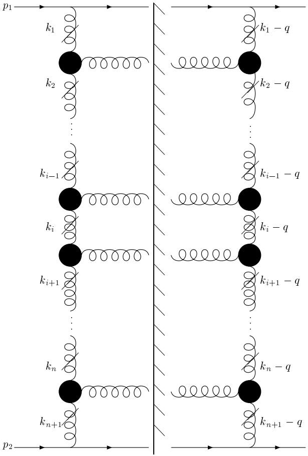
Up to NNLO we get

$$\mathcal{A}^{(8)}(\sqrt{s},\mathbf{q}) \;=\; \mathcal{A}_0^{(8)}(\sqrt{s},\mathbf{q})\left(1+\varepsilon_G(\mathbf{q}^2)\ln(s)+\frac{1}{2}\varepsilon_G^2(\mathbf{q}^2)\ln^2(s)\right)$$

Bootstrap - ladders within ladders

Suggests a self-consistency ansatz for colour octet exchange in multi-Regge region



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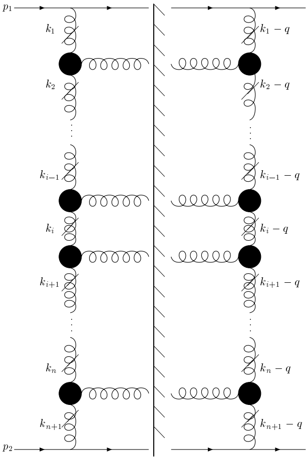

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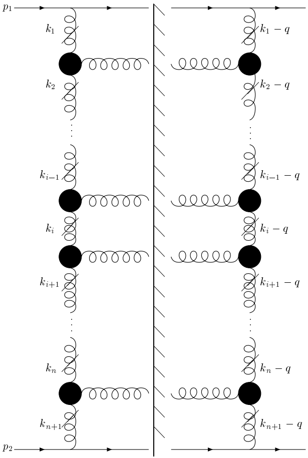

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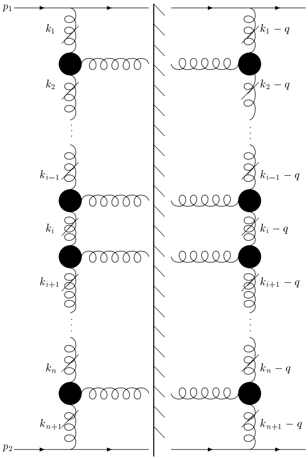

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N.B. in multi-Regge region

$$s_{i+1,i-1} \sim k_{i+1}^+ k_{i-1}^- \sim \mathbf{k_i}^2 \frac{k_{i-1}^+}{k_i^+}$$

Define $\mathcal{F}^{(8)}(s,\mathbf{k},\mathbf{q})$

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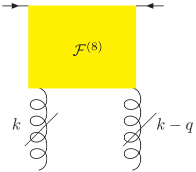
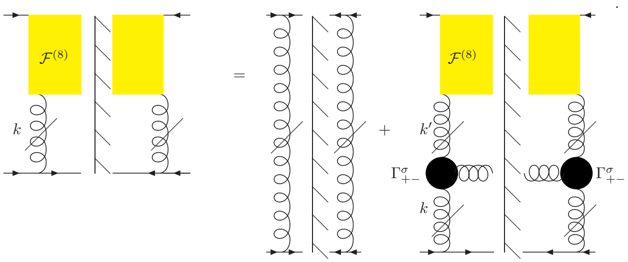
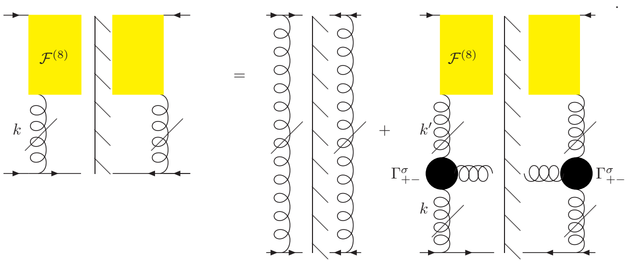




$\mathcal{F}^{(8)}$
k
=
+
$\mathcal{F}^{(8)}$
k'
Γ^{σ}_{+-}
k
Γ^{σ}_{+-}

.



$$\Im m \mathcal{F}^{(8)}(\sqrt{s}, \mathbf{k}, \mathbf{q}) = \Im m \mathcal{F}_0^{(8)}(\sqrt{s}, \mathbf{k}, \mathbf{q}) -$$

$$\frac{\alpha_s C_A}{4\pi^2} \int^{\sqrt{s}} \frac{dk^{+\prime}}{k^{+\prime}} \Gamma^{\sigma}_{+-}(k', k) \Gamma^{\sigma}_{+-}(k' - q, k - q) \frac{\Im m \mathcal{F}_0^{(8)}(k^{+\prime}, \mathbf{k}', \mathbf{q})}{\mathbf{k}'^2 (\mathbf{k}' - \mathbf{q})^2}$$

$$\times \left(\frac{k^{+\prime}}{k^{+}} \right)^{\varepsilon(\mathbf{k}^2) + \varepsilon((\mathbf{k} - \mathbf{q})^2)}$$

Take Mellin transform

$$\tilde{\mathcal{F}}^{(8)}(\omega, \mathbf{k}, \mathbf{q}) \;=\; \int_{s_0}^{\infty} s^{-\omega-1} \mathcal{F}^{(8)}(\sqrt{s}, \mathbf{k}, \mathbf{q}) ds$$

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$$\Im m\tilde{\mathcal{F}}^{(8)}(\omega,\mathbf{k},\mathbf{q}) = \Im m\tilde{\mathcal{F}}_0^{(8)}(\omega,\mathbf{k},\mathbf{q}) - \frac{\alpha_s C_A}{4\pi^2}\int d^2\mathbf{k}'\frac{\Im m\tilde{\mathcal{F}}^{(8)}(\omega,\mathbf{k}',\mathbf{q})}{\omega-\varepsilon_G(\mathbf{k}^2)-\varepsilon_G((\mathbf{k}-\mathbf{q})^2)}$$

$$\times\frac{1}{\mathbf{k}'^2(\mathbf{k}'-\mathbf{q})^2}\left(\mathbf{q}^2-\frac{\mathbf{k}^2(\mathbf{k}'-\mathbf{q})^2+\mathbf{k}'^2(\mathbf{k}-\mathbf{q})^2}{(\mathbf{k}-\mathbf{k}')^2}\right)$$

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Integrate both sides over $\mathbf{k}$ to get

$$\Im m\tilde{\mathcal{A}}^{(8)}(\omega,\mathbf{q}) = \Im m\tilde{\mathcal{A}}_0^{(8)}(\omega,\mathbf{q}) - \frac{\alpha_s C_A}{4\pi^2}\int d^2\mathbf{k}' d^2\mathbf{k} \frac{\Im m\tilde{\mathcal{F}}^{(8)}(\omega,\mathbf{k}',\mathbf{q})}{\omega-\varepsilon_G(\mathbf{k}^2)-\varepsilon_G((\mathbf{k}-\mathbf{q})^2)}$$

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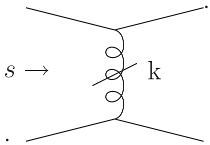
Solved by

$$\Im m\tilde{\mathcal{A}}^{(8)}(\omega,\mathbf{q}) \sim \frac{1}{\omega-\varepsilon_G(\mathbf{q}^2)}$$

This justifies the ansatz and shows that the reggeized gluon is given (to all orders in perturbation - in the leading log approximation)
Exchange of a colour octet in the Regge region $s \gg t$



$$\frac{1}{\mathbf{k}^2}\left(\frac{s}{\mathbf{k}^2}\right)^{\varepsilon_G(\mathbf{k}^2)}$$

where

$$\varepsilon_G(\mathbf{q}^2) \; = \; -\frac{\alpha_s C_A}{4\pi^2}\int d^2\mathbf{k}\frac{\mathbf{q}^2}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}$$

This is IR divergent, but since it is not a colour singlet process it is unphysical.

