3 Experiments that Shook the World of Physics.

At the beginning of the 20^{th} century we had a well-defined concept of "waves" and "particles" which displayed the different behaviours described above. However, a number of experiments were performed which gave results which were at variance with those expected from entities that were traditionally viewed as "waves" or "particles", i.e. entities that were considered to be particles turned out to behave like waves in certain experiments and vice versa. This was the beginning of Quantum Theory. Particles are "lumps" with well-defined masses (equivalent to "lumps" of energy by Einstein's relation [9], $E = mc^2$, between energy and mass) – so that if what we thought of as "waves" can behave like particles, their energy must come in such "lumps" or **"quanta"** as they are called. This immediately shatters the idea that the energy of a wave is determined by its amplitude and can take any value - the amplitude has to be such that the energy of the wave is an integer number of these quanta.

3.1 The Ultraviolet Catastrophe

When light or other electromagnetic radiation is incident on a surface at constant temperature, some of the light is reflected and the rest is absorbed and re-radiated. A body whose surface absorbs all the radiation incident upon it is called a **"blackbody"**. The number of different wavelengths of electromagnetic radiation, $N(\lambda)d\lambda$, with wavelengths in a small interval between λ and $\lambda+d\lambda$, ($d\lambda$ means a small interval in wavelength) can be shown to be proportional to $1/\lambda^4$, (i.e. the probability density of radiation with wavelength λ decreases as the fourth power of wavelength - there much less radiation for large wavelengths than for small wavelengths). For a surface at temperature T^0K (0^0C is equivalent to 273.16^0K), classical thermodynamics [11] predicts that the energy of each wave is proportional to T, with the constant of proportionality being the **Boltzmann constant**, written as k_B. If this is the case then we have an expression for the energy, $E(\lambda)d\lambda$, radiated from a blackbody in the wavelength interval λ to $\lambda + d\lambda$,

$$E(\lambda)d\lambda \ \propto \frac{T}{\lambda^4}d\lambda. \tag{3.1}$$

Such a behaviour of radiated energy plotted against wavelength, is shown as the black curve in Fig. 18 for a temperature, $T = 5000^0$K. The experimentally observed spectrum for $T = 5000^0$K. is shown as the blue line, with the results at lower temperatures shown in green and red. Note that the spectrum peaks at lower wavelengths as the temperature increases, explaining why a heated object glows at deeper colours - shorter wavelengths - as the temperature increases. We see that for sufficiently large wavelength the black line is a good approximation to the experimentally observed blue line but for small wavelengths the prediction of classical thermodynamics does not reproduce data even approximately. It is also a disaster, known as the "ultraviolet catastrophe" since it predicts that as the wavelength gets smaller (towards the **ultraviolet** end of the spectrum) the energy radiated

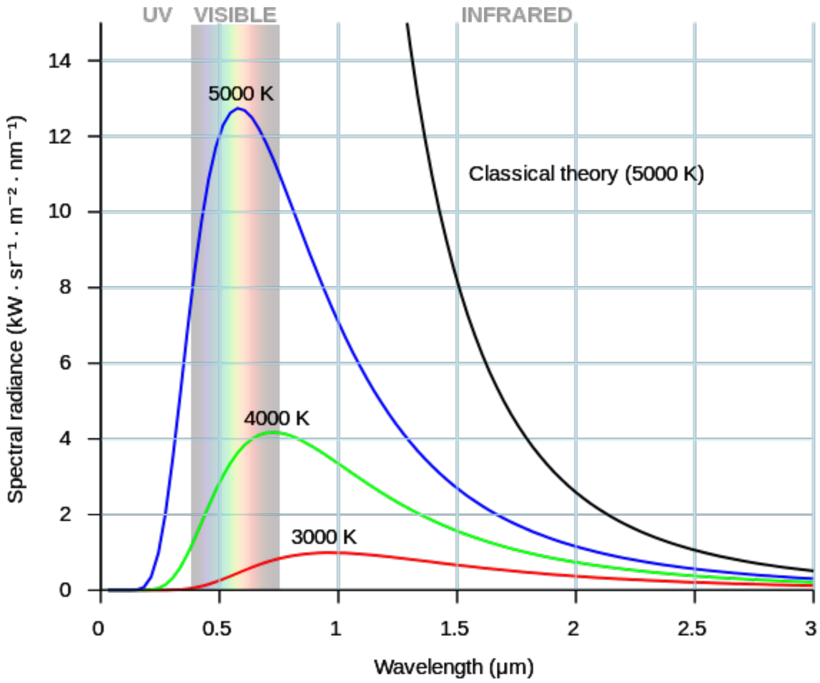

Figure 18: The intensity of radiation emitted from a blackbody in a small interval of wavelength between λ and $\lambda + d\lambda$, at temperatures of 5000^0K (blue line), 4000^0K (green line), and 3000^0K (red line). The black line is the result at 5000^0K using classical thermodynamics in which the average energy of each wave at temperature T, is $k_B T$ for any wavelength.

