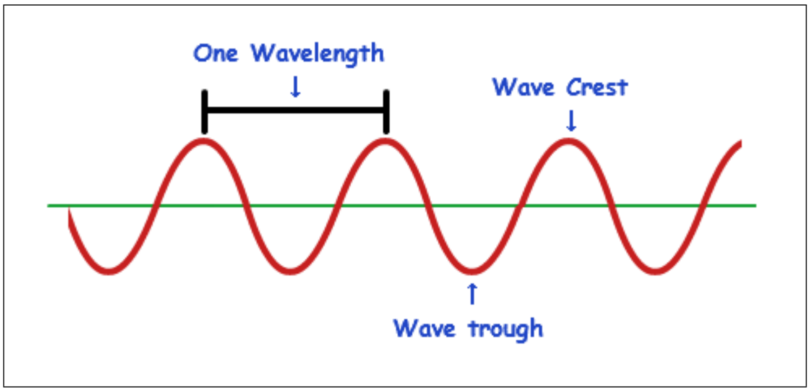

Figure 1: A pure sine wave. The wavelength is the distance between two adjacent peaks (or troughs). The number of peaks (or troughs) passing a given point per second is called the “frequency”. The wave velocity is the wavelength multiplied by the frequency.

2 Some Classical Physics

Before we can proceed to discuss Quantum Physics we need to understand a few things about classical physics. This should all be conceptually straightforward, since not only can we visualize the phenomena described, but we can set up experiments using macroscopic objects and observe them directly.

2.1 Waves

A wave is a quantity which oscillates between a maximum and minimum and travels in space. The number of times it reaches a maximum per second (at a fixed point) is called the **“frequency”** and the instantaneous distance between two maxima is called the **“wavelength”**. The maximum value of the oscillating quantity is called the **“amplitude”**. This is demonstrated in Fig. 1.

The quantity which travels in this way is known as a **“disturbance”**. Precisely what this disturbance is depends on the wave. For ocean waves it is the surface of the water. For sound waves it is the fluctuation in pressure, for light (and all other **electromagnetic radiation** such as microwaves, **X-rays** etc.) it is an electric field and a magnetic field which are perpendicular to each other and also perpendicular to the direction of the wave. The simplest waves can be generated by taking a long length of string under tension (such as a washing line) and shaking one end, so that it oscillates with a constant frequency.

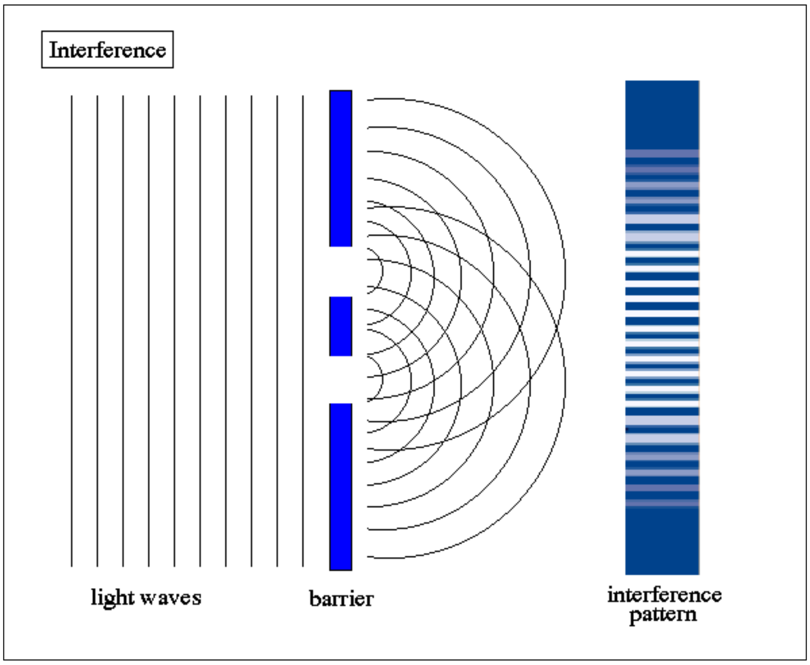

Figure 2: **Interference** of light emerging through two narrow slits. The light emerges from each slit as though the slits were the sources of the light. The light from the two slits travel a slightly different distance to a given point on the screen. If this distance is a whole number of wavelengths, the peaks and troughs coincide and we get a maximum, whereas half-way between these maxima the peaks from one slit coincide with the troughs for the other so that the wave motion from the two slits cancels out and we get a minimum. This is know as the **"Young's slits"** experiment after Thomas Young [4] who first performed the experiment in 1801.

2.1.1 Interference and Diffraction

Suppose a wave-front is incident on two slits which are close together. Waves are emitted from each of the two slits, as shown in Fig. 2.

If the resultant wave is observed on a screen at a point half-way between the two slits then the wave from each slit will have travelled the same distance and they will therefore be "in phase" i.e. they will have maxima together and minima together. This means that the total disturbance will be the sum of the two disturbances and we have what is called an "interference maximum". On the other hand, if the resultant wave is observed slightly away from the mid-point between the slits then the wave from one of the slits travels further than the wave from the other, and the waves from the two slits will no longer be in phase so that the resultant disturbance will be a wave whose amplitude is smaller than the sum of the two amplitudes. In the case where the extra distance travelled by one of the waves is half a wavelength the waves will cancel out exactly, leading to a minimum. This is demonstrated in Fig. 3.

Even further away from the mid-point the path difference between the waves from the two slits is again a whole number of wavelengths and the disturbances are again in phase – and so on. This is shown in Fig. 4. The result is an **interference pattern** consisting of series of light and dark fringes as shown on the right of Fig. 4.

We can do this experiment by shining a laser pen-torch on the surface of a DVD. The DVD consists of a large number of closely ruled grooves and the laser pen-torch produces light of a single wavelength. If the reflected light is observed on a screen whose plane is perpendicular to the DVD, then we see series of spots corresponding to the maxima in which light reflected from adjacent grooves on the DVD are exactly an **integer number** (i.e. whole number) of wavelengths apart so that the waves from all the grooves are in-phase and add up. The experimental setup is shown in Fig. 5. The rainbow which one can see when you look at a DVD and move it around arises because these maxima occur at slightly different angles for different wavelengths so that white light is dispersed, generating the rainbow.