The Pomeron in (perturbative) QCD

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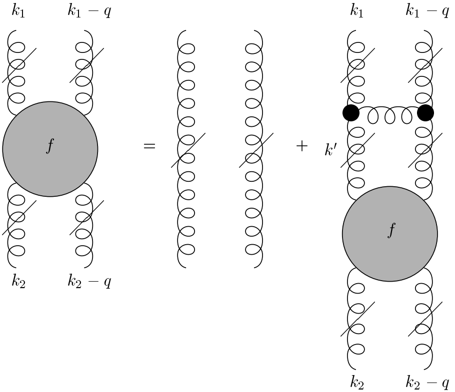
  - building ladders using the effective vertices Γ^{σ}_{+-}



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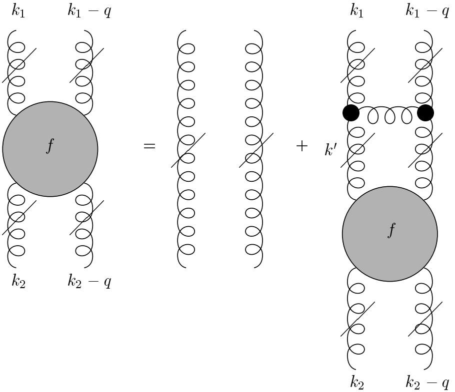

For $\mathbf{q} = 0$

$$f(\sqrt{s}, \mathbf{k_1}, \mathbf{k_2}) = \delta^2(\mathbf{k_1} - \mathbf{k_2}) - \frac{\alpha C_A}{2\pi^2} \int d^2\mathbf{k}' \int^{\sqrt{s}} dk^+ f(k^+, \mathbf{k}', \mathbf{k_2})$$

$$\times \left(\frac{k_1^+}{k^{+\prime}} \right)^{2\varepsilon_G(\mathbf{k}'^2)} \frac{\Gamma^{\sigma}_{+-}(k_1, k')\Gamma^{\sigma}_{+-}(k_1, k')}{\mathbf{k}'^4}$$

BFKL Equation at zero momentum transfer

After a HUGE amount of algebra we get (in Mellin space)

$$\omega\tilde{f}(\omega,\mathbf{k_1},\mathbf{k_2}) = \delta^2(\mathbf{k_1}-\mathbf{k2}) + \frac{\alpha C_A}{\pi^2}\int\frac{d^2\mathbf{k}'}{(\mathbf{k_1}-\mathbf{k}')^2}\left[\tilde{f}(\omega,\mathbf{k}',\mathbf{k_2}) - \frac{\mathbf{k_1}^2}{\mathbf{k}'^2+(\mathbf{k}'-\mathbf{k_1})^2}\tilde{f}(\omega,\mathbf{k_1},\mathbf{k_2})\right]$$

N.B. Integrand is finite as $\mathbf{k_1}\to\mathbf{k}'$

Integral is IR finite as expected for a colour singlet amplitude.

Solving the Equation

Write

$$\frac{\alpha C_A}{\pi^2}\int\frac{d^2\mathbf{k}'}{(\mathbf{k_1}-\mathbf{k}')^2}\left[\tilde{f}(\omega,\mathbf{k}',\mathbf{k_2})-\frac{\mathbf{k_1}^2}{\mathbf{k}'^2+(\mathbf{k}'-\mathbf{k_1})^2}\tilde{f}(\omega,\mathbf{k_1},\mathbf{k_2})\right]$$

as

$$\mathcal{K}_0\cdot\tilde{f}$$

Solution may be written as

$$\tilde{f}\omega,\mathbf{k_1},\mathbf{k_2})\ =\ \sum_i\frac{\phi_i(\mathbf{k_1})\phi_i^*((\mathbf{k_2})}{(\omega-\lambda_i)}$$

where

$$\mathcal{K}_0\cdot\phi_i\ =\ \lambda_i\phi_i$$

Eigenfunctions and Eigenvalues of BFKL Kernel

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$$\chi(\nu) \ = \ 2\Psi(1) - \Psi\left(\frac{(n+1)}{2} + i\nu\right) - \Psi\left(\frac{(n+1)}{2} - i\nu\right)$$

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General solution $(\mathbf{k} = (k, \theta))$

$$\tilde{f}(\omega, \mathbf{k_1}, \mathbf{k_2}) \;=\; \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi^2 \mathbf{k_1}\mathbf{k_2}} \left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}^2}\right)^{i\nu} \frac{e^{in(\theta_1-\theta_2)}}{\omega - \overline{\alpha_s}\chi_n(\nu)}$$

where

$$\overline{\alpha_s} \equiv \frac{\alpha_s C_A}{\pi}$$

At large s we need only the right-most singularity of the Mellin transform.

Set $n = 0$

Invert Mellin transform to get

$$f(\sqrt{s}, \mathbf{k_1}, \mathbf{k_2}) = \int \frac{d\nu}{2\pi \mathbf{k_1} \mathbf{k_2}} \left(\frac{\mathbf{k_1}}{\mathbf{k_2}} \right)^{i\nu} s^{\overline{\alpha_s} \chi_0(\nu)}$$

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Integral can be performed in saddle-point approximation (truncate χ_0 at $O(\nu^2)$)

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THE QCD POMERON

Properties of BFKL Pomeron

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- Does NOT resemble the Landshoff-Donnachie pomeron ($\sim s^{.08}$) in any way !!

BFKL Equation for $t \neq 0$

$$
\begin{aligned}
\frac{\partial}{\partial \ln(s)} f(s,\mathbf{k_1},\mathbf{k_2},\mathbf{q}) \quad = \quad & \delta^2(\mathbf{k_1}-\mathbf{k_2}) \\
& + \frac{\bar{\alpha}_s}{2\pi}\int d^2\mathbf{k}' \left[\frac{-\mathbf{q}^2}{(\mathbf{k}'-\mathbf{q})^2\mathbf{k_1}^2} f(s,\mathbf{k}',\mathbf{k_2},\mathbf{q}) \right. \\
& + \frac{1}{(\mathbf{k}'-\mathbf{k_1})^2}\left(f(s,\mathbf{k}',\mathbf{k_2},\mathbf{q}) - \frac{\mathbf{k_1}^2 f(s,\mathbf{k_1},\mathbf{k_2},\mathbf{q})}{\mathbf{k}'^2+(\mathbf{k_1}-\mathbf{k}')^2}\right). \\
& + \frac{1}{(\mathbf{k}'-\mathbf{k_1})^2}\left(\frac{(\mathbf{k_1}-\mathbf{q})^2\mathbf{k}'^2 f(s,\mathbf{k}',\mathbf{k_2},\mathbf{q})}{(\mathbf{k}'-\mathbf{q})^2\mathbf{k_1}^2} \right. \\
& \left.\left. \qquad - \frac{(\mathbf{k_1}-\mathbf{q})^2 f(s,\mathbf{k_1},\mathbf{k_2},\mathbf{q})}{(\mathbf{k}'-\mathbf{q})^2+(\mathbf{k_1}-\mathbf{k}')^2}\right)\right].
\end{aligned}
$$