grows rapidly and becomes indefinitely large as the wavelength becomes smaller and smaller with more and more energy radiated at lower and lower wavelengths. The total energy radiated must be finite (i.e. bounded) and so the black curve of Fig. 18 cannot be correct. A solution to this inconsistency was proposed by Max Planck in 1900 [12]. The waves of the radiation are due to oscillating electric and magnetic fields. Classically, an oscillator can have any energy, but Planck suggested that this was not quite correct and that the energy of an oscillator of frequency f, had to be an integer number of **"quanta"** whose energy was given by

$$E = h\, f. \tag{3.2}$$

The constant h is (naturally) known as **"Planck's constant"**. It is fantastically small: $h = 6.67 \times 10^{-34}$ **Joules** seconds, so that the smallest watch spring has an energy of many billions of these quanta. It is for this reason that the rather strange and certainly counter-intuitive proposal of Planck has no measurable effect on our own experience of oscillators in the macroscopic world. It is perfectly reasonable for us to assume that any oscillator that we can build can have any amplitude and hence any energy we choose. However for oscillators in the sub-microscopic world, such as the oscillations of **ions** about their **equilibrium** positions at the lattice sites of a crystal, the quantization of the energy of their oscillation plays an important part in the determination of the properties of the crystal, such as its specific heat at very low temperatures. This quantization of the energy of an oscillator with quanta whose magnitude is proportional to the frequency means that the classical thermodynamics assertion - namely that each oscillator has an average energy equal to $k_B T$ - does not work. For a high frequency (short wavelength) oscillator for which one quantum, $h\, f$, is very much larger than $k_B T$, it is very unlikely that that oscillator will be excited into any oscillating state - its energy is most likely to be zero. The upshot of this is that the average energy of such a high frequency oscillator is not $k_B T$, but much smaller since it is very unlikely that the oscillator will have any quanta of energy at all and therefore it usually has zero energy. On the other hand if $k_B T$ is much larger than the quantum, $h\, f$, the oscillator can have oscillating energy of many quanta, and it indeed turns out that the average energy is $k_B T$. Radiation of wavelength λ is generated by an oscillator whose frequency, f, is related to λ by the simple formula $f = c/\lambda$, (where c is the speed of light).

Fig. 19 shows the ratio of Planck's result for the average energy of a wave emitted from a blackbody at temperature $T = 5000^0$K to the classical thermodynamic result, $k_B T$, for different wavelengths λ. We see that for sufficiently large λ the classical result, $k_B T$, is a good approximation (the ratio is close to one), but in the ultraviolet region of small λ, the average energy is much smaller. When this ratio is used as a correction factor for the black line of Fig. 18 we get a perfect agreement between theory and experiment (we reproduce precisely the solid blue line of Fig. 18, and the other lines are obtained for lower temperatures). Therefore, the proposal that oscillators have quantized energies leads to a modified radiation spectrum which solves the problem of the ultraviolet catastrophe and reproduces the experimentally observed result for the spectrum of blackbody radiation.

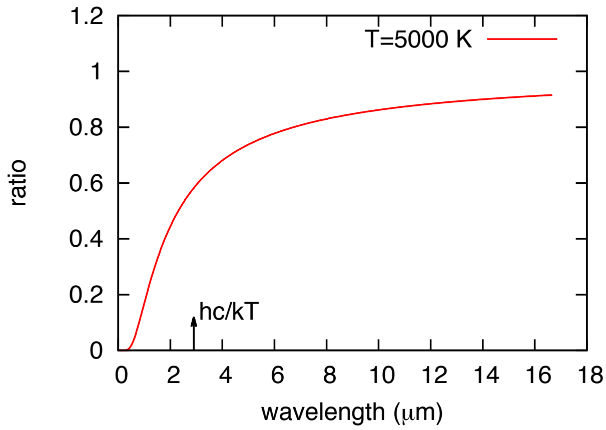

Figure 19: The average energy per unit volume for blackbody radiation with wavelength λ at a temperature of 5000^0K, divided by the classical value $k_B T$.

