QUANTUM FIELD THEORY 1

Christmas Problem

The solution to this problem should be written up in the format of a research paper as though it were a piece of original research.

The solution should be handed to me by Friday January 9, 2009.

Calculate the differential cross section with respect to t, $\frac{d\sigma}{dt}$ for the Compton scattering of a photon and an electron. t is the square momentum-transfer between the initial-state and final-state photons.

Carry out the calculation of the square matrix-element in Feynman gauge.

Now calculate the differential cross section with respect to scattering angle, $\frac{d\sigma}{d\cos\theta}$ in the rest frame of the incident electron, by *two* different methods:

1. By returning to the phase-space integral and carrying this out in the rest frame of the incident electron rather than the centre-of-mass frame.
2. By direct conversion of dt into $d\cos\theta$. (Take care to account for the fact that in this case the energy of the final-state photon and the scattering angle are *not* independent variables).

Verify that both methods yield the same results.

For the case of $\frac{d\sigma}{dt}$, it is more convenient to express your answers in terms of s and u, rather than s and t.

For the case of the differential cross-section with respect to scattering angle in the rest frame of the incident electron, express your answer in terms of the scattering angle, θ of the photon and the ratio ω of the final-state photon energy to the incident photon energy. This is the Klein-Nishina formula.

For three different values of the centre-of-mass energy, $\sqrt{s}$, $(s-m_e^2) << m_e^2$ (note that we must have $\sqrt{s} > m_e$), $(s-m_e^2) \sim m_e^2$, and $s \gg m_e^2$, plot the differential cross-section $\frac{d\sigma}{dt}$ against t and the differential cross-section $\frac{d\sigma}{d\cos\theta}$ against $\cos\theta$. Comment on the results.

The traces involved in this calculation can be performed by hand provided the sums over the polarisations of the incoming and outgoing photon are performed *before* the trace is carried out.

Alternatively, you may wish to use this opportunity to familiarise yourself with one of the several available algebraic manipulation computer packages, which can carry out such traces over Dirac matrices. The most convenient of these is FORM, written by J. Vermaseren. After receiving all the solutions, I will circulate a sample FORM file which performs this calculation.