Solutions for $t \neq 0$

Solution for $t = -\mathbf{q}^2$, is still of the form

$$f(s, \mathbf{k_1}, \mathbf{k_2}, \mathbf{q}) = \int d\nu \, s^{\overline{\alpha_s}\chi_0(\nu)} f_\nu(\mathbf{k_1}, \mathbf{k_2}, \mathbf{q})$$

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$$\tilde{f}^\nu(\mathbf{b_1},\mathbf{b}_1',\mathbf{b_2},\mathbf{b}_2') = \int d^2\mathbf{c} \left(\frac{(\mathbf{b_1}-\mathbf{b}_1')^2(\mathbf{b_2}-\mathbf{c})^2(\mathbf{b}_2'-\mathbf{c})^2}{(\mathbf{b_2}-\mathbf{b}_2')^2(\mathbf{b_1}-\mathbf{c})^2(\mathbf{b}_1'-\mathbf{c})^2} \right)^{i\nu}$$

Cut with (t-independent) branch-point $4\overline{\alpha_s}\ln(2)$.

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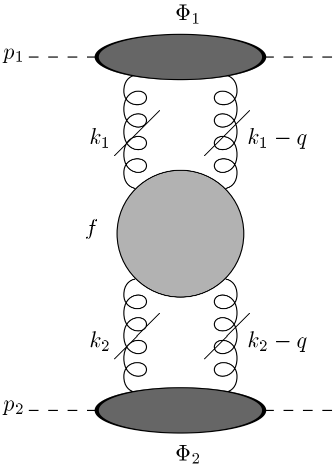
t- dependence only in the eigenfunctions.

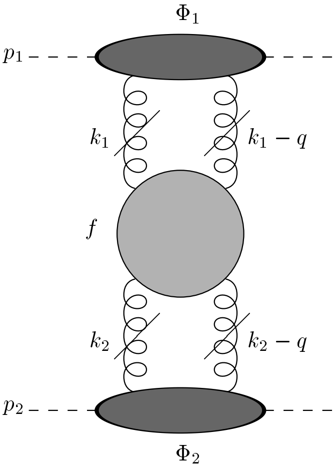
Impact factors



In order to calculate the amplitude for a physical process, the gluons at the top and bottom of the BFKL ladder must be attached to physical initial and final states. The amplitude for this process is

$$\mathcal{A} = \int \frac{d^2\mathbf{k_1}}{\mathbf{k_1}^2}\frac{d^2\mathbf{k_2}}{\mathbf{k_2}^2}\Phi_1(\mathbf{k_1})\Phi_2(\mathbf{k_2}) \times f(s,\mathbf{k_1},\mathbf{k_2},\mathbf{q})$$

Impact factors



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Φ_1 and Φ_2 are process dependent impact factors, but they can be calculated in perturbation theory.

For the perturbation expansion to be totally reliable, the impact factors should only have support for $\mathbf{k} \gg \Lambda_{QCD}$

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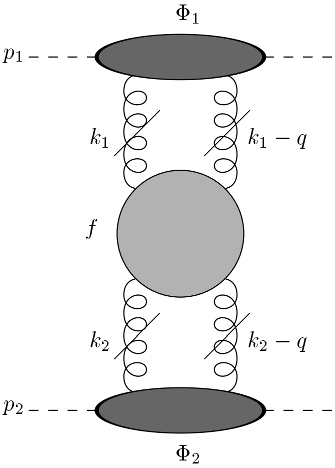

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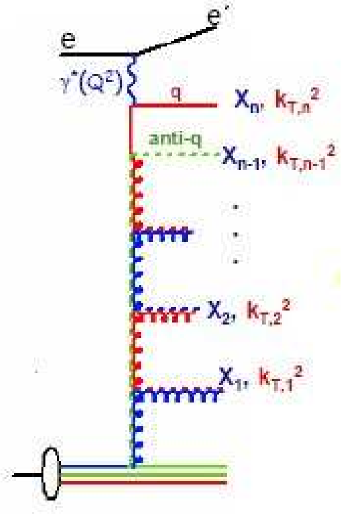
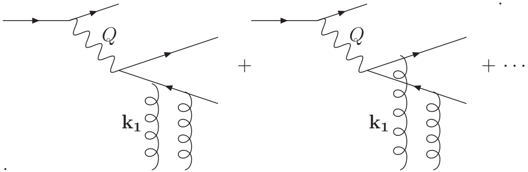
This is not usually possible - but it is often possible that one of the impact factors is in the perturbative regime.

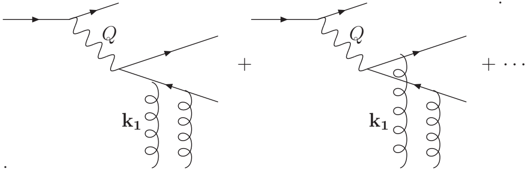
Deep Inelastic Scattering at Low-x



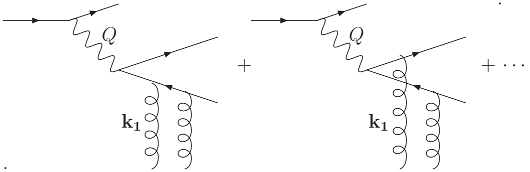
$$s = Q^2 \frac{(1-x)}{x}$$



Q
$\mathbf{k_1}$
+
Q
$\mathbf{k_1}$
+ ···



$$\Phi_2^{\gamma^*} = 4\pi\alpha\alpha_s \sum_f Q_f^2 \int_0^1 d\rho d\tau \frac{1-2\rho(1-\rho)-2\tau(1-\tau)+12\rho(1-\rho)\tau(1-\tau)}{Q^2\rho(1-\rho)+\mathbf{k_1}^2\tau(1-\tau)}$$



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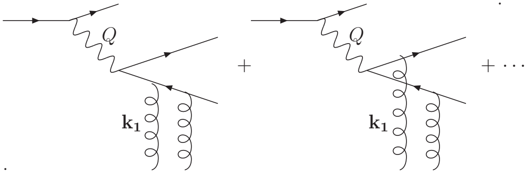
$$\mathcal{F}(x,\mathbf{k_1}^2) \equiv \frac{1}{2\pi^3}\int \frac{d^2k_2}{\mathbf{k_2}^2}\Phi_p(\mathbf{k_2}) f(Q^2/x,\mathbf{k_1},\mathbf{k_2})$$



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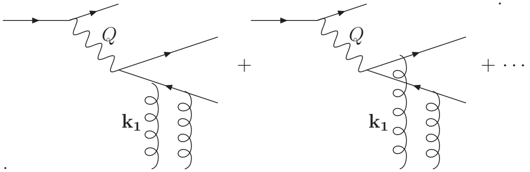

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$$F_2(x,Q^2) \ = \ \frac{Q^2}{4\pi\alpha}\int \frac{d^2\mathrm{k}_1}{\mathbf{k_1}^4}\mathcal{F}(x,\mathbf{k_1}^2)\Phi_2^{\gamma^*}(Q^2,\mathbf{k_1})$$