3.2 The Photoelectric Effect.

When light (visible or ultraviolet) is incident on certain metals, electrons can be emitted. This is called the **"photoelectric effect"**. It is the mechanism by which a light meter for a camera works. It was discovered by Heinrich Hertz [13] in 1887. We can understand this effect in terms of the wave nature of light. At any fixed point, an electromagnetic wave (e.g. visible light) is an oscillating electric and magnetic field. When incident upon a metal, which contains loosely bound electrons, the electric field applies a force on these electrons which can sometimes be large enough to liberate them from their binding in the metal. However, when we consider the quantitative effect of such a wave, we predict results which are at variance with experiment. The higher the frequency of the electromagnetic radiation the more often we would expect an electron to be emitted. The maximum kinetic energy of the emitted electrons is expected to increase with increasing intensity of the incident light. Furthermore for very low intensity incident light one would expect a delay before observing the emission of such electrons (known as "photoelectrons") in order to allow the metal to absorb enough energy to free the electrons from the metal.

The first quantitative experiment on the photoelectric effect was conducted in 1902 by Philipp Lenard [14]. The maximum energy of the emitted photoelectrons was determined by applying a retarding potential to the emitted electrons and adjusting that potential until it reached the "stopping potential", V_0, for which no electrons were collected at the collector electrode. A diagram of the experimental setup is shown in Fig. 20. Lenard found that when he increased the frequency of the incident light, the stopping potential increased, i.e. the maximum kinetic energy of the photoelectrons increased. The rate at which the electrons were emitted was found to increase with increasing intensity of incident light. On the other

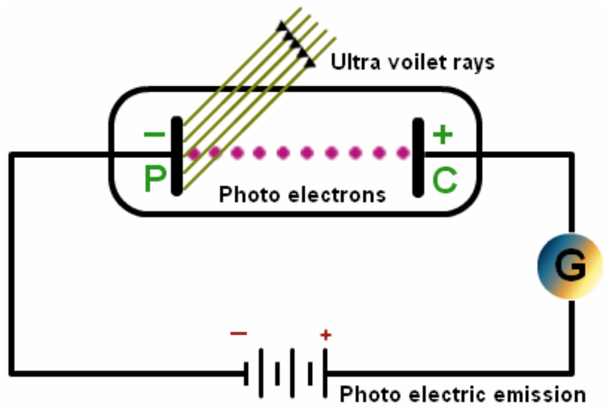

Figure 20: The experimental setup for the photoelectric effect. A voltage is applied so that the emitted electrons are accelerated towards the collecting **anode**, C. The photoelectric current is measured by the galvanometer, G. The stopping potential is determined by reversing the potential so that it retards the (negatively charged) electrons.

hand, the electron emission rate was found to be (surprisingly) independent of the frequency of the illuminating light, whereas the classical wave interpretation would imply that at higher frequencies the rate of electron emission would be higher as the electrons were "shaken" with higher frequency. Furthermore, even with the lowest intensity light sources, there was no delay between the initial incidence of the radiation and the emission of the photoelectrons. The explanation of this enigmatic behaviour was provided by Albert Einstein in 1905 [15][4]:

Light (or any other electromagnetic radiation) comes in packets or "quanta" of energy, E, given by

$$E = h\,f \tag{3.3}$$

where f is the frequency of the light and h is Planck's constant. These "quanta" behave like particles (they are now called **"photons"** the "-on" suffix is used for the names of particles). These particles scatter off the electrons in the metal. Some of the energy, ϕ , is used to free the electrons from the metal. This energy is called the **"work function"** and it is a property of the metal from which the electrons are emitted. Some or all of the remaining energy of the photon can be transferred to the emitted electrons, so the maximum energy which the photoelectrons can have is the difference $h\,f - \phi$. This maximum is the stopping potential multiplied by the charge, e, of an electron. We therefore have the simple formula relating the frequency, f, of the incident electromagnetic radiation to the stopping potential V_0

$$h\,f = \phi + eV_0 \tag{3.4}$$

[4]Together with Planck's theory of blackbody radiation, Einstein's theory of the photoelectric effect constitute the birth of Quantum Physics. It was *this* theory that won Einstein the Nobel prize in Physics in 1921 and *not* the Theory of Relativity.