Something similar happens if we illuminate a small object (such as a pin) with **monochromatic** (i.e. single wavelength) light. A set of closely packed fringes can be seen either side of the object, owing to interference between light waves from the two sides of the object. This is actually called **"diffraction"** (but I have never seen an adequate definition of "interference" and "diffraction" which distinguishes between the two).

An example is shown in Fig. 6, which shows the diffraction pattern of a pin-head illuminated with monochromatic (red) light. There is interference between the wave that passes around the object in one direction with the wave that passes around the object in the other direction, leading to fringes whose maxima and minima correspond to directions in which these two waves are in phase or out of phase. The path difference between these two waves is proportional to the size of the object, so that the separation of the fringes is proportional to the ratio of the wavelength of the light to the object size. If the object is much larger than the wavelength of the light used, then we still get a sharp image of the object, together with closely packed diffraction fringes. On the other hand, if we look at even smaller objects which are of the order of magnitude of the wavelength of the light, the fringes become more and more widely separated and eventually we just see the fringes but no sharp image of the object.

2.1.2 Wavepackets

A pure wave with a single wavelength/frequency oscillates for ever - the wave is infinitely long. In practice a wave only exists for a finite time interval. In the case of the sound from an object landing on the floor this could be about $1/20^{th}$ of a second.

The wave would look something like the wave shown in Fig. 7, in which we have a wave of some frequency, f_0, with an amplitude which grows and then shrinks again. The total length of the **wavepacket** is the distance over which the amplitude grows plus the distance over which it shrinks. Thus, whereas a classical point-particle is totally localized, i.e. it

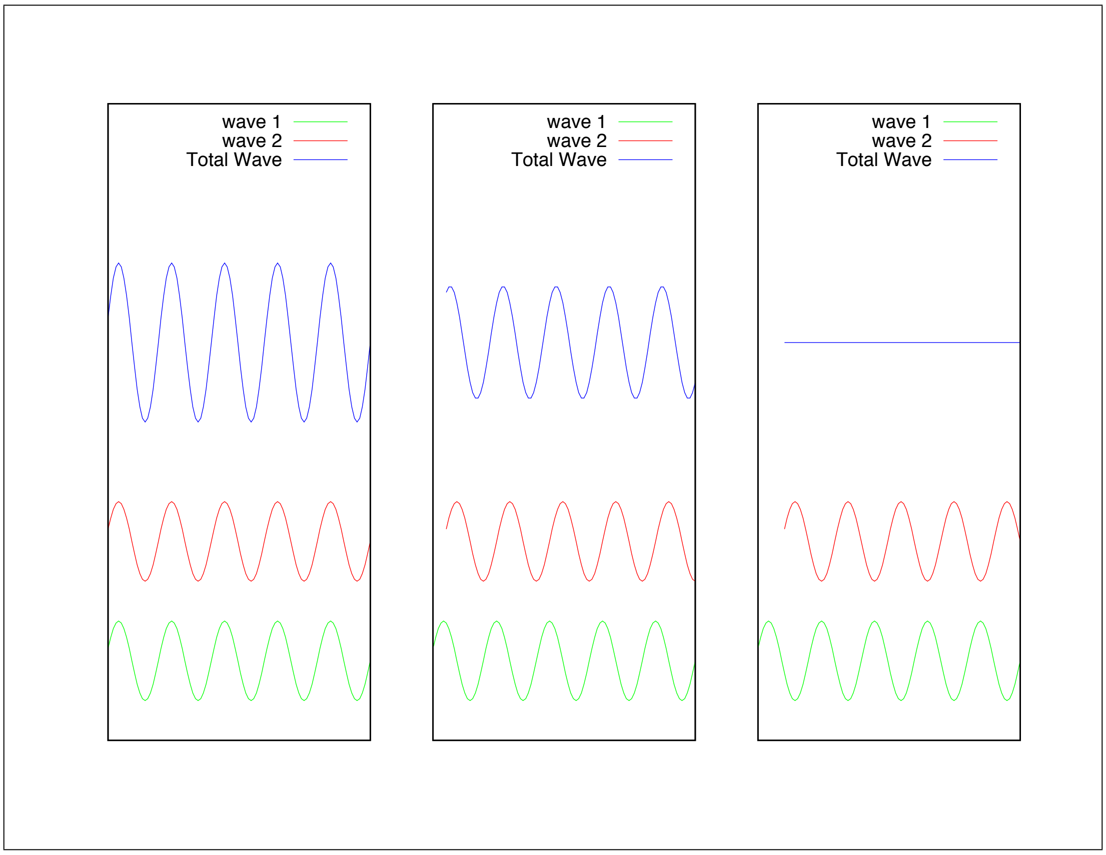

Figure 3: The left panel shows the lower two waves travelling the same distance and therefore being in phase so that the resultant disturbance is a wave with double the amplitude – an interference maximum. The centre panel, shows the lower wave travelling a extra distance of one quarter of a wavelength relative to the upper wave and the resultant disturbance is a wave with a reduced amplitude. In the right panel the lower wave travels an extra distance of one half of a wavelength so that the two disturbances are exactly out of phase - one has a maximum at the point where the other has a minimum and vice versa so that the resultant disturbance is zero and we have an interference minimum.