Insert expression for $f(Q^2/x, \mathbf{k_1}, \mathbf{k_2})$

$$F_2(x, Q^2) \sim \left(\frac{1}{x}\right)^{4\overline{\alpha_s}\ln(2)} \frac{1}{\sqrt{-\ln(x)}} \exp\left\{\frac{\ln^2(Q^2/\mu^2)}{56\zeta(3)\overline{\alpha_s}\ln(x)}\right\}$$

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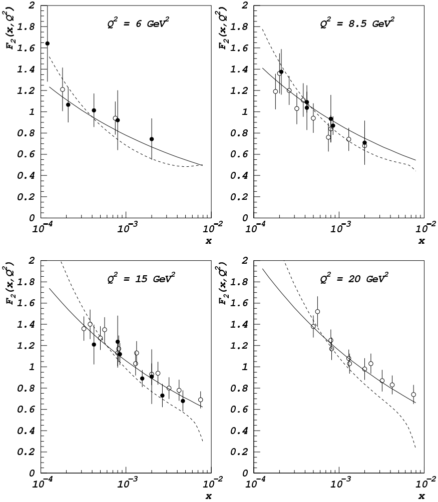

dashed line $\alpha_s = 0.1$
solid line $\alpha_s = 0.2$

Relationship with DGLAP

Moments of structure functions:

$$F_N(Q^2) = \int_0^1 x^{N-1} F_2(x,Q^2) dx = \frac{Q^2}{4\pi\alpha} \int \frac{d^2\mathbf{k_1}}{\mathbf{k_1}^2} \mathcal{F}_N(\mathbf{k_1}) \Phi_2^{\gamma^*}(Q^2,\mathbf{k_1})$$

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Integral over γ picks up a pole at

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to get DGLAP gluon splitting function near N=0 to all orders in α_s

$$\lim_{N\to 0}\gamma_N = \overline{\gamma} = \left(\frac{\overline{\alpha_s}}{N}\right) + 2\zeta(3)\left(\frac{\overline{\alpha_s}}{N}\right)^4 + \ \cdots$$

Mono-jets

Is it possible to isolate a region of phase-space which has a "clean" BFKL pomeron - no non-perturbative model.

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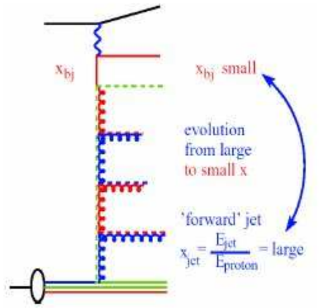

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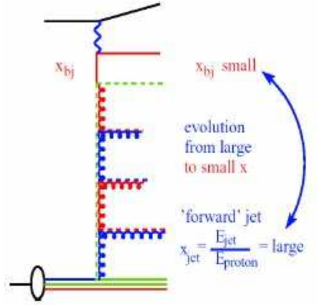

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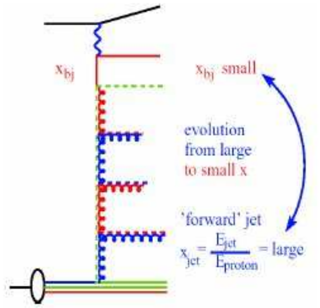

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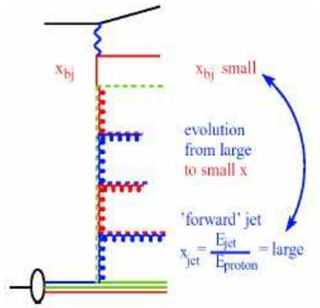

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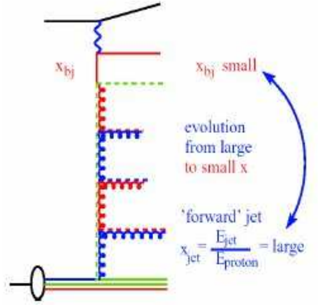

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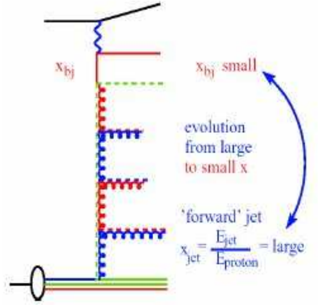

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Experimentally difficult to observe - jet in forward direction

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Vector Meson Production

Another possible way of guaranteeing that the top and bottom of BFKL ladder has $\mathbf{k} \gg \Lambda_{QCD}$ is in vector meson production

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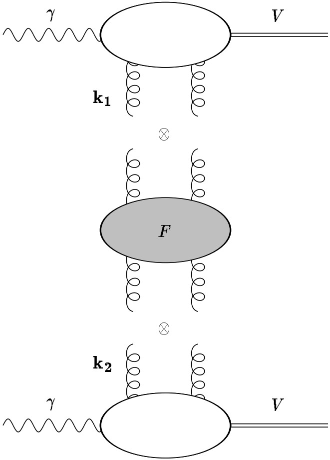

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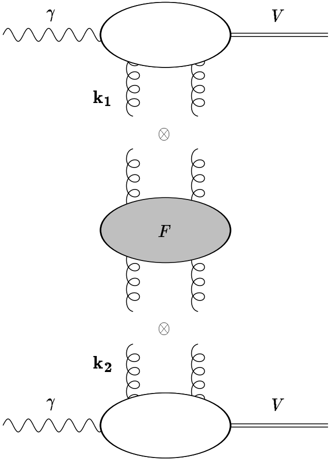

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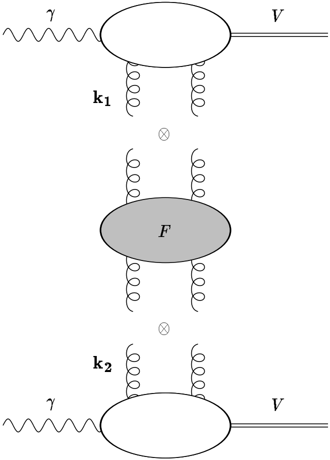

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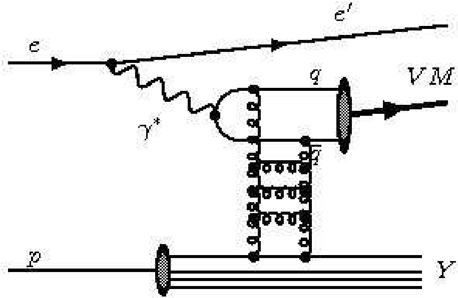
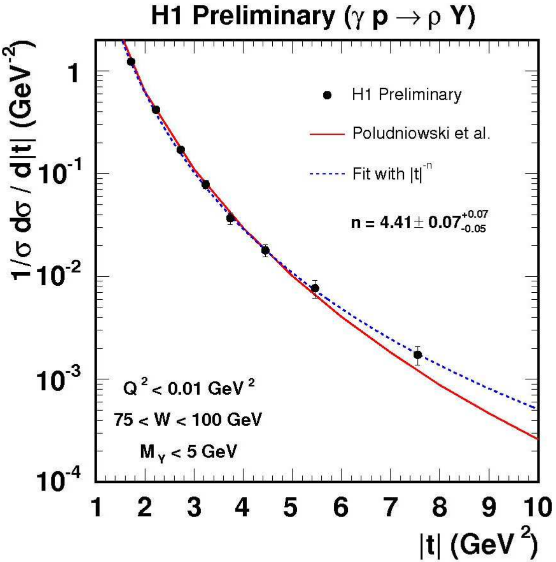
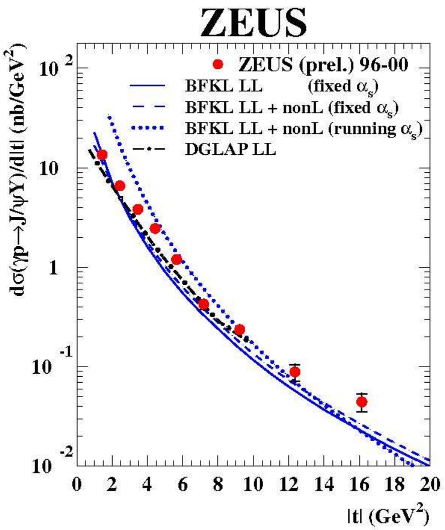
Or:
V is emitted with transverse momentum, $-t >> \Lambda^2_{QCD}$
but $-t \ll s$