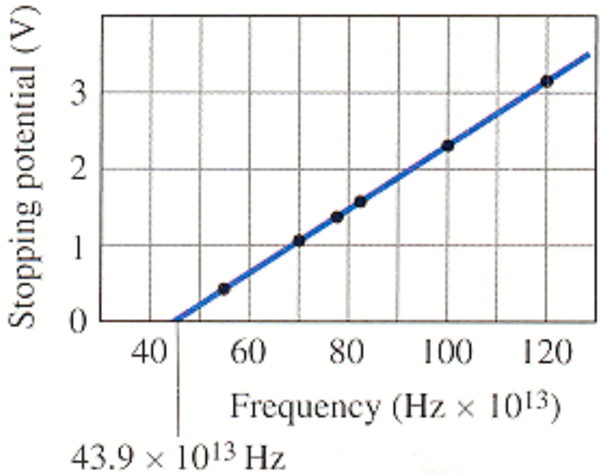

Figure 21: The result of Millikan's experiment on the photoelectric effect showing the stopping potential against the frequency of incident radiation. The slope, $\Delta V/\Delta f$ is Planck's constant divided by the electric charge of the electron. The intercept is equal the work function divided by Planck's constant.

This formula was tested in detail by Robert Millikan in 1916 [16]. His results for the stopping potential for various different frequencies is shown in Fig. 21. The points lay on a very good straight line whose slope was proportional to Planck's constant. This was the most accurate measurement of Planck's constant at that time yielding $h = (6.57 \pm 0.03) \times 10^{-34}$ Joule seconds. The intercept, i.e. the value of frequency, f_0, at which the line crosses the x-axis can be used to determine the work function, since $h\, f_0$ is the energy of a photon which causes the emission of an electron with zero kinetic energy.

The increase of the rate of electrons emitted with increasing intensity of incident light is explained by the fact that increased intensity of light means a larger flux of incident photons – and therefore a larger number of interactions which can free an electron from the metal.[5]

3.3 Compton Scattering

In 1920 Arthur Compton [17] performed an experiment in which he scattered monochromatic (single wavelength) X-rays with wavelength λ against various different targets. He discovered that as well as an emitted X-ray line with the same wavelength as the incident X-ray, there was also a further line with a larger wavelength, λ', and that the difference between the incident wavelength, λ, and the wavelength, λ' , of the supplementary line increased with

[5]Einstein's explanation of the photoelectric effect can also be used to explain the observation made by Philipp Lenard in 1900 that light can ionize gases, for which he received the Nobel Prize in 1905. This is very ironic. Lenard was a staunch supporter of National Socialism and condemned the Theory of Relativity on the grounds that it was "Jewish physics."

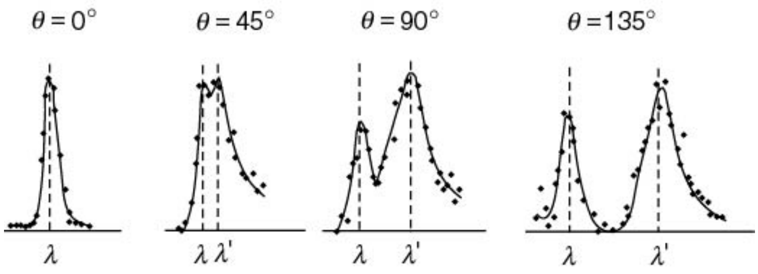

Figure 22: Incident and scattered X-ray wavelengths for various different scattering angles.

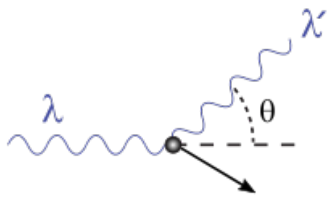

Figure 23: A photon with wavelength λ scatters off an electron with scattering angle θ between the directions of the incident photon and the scattered photon. The scattered photon has a longer wavelength λ' since some of its momentum has been imparted to the target electron.

increasing scattering angle, θ. The result is shown in Fig. 22.

Compton's explanation of this effect again exploits the fact that electromagnetic radiation (in this case X-rays rather than visible or ultraviolet light) comes in quanta and that these quanta (photons) are particles, which sometimes scatter off the electrons in the target material. These photons have energy $h\,f$, where f is the frequency of the X-rays (recall the relation between frequency and wavelength, $f = c/\lambda$, where c is the speed of light) and momentum[6] $h\,f/c = h/\lambda$.