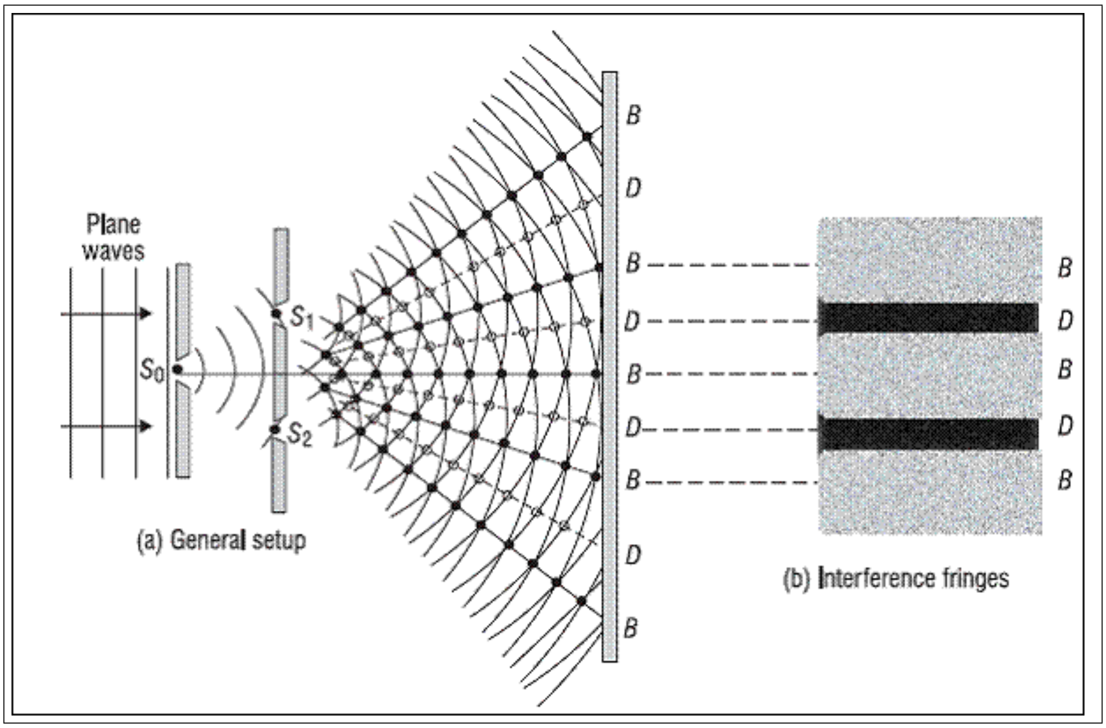

Figure 4: The double slit experiment (a). The interference from the light from the two slits leads to a set of fringes on the screen (b)

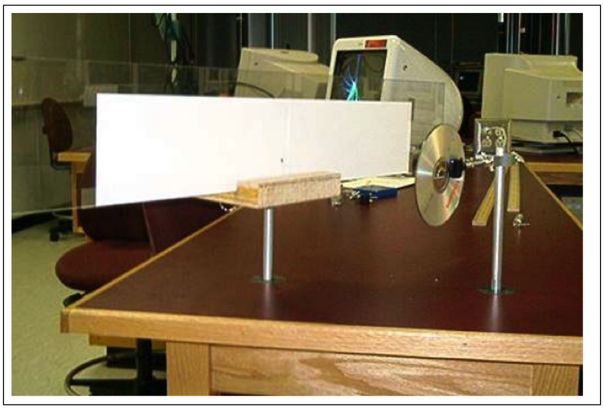

Figure 5: Using a DVD as a diffraction grating with a laser pen-torch held on a stand, a CD or DVD and a vertical screen, spots corresponding to interference maxima appear on the screen.

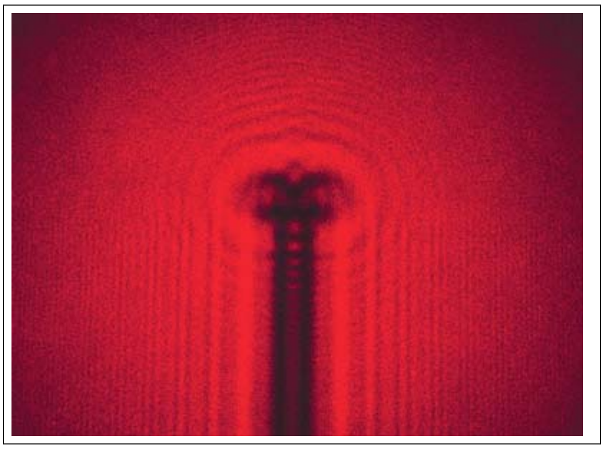

Figure 6: The diffraction pattern obtained from illuminating a pin-head with monochromatic (red) light. The diffraction pattern arises from interference between light waves which go around the pin in one direction and those that go around the pin in the other direction. For light emerging at different angles these two waves will have different phase differences because they travel different distances. Maxima are found when the difference in distance travelled by the two waves is a whole number of wavelengths so that the waves are in phase and add up. Because the width of the pin is very much larger than the wavelength of the light used to illuminate it, the fringes are closely packed and there is a reasonably sharp image of the pin at the centre.

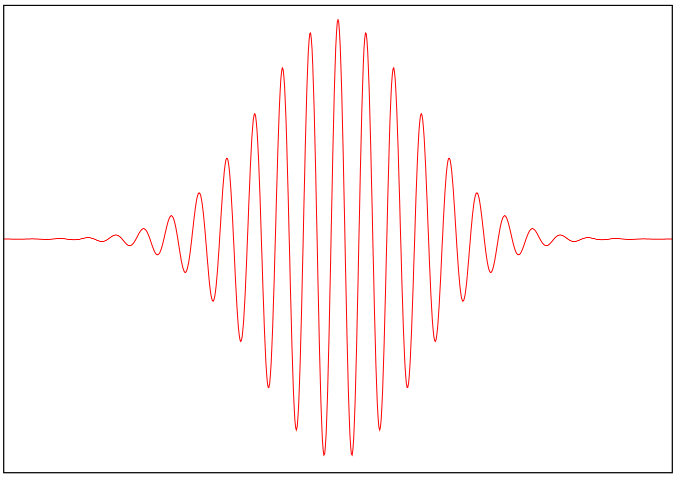

Figure 7: An example of a wavepacket representing a wave that lasts for a finite time. The amplitude begins very small and then grows to a maximum and then shrinks again.

has a well-defined exact position in space, a wavepacket is spread over a finite distance and therefore cannot be said to have a definite (i.e. localized) position. The wavepacket exists *everywhere* although the amplitude has a maximum at a specific position (at a given time). The amplitude decreases as one moves away from that specific position and is negligibly small far away from it. It is a mathematical fact that such a wavepacket is actually the sum of many pure frequency waves as shown in Fig. 8 with different amplitudes and frequencies slightly different from f_0. The largest amplitude is the wave with frequency, f_0 but there are other frequencies ranging from a little below, i.e. $f_0 - \frac{1}{2}\Delta f$, to a little above, i.e. $f_0 + \frac{1}{2}\Delta f$. The range of frequencies required to construct the wavepacket is Δf. Waves with different frequencies and amplitudes for which, when the disturbances of the individual waves added together, generate a total disturbance given by the wavepacket of Fig. 7.