More realistically only one vector-meson is produced




H1 Preliminary (γ p → ρ Y)
1/σ dσ / d|t| (GeV^-2)
1
10^-1
10^-2
10^-3
10^-4
1 2 3 4 5 6 7 8 9 10
|t| (GeV^2)
H1 Preliminary
Poludniowski et al.
Fit with |t|^-n
n = 4.41 ± 0.07 +0.07 -0.05
Q^2 < 0.01 GeV^2
75 < W < 100 GeV
M_Y < 5 GeV


ZEUS
ZEUS (prel.) 96-00
BFKL LL (fixed αs)
BFKL LL + nonL (fixed αs)
BFKL LL + nonL (running αs)
DGLAP LL
dσ(γp→J/ψY)/d|t| (nb/GeV2)
|t| (GeV2)
10 2
10
1
10 -1
10 -2
0
2
4
6
8
10
12
14
16
18
20

Diffraction

Processes for which $-t \ll s$

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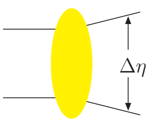
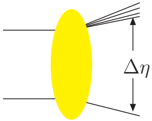
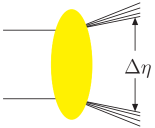


Elastic diffraction



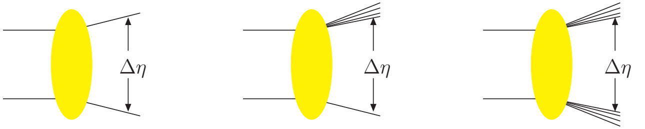
Diffractive Dissociation



Double Diffractive Dissociation

Diffraction

Processes for which $-t \ll s$



Elastic diffraction   Diffractive Dissociation   Double Diffractive Dissociation

In each case the "jets" have large positive or negative rapidity

$$\eta = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right)$$

Large rapidity gap between the two jets

$$\Delta\eta \sim \ln \left(\frac{s}{-t} \right)$$

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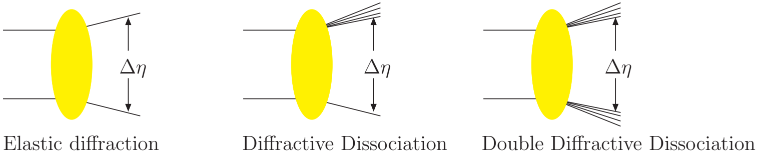

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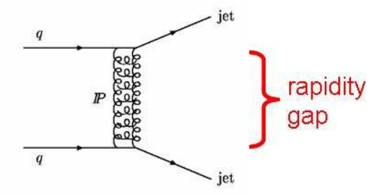
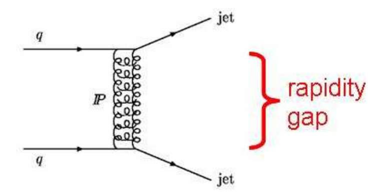
Such rapidity gaps are possible is object exchanged between scattering particles is a colour singlet (a pomeron)


q
q
$I\!P$
jet
jet
rapidity
gap

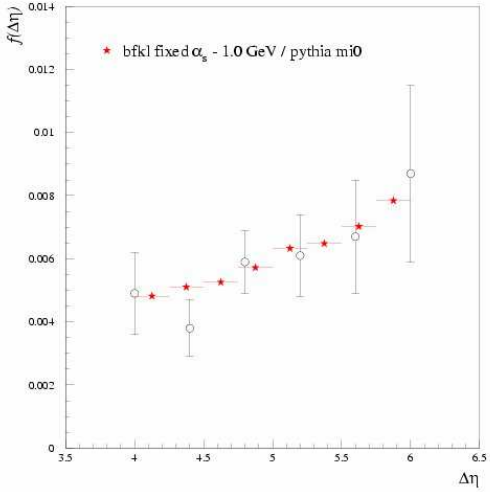


BFKL amplitude grows as

$$e^{4\overline{\alpha_s}\ln(2)\Delta\eta}$$

Expect the number of events with rapidity gap $\Delta\eta$ to grow with $\Delta\eta$

Gap fraction $f(\Delta\eta)$

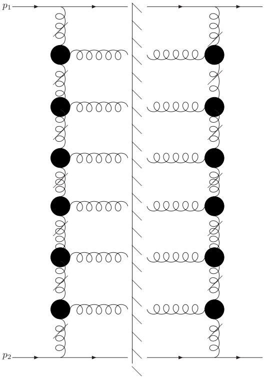


Rapidity Gap Survival

The theoretical gap fraction calculated using BFKL should be larger than that observed

The spectator partons which do NOT partake in the BFKL evolution can nevertheless radiate gluons which can populate the rapidity gap between the primary jets.

Mini-jets



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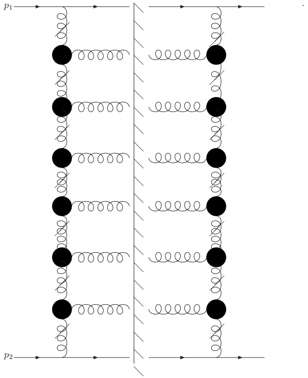

Optical Theorem

Imaginary part of BFKL amplitude is probability to find two primary jets and any number of low-energy “mini-jets”

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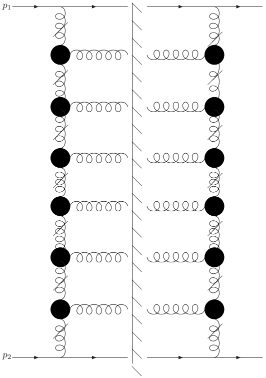

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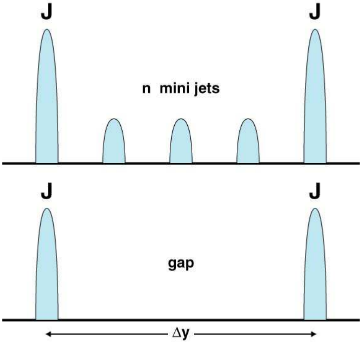

Write the BFKL amplitude as

$$f(s, \mathbf{k_1}, \mathbf{k_2}) = \sum_n P_n \ln^n(s)$$

P_n is the probability that the ladder has n rungs (equal to the probability that there are n mini-jets)

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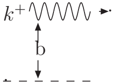
Can get $< n > \sim 3-4$ at Tevatron - MORE AT LHC!!!