When such particles scatter off electrons at rest, (as shown diagrammatically in Fig. 23) some of their momentum is imparted to the electron and, since the total momentum of the photon and the electron is conserved in such a scattering, the momentum of the photon is reduced – leading to a larger wavelength, λ' (the momentum of the photon is inversely proportional its wavelength, so that a reduction in momentum implies an increase in wave-

[6]Photons travel with the speed of light. According to Einstein's theory of special relativity a particle that travels with the speed of light has zero mass. This does not mean that the momentum, p, is zero. The formula $p = mv$ is only valid for particles whose speed is much less than the speed of light. For a massless particle the relation between momentum, p, and energy, E is $E = pc$. Therefore if a photon has energy $h\,f = h\,c/\lambda$, and momentum $p = h/\lambda$.

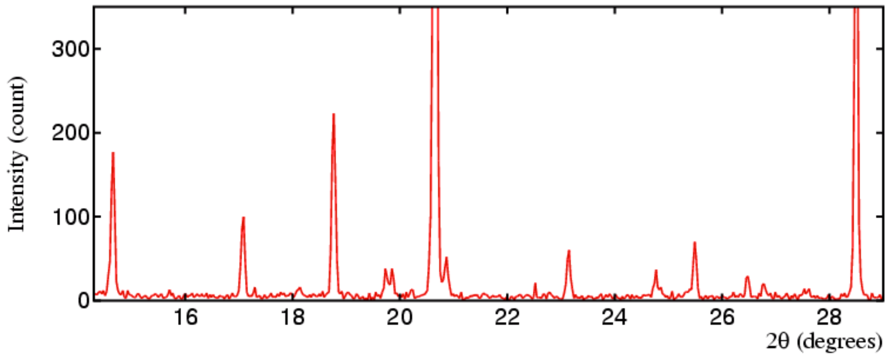

Figure 24: Diffraction maxima from X-rays diffracted from a regular crystal.

length). Using conservation of energy and momentum (in this case the electrons recoil with velocities close to the speed of light so that it is necessary to use relativistic dynamics rather than Newtonian dynamics), the fraction of the initial momentum imparted to the electron increases with scattering angle, leading to an increase in the change of the wavelength of the photon. In case you are interested the exact relation between incident wavelength, λ, wavelength, λ' , of the scattered photon and the scattering angle, θ is

$$\lambda' - \lambda \;=\; \frac{h}{m_e\, c}\,(1 - \cos\theta)\,, \tag{3.5}$$

where m_e is the mass of the electron.

3.4 Bragg X-ray Diffraction

This is actually an experiment that is explained using the classical, i.e. wave-like, properties of X-rays. The results of the experiment are nevertheless an important for the development of Quantum Theory. In 1914 W.H.Bragg and his son, W.L.Bragg [18] scattered X-rays from various different crystals. They found diffraction maxima (sharp peaks in intensity) at certain X-ray scattering angles as shown in Fig. 24

A crystal consists of a regular array (lattice) of ions from which the wave is reflected. The explanation for these peaks is that the wave reflected from adjacent planes of ions in the crystal have a path difference which depends on the reflection angle and the distance between the planes of ions, as shown in Fig. 25. For certain reflection angles, this path difference is an integer number of wavelengths so that there is constructive interference between the wave reflected from adjacent planes, leading to a diffraction maximum. Bragg's law tells us that there will be maxima for X-rays of wavelength λ for a diffracted angle of 2θ whenever

$$2d\sin\theta = n\lambda, \quad \text{(n integer)}. \tag{3.6}$$

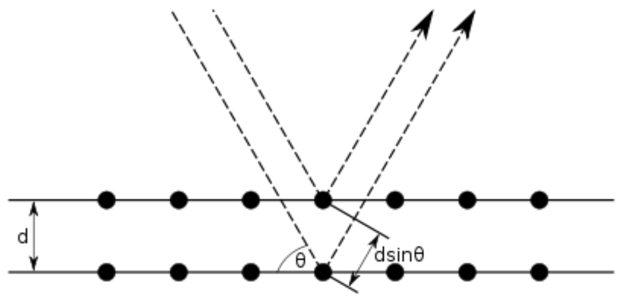

Figure 25: The path difference between waves reflected from adjacent planes of a crystal with separation, d, is (using a little trigonometry) $2d\sin\theta$, where θ is the reflection (equal to the incident) angle, as shown.