In Fig. 9 we show a bar-chart of the amplitudes of the pure waves against frequency (it is easy to write a simple computer program which verifies that this works.) So, contrary to common belief, any wave which lasts a finite time, Δt, is never a pure single-frequency (monochromatic) wave with frequency f_0, but a **superposition** of waves with the largest amplitude for frequency f_0, but with waves with frequencies below and above f_0, albeit with smaller amplitudes. This is extremely important. It means that if we hear a short sound such as an object knocking against another object or a balloon popping, or see a very short flash of light, it is meaningless to ask the question "What was the frequency of that noise, or that flash", because a short wavepacket does not possess a well-defined (exact) frequency - it is a superposition of many different frequencies.

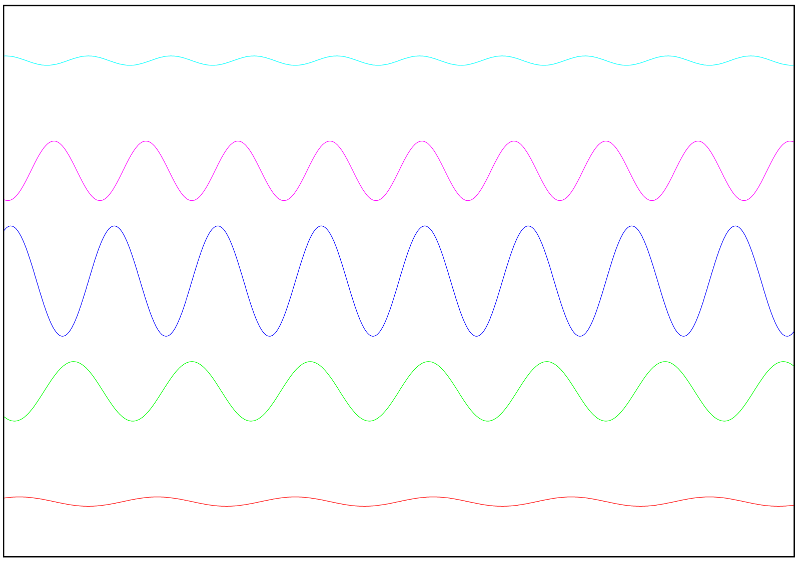

Figure 8: Waves with different frequencies and amplitudes for which, when the disturbances of the individual waves added together, generate a total disturbance given by the wavepacket of Fig. 7.

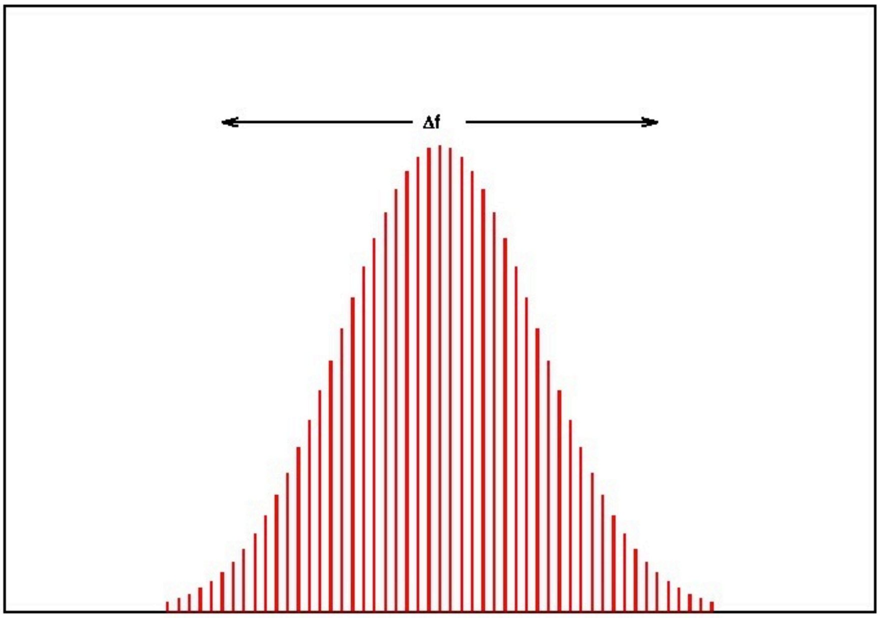

Figure 9: A bar-chart showing the frequencies and amplitudes which constitute the wavepacket of Fig. 7