Diffraction Quantum Optics



Describe a photon in terms of longitudinal momentum k^+ and impact parameter $\mathbf{b}$, $|k^+,\mathbf{b}\rangle$
("energy" eigenstate in light-cone quantisation)

$$|in\rangle = \int dk^+ d^2\mathbf{b}\, \phi_{in}(k^+,\mathbf{b}) |k^+,\mathbf{b}\rangle$$

$|k^+,\mathbf{b}\rangle$ are eigenstate of diffraction operator T

$$T|k^+,\mathbf{b}\rangle = t(k^+,\mathbf{b})|k^+,\mathbf{b}\rangle$$

$$|out\rangle = \int dk^+ d^2\mathbf{b}\, \phi_{out}(k^+,\mathbf{b}) |k^+,\mathbf{b}\rangle$$

e.g amplitude to scatter into state with transverse momentum **k**

$$\mathcal{A} = \int dk^+ d^2\mathbf{b}\, \phi_{in}(k^+, \mathbf{b})\, t(k^+, \mathbf{b}) \langle k^{+\prime}, \mathbf{k} | k^+, \mathbf{b} \rangle = \int d^2\mathbf{b}\, \phi_{in}(k^+, \mathbf{b})\, t(k^+, \mathbf{b})\, e^{i\mathbf{k}\cdot\mathbf{b}}$$

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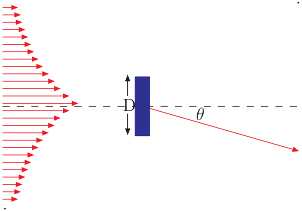
$$\phi_{in} = e^{-\alpha b} \quad (k^+ \gg \alpha)$$

e.g. Diffraction due to opaque slab



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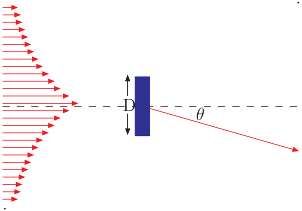

$$\mathcal{A} = \int d^2\mathbf{b} e^{-\alpha b} \theta(b - D/2) e^{ik^+ b \sin\theta}$$

$$= 4 \frac{e^{-\alpha D/2}}{k^+ \sin\theta} \cos(\frac{1}{2} k^+ D \sin\theta)$$

Diffraction pattern

Diffraction in Particle Scattering

In particle physics it is also the case that for diffractive processes an incoming particle (helicity λ) described in terms of k^+, $\mathbf{b}$, λ is an eigenstate of the diffractive scattering operator.

$$T|k^+,\mathbf{b},\lambda\rangle = t(k^+,\mathbf{b}\lambda)|k^+,\mathbf{b},\lambda\rangle$$

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Incoming state with given k^+ may be written

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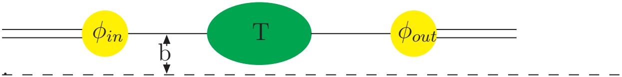
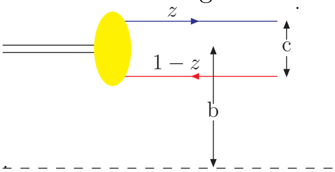
The diffractive scattering process is



Colour Dipole Approach

In QCD the diffraction eigenstates are colour dipoles of transverse size **c** with (average) impact parameter **b**, with the coloured particles carrying fractions z and $1-z$ of the longitudinal momentum k^+.



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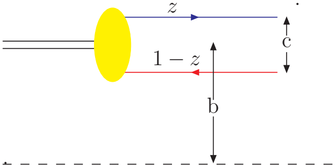

The amplitude for an incoming particle to split into such a dipole depends on z and **c** and likewise the amplitude for the dipole to form a final state also depends on these variables so the diffraction amplitude is given by

$$\int d^2\mathbf{c}\, dz\, \phi_{in}(z,\mathbf{c})\, t(k^+,\mathbf{b},\mathbf{c})\, \phi_{out}(z,\mathbf{c})$$

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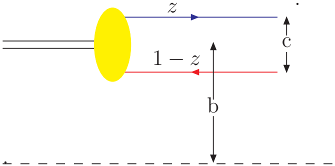

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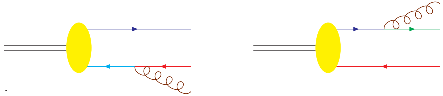
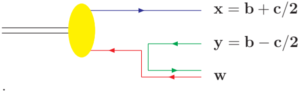
$\phi_{in}(z,\mathbf{c})\phi_{out}(z,\mathbf{c})$ plays then role of the impact factor (Fourier transformed) and $t(k^+,\mathbf{b},\mathbf{c})$ plays the part of the (process independent) BFKL evolution.

Dipole Evolution

As an incoming colour dipole propagates it can emit gluons,



thereby splitting into two colour dipoles.



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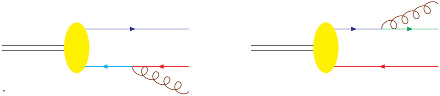

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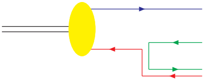


Probability to emit dipole into rapidity interval $d\eta$ and impact parameter interval $d^2\mathbf{w}$ is

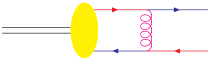
$$dP = \frac{\overline{\alpha_s}}{2\pi} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{w})^2(\mathbf{w}-\mathbf{y})^2} d^2\mathbf{w}\, d\eta$$

This increases the effective incoming dipole flux, thereby increasing the cross-section.

Rapidity dependence of dipole scattering cross-section

$$\frac{\partial\sigma(\mathbf{x},\mathbf{y},\eta)}{\partial\eta} = \frac{\overline{\alpha_s}}{2\pi}\int d^2\mathbf{w}\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{w})^2(\mathbf{w}-\mathbf{y})^2} \times\left(\sigma(\mathbf{x},\mathbf{w},\eta)+\sigma(\mathbf{y},\mathbf{w},\eta)-\sigma(\mathbf{x},\mathbf{y},\eta)\right)$$

The first two terms represent the scattering of either of the two dipoles from the target, the third represents the case in which the



original dipole is destroyed (virtual corrections) .

This is the BFKL equation in impact parameter space.

From the Optical Theorem

$$\sigma(\mathbf{x}, \mathbf{y}, \eta) = \frac{1}{s} \Im m(t(\sqrt{s}, \mathbf{b}, \mathbf{c})$$

$$\mathbf{b} = \frac{\mathbf{x} + \mathbf{y}}{2}, \quad \mathbf{c} = \mathbf{x} - \mathbf{y}, \quad \eta \sim \ln(s)$$

Rapidity dependence of $\boldsymbol{\sigma}$ gives rapidity dependence of diffractive scattering eigenvalue.

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In the standard BFKL approach, impact factor is probability for dipole production/decay - the rapidity dependence is encoded in the gluon ladder.

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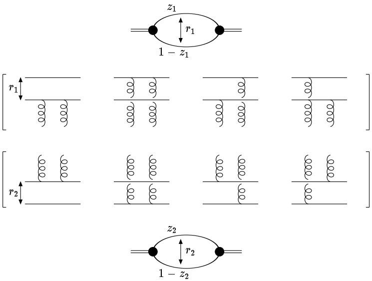

In dipole approach the gluon ladder is replaced by 2 gluons (LO colour singlet exchange) - rapidity dependence is absorbed into the impact factor leading to an incoming dipole density that grows with rapidity.

