QUANTUM FIELD THEORY 1

Problem sheet 5

1. Verify that the representation for the Dirac γ-matrices

$$\gamma^0 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

   is a representation of the Clifford algebra.

2. Show that in the above representation, the spinors

$$u(p,\lambda) = \sqrt{E+m} \begin{pmatrix} \chi^\lambda \\ \frac{\sigma\cdot\mathbf{p}}{(E+m)}\chi^\lambda \end{pmatrix}, \quad v(p,\lambda) = \sqrt{|E|+m} \begin{pmatrix} \frac{\sigma\cdot\mathbf{p}}{(|E|+m)}\chi^\lambda \\ \chi^\lambda \end{pmatrix},$$

   are solutions to the equations

$$(\gamma\cdot p - m)\,u = 0,$$
$$(\gamma\cdot p + m)\,v = 0,$$

   respectively. (χ^λ is a two-component spinor for a a spin-$\frac{1}{2}$ particle with helicity λ.)

3. Use the above form of the spinors to show that

$$\sum_\lambda u_\alpha(p,\lambda)\bar{u}_\beta(p,\lambda) = (\gamma\cdot p + m)_{\alpha\beta}$$

$$\sum_\lambda v_\alpha(p,\lambda)\bar{v}_\beta(p,\lambda) = (\gamma\cdot p - m)_{\alpha\beta}$$