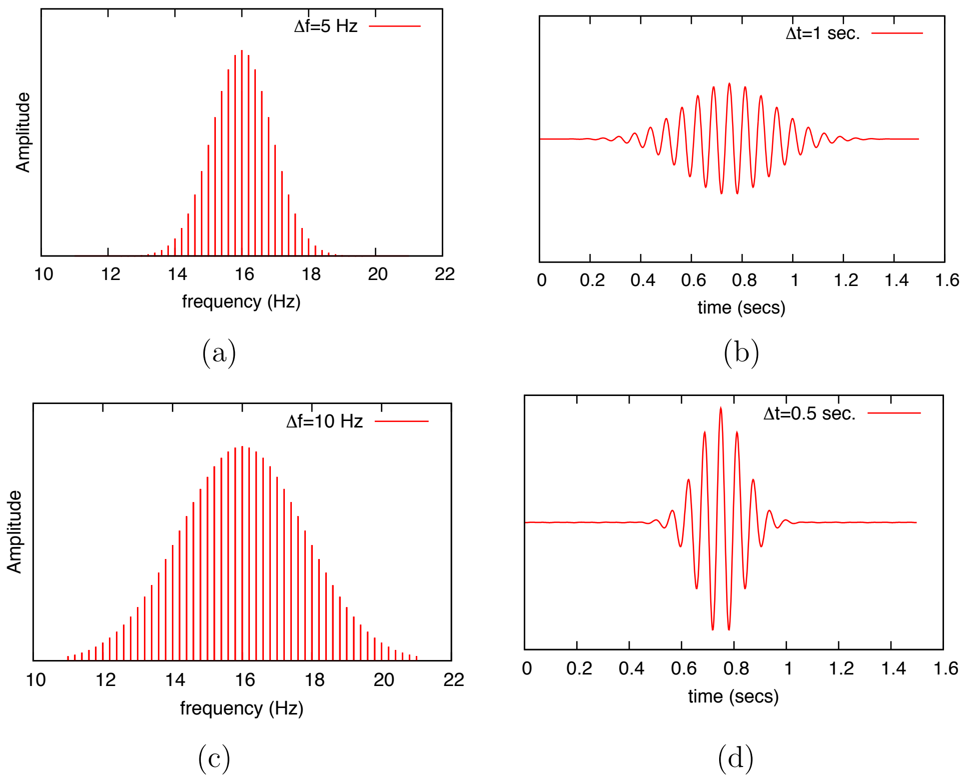

Figure 10: (a): the spectrum of frequencies with range of 5 Hz; (b): The resulting wavepacket with time range of 1 sec. (c): the spectrum of frequencies with range of 10 Hz (d): The resulting wavepacket with time range of 0.5 sec. Note that we have chosen to plot the disturbance at a given point as a function of time. We could have chosen to plot the spatial distribution of the disturbance at a given instant (same time). The shape of the graphs on the right would have been exactly the same. The graphs on the left would have been replaced by a distribution in wavelengths rather than frequencies. The lower graphs would have a wider range of wavelengths than the upper graphs.

In Fig. 10, we show two more such bar-charts and the wavepackets that are produced when we add the waves with the amplitudes and frequencies shown in the bar-charts. In this case, we show the relevant numbers. In the upper figure the frequency ranges from about 13.5 to 18.5 **Hertz** (abbreviated to 'Hz', meaning cycles per second) and the resulting wavepacket has a width (time interval) of around 1 second. In the lower figure, the range of frequencies is significantly larger - from about 10 to 20 Hz, but the wavepacket is narrower, with a width of around 0.5 seconds. In both cases, if we multiply the range of frequencies by the width of the wavepacket we get approximately the same result (around 5). This is a result which can be derived from the mathematical properties of wavepackets and tells us that the width of the wavepacket is inversely proportional to the range of frequencies which are used to construct that particular wavepacket. This result is the basis of the Heisenberg **uncertainty principle**. We will see later that the width of the wavepacket is proportional to the uncertainty in the position of a particle and the range of frequencies is proportional to the uncertainty in its **momentum**.

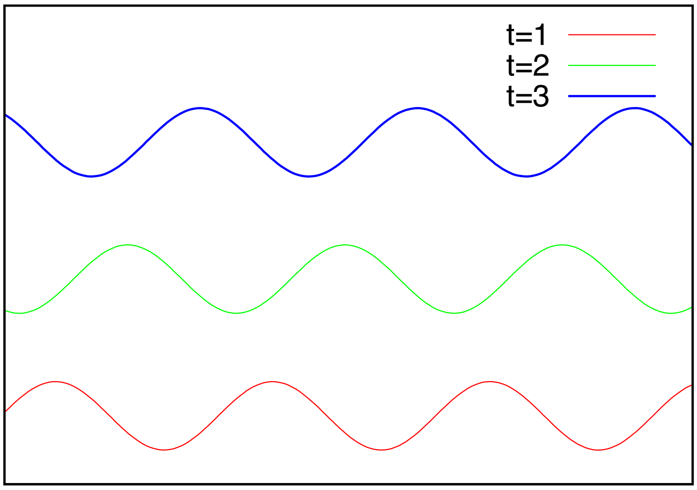

Figure 11: A travelling wave at three different times showing how the peaks and troughs propagate through space as time advances.

2.1.3 Travelling and Standing Waves

A travelling wave, such as the wave on the surface of the sea, advances as time evolves. For a single wavelength wave, if we take a snapshot of the disturbance we see a disturbance which oscillates in space with peaks and troughs. At a later time a particular peak has moved on (either to the right or to the left depending on the direction of the wave). This is demonstrated in Fig. 11 where we show for three such snapshots at different times and note that a particular peak (or trough) advances with time.

The other type of wave is called a **"standing wave"**. This occurs if there is some restriction on the disturbance at the ends. An example is a violin string for which the disturbance of the string is zero at the bridge of the violin and also zero at the point on the neck where a finger presses on the string so that there is no displacement. In this case the wave does not propagate, but remains with the same shape with an amplitude that oscillates with time as shown in Fig. 12. For such waves only certain wavelengths are allowed. The reason for this is that the disturbance vanishes at two points. The allowed wavelengths are those for which the disturbance can be zero at both these points - i.e. the distance between these two points must be a whole number of half-wavelengths (see Fig. 13). This in turn means that the longest wavelength, λ_1, that a standing wave can have is $2L$, where L is the distance between the two points at which the disturbance is constrained to be zero. Such a wave only has zeros at the ends ($x = 0$ and $x = L$).