Saturation

Dipole density grows as

$$e^{4\overline{\alpha_s}\ln(2)\eta}$$

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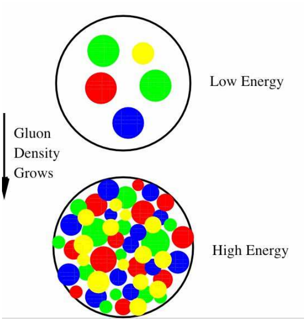
As rapidity grows dipole density increases so that dilute dipole approximation (single dipole scattered by target) breaks down.



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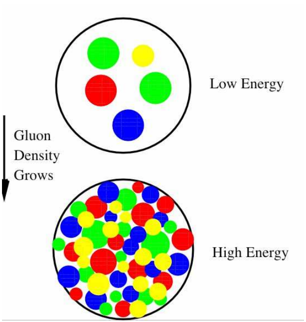

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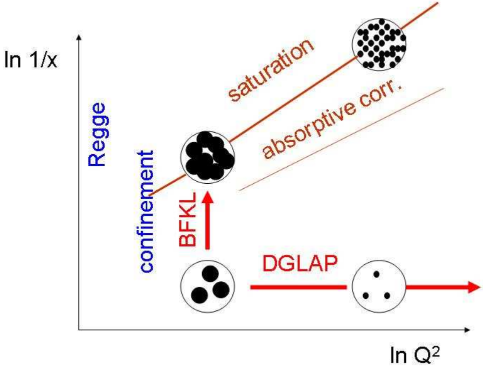
This is the basis of the

Colour Glass Condensate

Effective theory that describes QCD in limit of high gluon density.
First applied to heavy ion collision (RHIC) but can also be applied to DIS at sufficiently low x or diffractive scattering at sufficiently large rapidities.


ln 1/x
saturation
absorptive corr.
Regge
confinement
BFKL
DGLAP
ln Q2

Balitsky-Kovchegov Equation

In terms of the dipole evolution multiple scatterings are introduced by adding a term that account for both dipoles to scatter form the target simultaneously

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Non-linear equation (difficult to solve)

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Non-linear equation (difficult to solve)
Analogous to

- Langevin equation (stochastic processes)
- Fisher, Kolmogov, Petrovski, Pisconov equation

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Non-linear term moderates the growth so that σ saturates.

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Expect a solution of the form

$$\sigma(\mathbf{x},\mathbf{y},\eta)\sim\left[1-\exp\left(-(\mathbf{x}-\mathbf{y})^2 e^{4\overline{\alpha_s}\ln(2)\eta}\right)\right]$$

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Unitarity is respected in terms of dipole density.
BUT to get a physical cross-section need F.T. in $\mathbf{b}=(\mathbf{x}+\mathbf{y})/2$.
Region itself in **b**-space over which $\boldsymbol{\sigma}$ is non-zero (after saturating) grows exponentially with rapidity groon - thereby violating the Froissart-Martin bound)

$$\frac{\partial\sigma(\mathbf{x},\mathbf{y},\eta)}{\partial\eta} = \frac{\overline{\alpha_s}}{2\pi}\int d^2\mathbf{w}\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{w})^2(\mathbf{w}-\mathbf{y})^2}$$
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Non-linear term moderates the growth so that σ saturates.
Expect a solution of the form

$$\sigma(\mathbf{x},\mathbf{y},\eta) \sim \left[1-\exp\left(-(\mathbf{x}-\mathbf{y})^2 e^{4\overline{\alpha_s}\ln(2)\eta}\right)\right]$$

Unitarity is respected in terms of dipole density.
BUT to get a physical cross-section need F.T. in $\mathbf{b}=(\mathbf{x}+\mathbf{y})/2$.
Region itself in $\mathbf{b}$-space over which σ is non-zero (after saturating) grows exponentially with rapidity groon - thereby violating the Froissart-Martin bound)
The unitarization of the BFKL amplitude even with BK modification is still not fully resolved.

Cuts not Poles

The Mellin transform of the BFKL amplitude involves an integral over ν.

$$f(\omega, \mathbf{k_1}, \mathbf{k_2}) \sim \int d\nu \left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}} \right)^{i\nu} \frac{1}{\omega - \overline{\alpha_s}\chi(\nu)}$$

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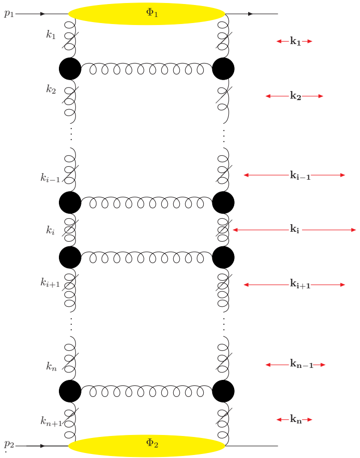
Regge theory predicts a pole !

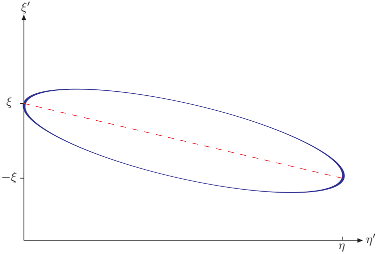
Diffusion

As we go down the BFKL ladder away from the impact factor the spread of **k** for which the integral has support, increases (until we start getting near the bottom of the ladder)



Bartels' Cigar

The BFKL equation is a diffusion equation in $\eta \sim \ln(s)$ and $\xi = \ln(\mathbf{k}^2/\Lambda^2_{QCD})$.



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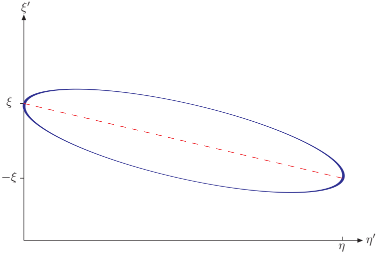

Eigenvalue equation

$$\mathcal{K}_0 e^{i\nu\xi} = \overline{\alpha_s}\chi(\nu) e^{i\nu\xi} = \overline{\alpha_s}\chi\left(-i\frac{\partial}{\partial\xi}\right) e^{i\nu\xi}$$

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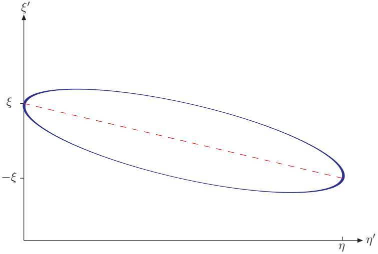

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BFKL equation (as a diffusion equation)

$$\left[\frac{\partial}{\partial\eta} - \overline{\alpha_s}\chi\left(-i\frac{\partial}{\partial\xi}\right)\right] f(\eta,\xi) = \delta(\xi - \xi_0)$$

What value should be used for $\overline{\alpha_s}$?

$$\overline{\alpha_s}(\xi) \sim \frac{C_A \beta_0}{\pi \xi}$$

Eigenfunction with eigenvalue ω

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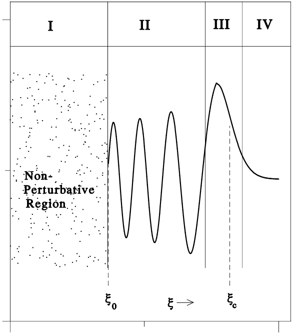
[For $\xi > \xi_c$, ν is imaginary]


I
II
III
IV
Non-
Perturbative
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ξ_0
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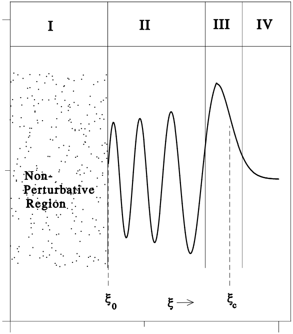
- Region II: $\xi \ll \xi_c$ Oscillating solution



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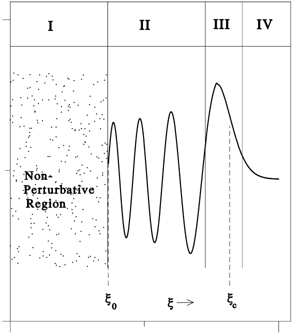

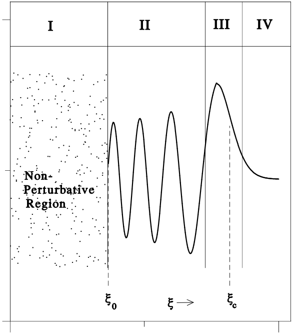

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Expanding χ to order ν^2

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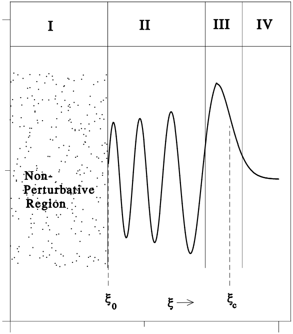
Airy functions chosen to match functions and their derivatives in regions II and IV (fixes phase at II-III boundary).



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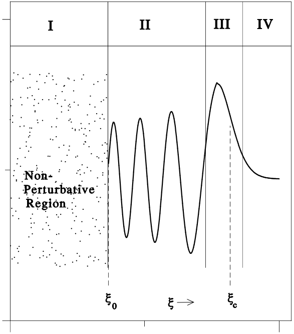
- Region 1: $\xi < \xi_0$, too small for perturbation theory to be reliable.



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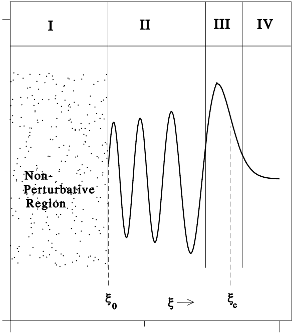

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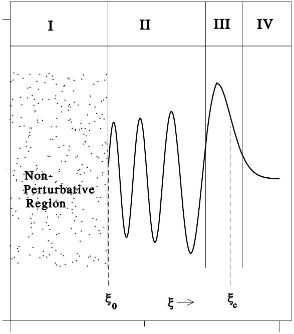

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This gives separate poles, rather than a cut for the BFKL amplitude - consistent with the predictions of Regge theory.

Hard Pomeron:

Solution to BFKL equation with leading energy dependence

$$s^{\omega}$$

$$\omega = 4\ln(2)\overline{\alpha_s} + O(\overline{\alpha_s}^2)$$

Rapid rise with increasing s.

Controversial evidence for BFKL behaviour in low-x DIS, diffractive processes etc.

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Regge trajectory $\alpha_P(t)$ with vacuum quantum numbers.
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The “two” pomerons seem to be mutually incompatible.

[problem has existed for 30 years !!]

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To soften the s^{ω} behaviour of the perturbative (hard) pomeron, these infrared effects must somehow exactly cancel the perturbative contributions (at least below the saturation scale)

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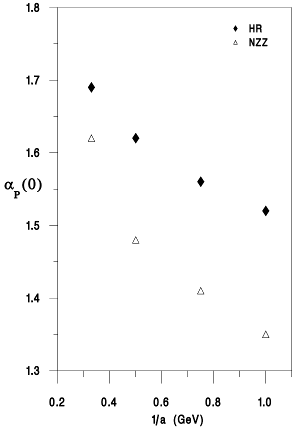
Studies of Dyson-Schwinger equation show that this does NOT but gluon acquires an effective mass at the IR scale a

$$D(\mathbf{k}^2) \sim \frac{a^2}{1 + a^2\mathbf{k}^2}$$



HR: insert non-perturbative propagators into BFKL equation and solve numerically.
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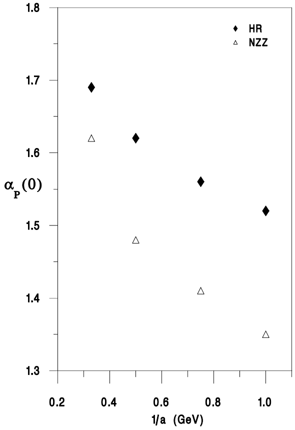

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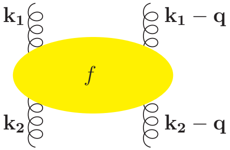
Find a modest decrease in pomeron intercept $\alpha_P(0)$, but not sufficient.

Heterotic Pomeron



$$f(s,\mathbf{k_1},\mathbf{k_2},\mathbf{q}) = \int d^2\mathbf{b}\, e^{i\mathbf{q}\cdot\mathbf{b}} \tilde{f}(s,\mathbf{k_1},\mathbf{k_2},\mathbf{b}).$$

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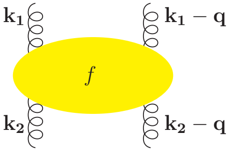

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Diffusion in s, $\mathbf{k_1}$ and $\mathbf{b}$.

$$\frac{\partial \tilde{f}}{\partial s} = \int \frac{ds'}{s'} d^2\mathbf{k}' \, d^2\mathbf{b} \, \mathcal{K}\left(\frac{s}{s'}, \mathbf{k}', \mathbf{k_2}, (\mathbf{b} - \mathbf{b}')\right) \tilde{f}(s', \mathbf{k}', \mathbf{k_2}, \mathbf{b}')$$

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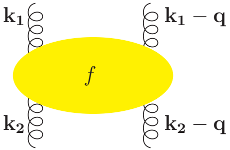

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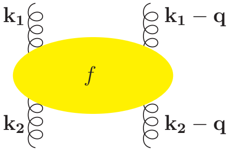

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For $\mathbf{k_1},\mathbf{k_2} \leq \Lambda_{QCD}$ - soft pomeron

$$\mathcal{K}\left(\frac{s}{s'},\mathbf{k}',\mathbf{k_2},(\mathbf{b}-\mathbf{b}')\right) = \delta\left(\mathbf{k}'-\mathbf{k_1}\right)\left(\frac{s}{s'}\right)^{\alpha_0-1} B(\mathbf{b}-\mathbf{b}')$$

e.g. $B(\mathbf{b}-\mathbf{b}')$ is a random walk diffusion

$$B \sim \exp\left\{-\frac{(\mathbf{b}-\mathbf{b}')^2}{4\alpha'\ln(s)}\right\}$$

This is the Fourier transform of

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$\alpha_0 = 1.08,\ \alpha' = .25$ matches soft pomeron e.g. $B(\mathbf{b}-\mathbf{b}')$ is a random walk diffusion

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The full $\mathcal{K}\left(\frac{s}{s'}, \mathbf{k'}, \mathbf{k_2}, (\mathbf{b}-\mathbf{b}')\right)$ interpolates between hard and soft extremes.

The Pomeron and Gauge-String Duality

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For positive t, there are trajectories for which integer values of the power of s correspond to glue-ball masses.

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Even if low r contributions are heavily attenuated for total hadronic cross-section and low-t processes, the hard pomeron should eventually dominate at large enough s - size of R must decrease with energy.

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- Over the years many attempts have been made to reconcile hard and soft pomeron behaviour (most recently based on gauge-string duality)