In the case of a violin string, the fact that this is a specific wavelength means that the

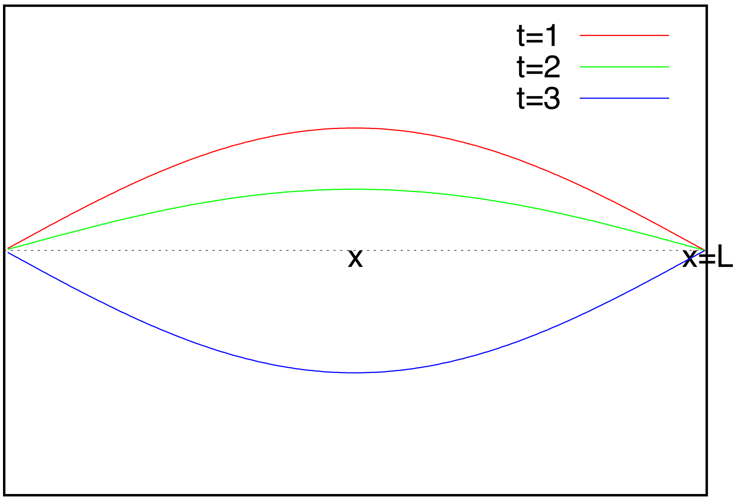

Figure 12: The time development of a standing wave. This wave does not propagate through space but simply oscillates, maintaining the constraint that the disturbance is zero at the end points, $x = 0$ and $x = L$.

string vibrates with a specific frequency producing a sound with a given pitch. The next possible wavelength, λ_2, is L, which has zeros at the ends and also a zero in the middle. After that we have $\lambda_3 = \frac{2}{3}L$ which has zeros at the ends and two points between the ends where the disturbance is also zero. These three cases are demonstrated in Fig. 13. In general, a standing wave in which the disturbance at $x = 0$ and $x = L$ is zero can have a wavelength λ_n, given by

$$\lambda_n = \frac{2L}{n} \quad \text{where } n \text{ is a positive integer, (i.e a whole number e.g. 1,2,3,4 \ldots\ldots).} \qquad (2.1)$$

2.2 Particles

In classical physics particles behave like tiny billiard balls (so tiny that they may be considered to be points). Their motion is determined by Newton's three laws of motion [5]. A moving particle of mass m moving with velocity $\mathbf{v}$ (assumed to be very small compared with the speed of light – so that relativistic effects may be neglected) has a **kinetic energy** $T = \frac{1}{2}mv^2$ and momentum $\mathbf{p} = m\mathbf{v}$. $\mathbf{p}$ and $\mathbf{v}$ are written in bold letters because they represent **"vectors"** i.e. they have direction as well as magnitude or alternatively they have three **components** - one for each direction in space. It is insufficient to specify the speed, v, of a particle in order to describe its motion. We also need to know the direction in which it is moving. In an elastic collision (no loss of energy into heat or sound) the total kinetic energy

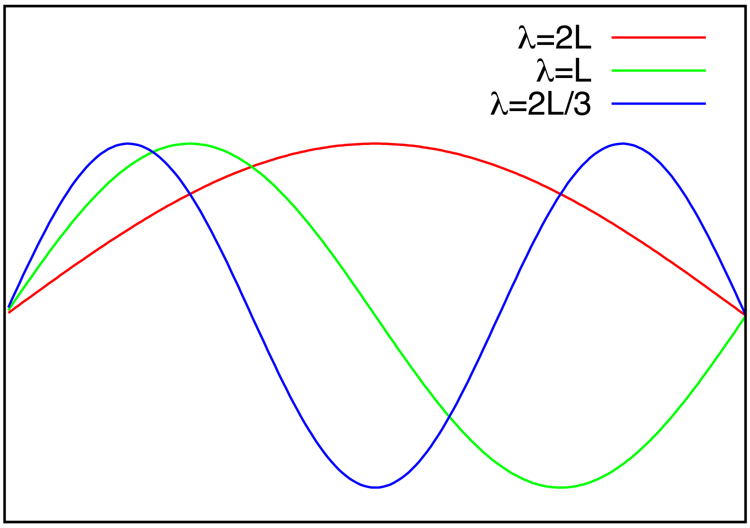

Figure 13: The longest ($\lambda = 2L$), next longest ($\lambda = L$) and third longest ($\lambda = \frac{2}{3}L$) wavelength for standing waves in which the displacement is constrained to be zero at $x = 0$ and at $x = L$.

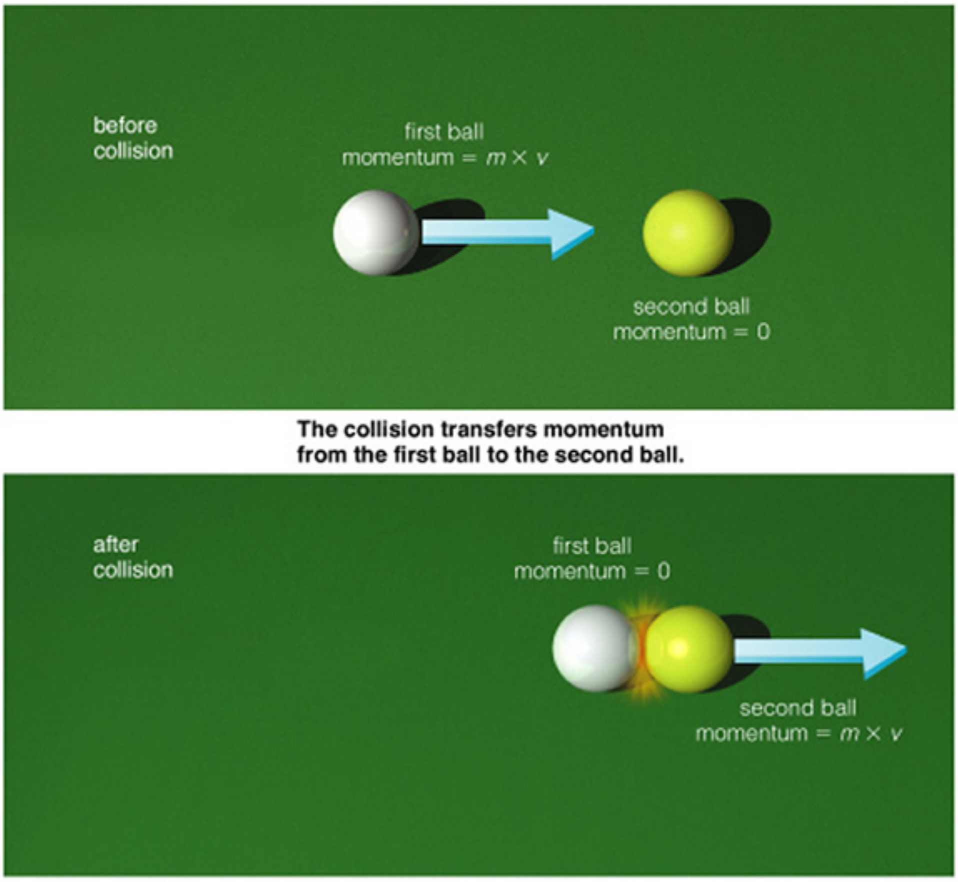

Figure 14: When a ball of mass m collides head-on with a stationary ball with the same mass, conservation of momentum tells us that the target ball then moves with the initial velocity of the incident ball and the incident ball becomes stationary.

and the three components of momentum are conserved. For a given scattering between two particles with given masses and initial velocities these conservation laws are sufficient to derive a unique relation between the final speeds of the two particles and their scattering angles (i.e. the angle between the initial and final directions of motion).

An example is shown in Fig. 14 in which a particle (1) with initial velocity u_1 collides with an initially stationary particle (2). The final speeds v_1 and v_2 and the angles of the particles after the collision are related to each other. As any good billiards player knows, the scattering angle of the incident billiard ball is determined by the offset of the centres. If the projectile billiard ball had a velocity whose direction is through the centre of the target billiard ball, then after the collision the two billiard balls will continue to move in the same direction, but if the billiard balls are not concentric then the balls will scatter at an angle determined exactly by the offset of the two centres. Thus an expert billiards player can

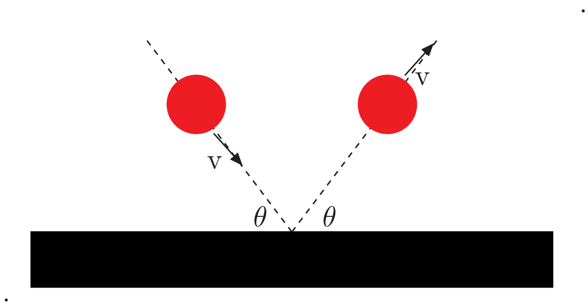

Figure 15: For a ball bounced of a hard surface for which there is no loss of energy The initial speed is the same as the final speed (because the kinetic energy after the bounce is the same as the kinetic energy before) and the angle the trajectory of the ball makes with the surface is also conserved because the component of momentum tangential to the surface is conserved.

shoot the projectile ball in such a way that (s)he can precisely predict the final velocities (speeds and directions) of the two billiard balls. In particular, for the ball bounced off a hard surface (no energy loss), the conservation of momentum and kinetic energy is sufficient to tell us that the angle of reflection is equal to the angle of incidence as shown in Fig. 15. The component of momentum parallel to the hard surface, before the impact is equal to the same component of momentum after the impact which leads to the equality of the incident and reflected angle. We will see that in the sub-microscopic world this is not necessarily the case because what we traditionally think of as particles can also behave like waves and produce diffraction.

Another example of the application of classical physics to particles was Ernest Rutherford's calculation of the path of an **α-particle** of given energy moving initially along a line with a distance b from a nucleus [6]. This is shown in Fig. 16. The electrostatic repulsion between the nucleus and the incident α-particle, causes a deflection in the path of the α-particle by an angle, θ, (called the scattering angle), which depends on the distance, b, of the initial direction of the incident particle from a parallel line passing through the nucleus (called the **"impact parameter"**). The laws of electrostatics (the inverse square law which tells us that the electric field due to a charged particle is inversely proportional to the square of the distance from the charged particle) together with Newton's second law of motion which tells us how the momentum of a particle changes under the influence of a force, can be used together to calculate this scattering angle. In the experiment of Hans Geiger and Ernest Marsden [7], α-particles were projected through a thin gold foil and were scattered through angles in agreement with the calculation of Rutherford – this confirmed that the gold foil was a **lattice** of positively charged nuclei.

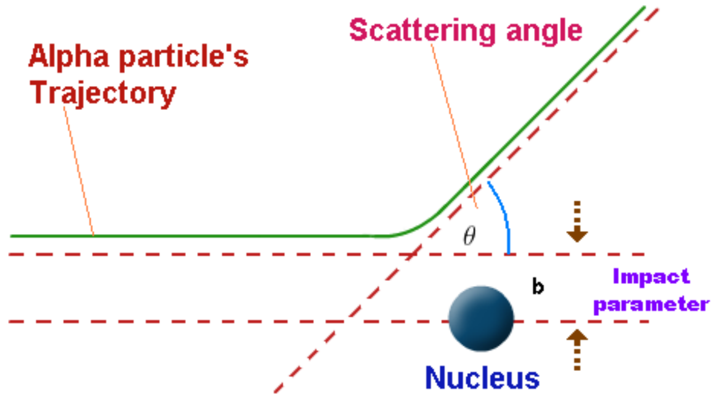

Figure 16: A diagram showing Rutherford's calculation of the scattering angle, θ, for the scattering of an α-particle off a gold nucleus. The scattering angle can be calculated, using Newton's laws of motion, as a function of the momentum of the incident α-particle and the impact parameter, b, which is the perpendicular distance between the initial direction of the α-particle and a parallel line through the target nucleus.

The Smallest Size we Can See

When I was a boy, I had a toy microscope. The maximum magnification was 300×. I discovered that for professional microscopes the maximum magnification was 500× (optical microscopes with magnification up to 1000× are now available). I assumed that this was limited by the technology of manufacturing lenses and that advances in engineering would increase this indefinitely. However, this is not the case.

We have seen in the section on interference of waves that when a sufficiently small object is viewed with monochromatic light, we observe diffraction fringes at the edges of the image. The separation of the interference fringes is proportional to the ratio of the wavelength of the illuminating light to size of the object viewed. If the size of the object viewed is much larger than the wavelength of the light, there is still a sharp image of the object itself, albeit with fringes at the edges. However, as the size of the object is decreased to a few times the wavelength of light, the diffraction fringes become wider and wider and if the size of the object is less than the wavelength of light, then all one can see is the central diffraction fringe which covers the entire field of vision and the image of the object is totally lost. The shortest wavelength that the human eye can see is about 0.4 **microns** (a micron is a millionth of a meter). So this is the smallest object that we can see directly. The naked eye can see an object of size 100 microns, so there is little advantage in building microscopes with magnification exceeding around 500. Some animals can detect UV radiation with somewhat shorter wavelengths - so their optical microscopes may be a little more powerful than ours - but not much!

If we can't see an object directly with visible light, could we not use electromagnetic

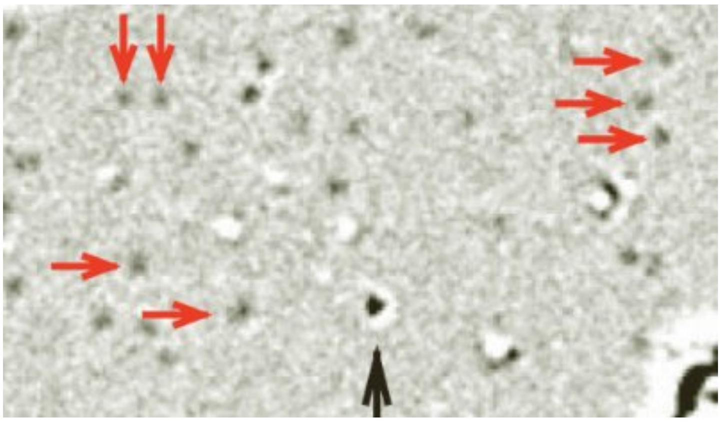

Figure 17: The arrows point to individual hydrogen atoms, within a graphene lattice. The image is created using an electron microscope.

radiation with much shorter wavelengths to make an image of an object? An example that comes to mind is X-rays which have wavelengths down to 0.00001 microns – so we could see things of the size of an atom with these. X-rays are the same as light waves but with a much smaller wavelength. The trouble is that we have no way of focusing a beam of X-rays (we do not know how to build a lens for X-rays). Fortunately Quantum Physics comes to our rescue here. We will see later that what are classically regarded as particles, can also behave like waves - a particle has a wavelength associated with it – and this wavelength is inversely proportional to the particle's momentum. An electron accelerated through a **potential** of 100000 **Volts** has a wavelength of .000004 microns. With such wavelengths individual atoms can be detected. Such an image, which shows hydrogen atoms in **graphene** [8] is shown in Fig. 17.

However, there is a price to pay for such small wavelengths leading to such high resolution. The high energy electron also has a very high momentum. When it scatters off a hydrogen atom it imparts some of its momentum to the hydrogen atom, which then begins to move with a velocity of around 100000 meters per second. This is why the images of the hydrogen atoms in Fig. 17 are rather blurred. The energy of the incident electrons are far larger than the **binding energies** of the electron inside the hydrogen atom, so that in a collision between the incident electrons and the electron in the hydrogen atom, the electron originally bound to the hydrogen nucleus is completely knocked out. This would also happen if we could use X-rays - we will also see that what is classically regarded as a wave comes in discrete "lumps", known as "quanta", which have energy and momentum associated with them, just like particles. It is therefore impossible to study directly the internal structure of atoms. Either the wavelength is larger than the size of the atoms so that they cannot be resolved, or the momentum associated with the wave is so large as to destroy the atomic structure that we are trying to study.

It is tempting to imagine that the sub-microscopic (i.e. sub-atomic) world that we cannot see, even indirectly, is a scaled down copy of the world that we can see, so that although we cannot see individual electrons within an atom, they have well-defined positions and velocities (momenta) in orbits around the nucleus, in analogy to the planets moving in well-defined orbits around the sun. Unfortunately, this is not the case. We have to learn to live with the fact that an electron inside an atom behaves very much like a wave in a wavepacket and simply does not possess a well-defined position and momentum. As we have no experience of such a system, our imagination is limited in such a way that we cannot "understand" this – i.e. we cannot draw a picture in our mind which represents what is happening. Let me remind you that the objective of theoretical physics is to develop a theory which allows us to calculate quantities which can be measured experimentally, and not to allow us to draw pictures in our minds which portray systems with quantities whose values cannot, even in principle, be measured. As we shall see from Heisenberg's uncertainty principle, it is impossible to measure, simultaneously, the position and momentum of a particle to better than a certain accuracy. In our macroscopic world, with which we are familiar, this accuracy is so good that any theoretical error is negligible compared to the accuracy with which we can perform a measurement - so that we have no concept of quantum effects in our everyday life.[2]

Probability Density

The idea of **probability density** will crop up often in our discussion of Quantum Physics – so it is worthwhile spending a little time explaining this. The probability that somebody's birthday is May 9 is about one in 365[3]. What this really means is that this is the probability that somebody was born between midnight on May 8 and midnight May 9 or between time $T = 129$ and $T = 130$ (measured in days from the beginning of the year). The probability of somebody being born between midnight on May 8 and noon on May 9 is only half of this and the probability of their being born in an even smaller time slice is even less. The symbol dT is used to denote a small interval in time T, and the probability of somebody being born between time T and time $T + dT$ is given by $P(T)\, dT$, where $P(T)$ is called the "probability density". In the case we are discussing this density is (approximately) 1/365 per day – so that the probability of being born between midnight on May 8 and noon on May 9 is $(1/365) \times (1/2) = 1/730$ whereas the probability of being born between 9 a.m. and 10 a.m. on May 9 ($dT = 1$ hour $= 1/24$ day) is $(1/365) \times (1/24) = 1/8760$.

[2]This is something that is indeed limited by the technology of measurements. It is perfectly conceivable that as experimental methods improve, we will be able to detect quantum effects in larger systems, but we will never be able to evade the limitation of the accuracy to which simultaneous measurements of position and momentum can be made.

[3]The distribution of birthdays is not quite uniform throughout the year because even we humans have preferred "mating seasons" so the actually probability may be a little different.