This is the explanation when applied to a regular cubic lattice. For more complicated lattices the maxima will be at different angles for different orientations and this can be used to deduce the crystal structure. It can also be used to deduce the structure of large molecules and was indeed the technique that was used by Franklin, Wilkins, Crick and Watson [19] to determine the double helix structure of the DNA molecule.

3.5 The Davisson-Germer Experiment

We have seen so far that X-rays (or visible light or any other electromagnetic radiation) are both waves and particles, and that the wave or particle properties manifest themselves in different experiments. What about electrons or other entities that have traditionally been treated as particles? Between 1923 and 1927 Clinton Davisson and Lester Germer [20] performed a series of experiments in which electrons were accelerated through a particular voltage and then scattered off a regular crystalline solid (similar to the scattering experiment conducted by Bragg with X-rays.) but in their case the target was made from nickel. The experimental setup is shown in Fig. 26

The results were displayed in a slightly unconventional way in Fig. 27. For each scattering angle, ϕ, a point is drawn which makes that angle to the vertical axis and the distance between that point and the origin is proportional to the intensity of the scattered electrons (basically the scattered intensity and scattering angle are treated as the radius and angle in polar coordinates). As the scattering angle, ϕ, is varied a curve is described in this polar coordinate graph.

In 1924, Louis de Broglie [21] had postulated that, in analogy with X-rays that possess momentum p as well as wavelength, λ, related by

$$\lambda = \frac{h}{p}, \quad \text{where } h \text{ is Planck's constant,} \tag{3.7}$$

particles possess wavelength as well as momentum with the relation between the two also

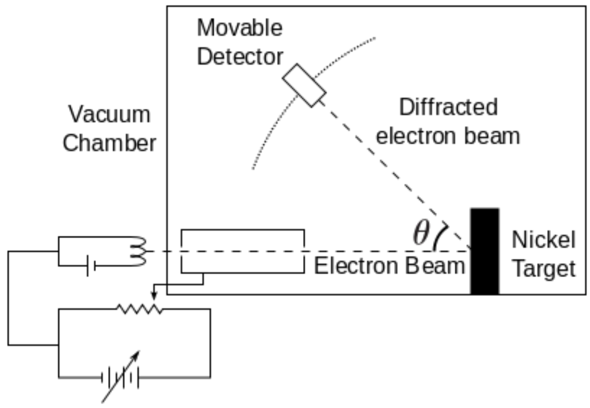

Figure 26: The Davisson-Germer experiment in which electrons are accelerated through a given voltage and then scattered from a nickel target. The intensity of the scattered electrons is measured by a detector which was moveable so that different scattering angles can be probed.

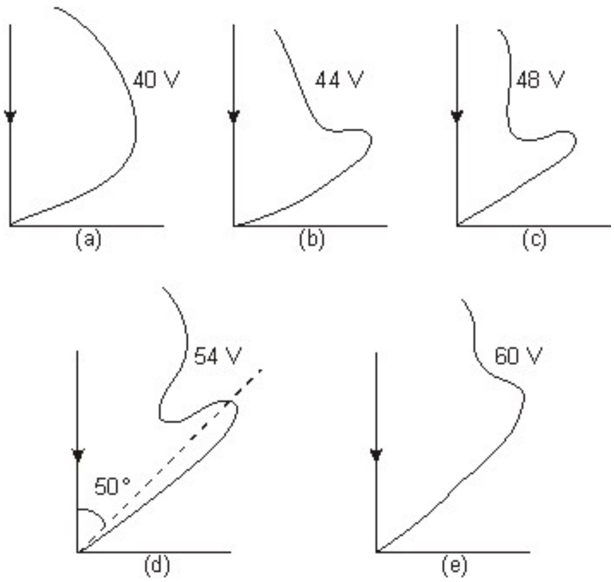

Figure 27: The results of the Davisson-Germer experiment for several different accelerating potentials. The distance of the line from the origin at some angle, ϕ, to the vertical is proportional to the intensity of the scattered electrons at a scattering angle ϕ. The bulges occur at angles that indicate the scattering angle corresponding to the first diffraction maximum of the matter waves of the electrons scattered off a nickel crystal target.

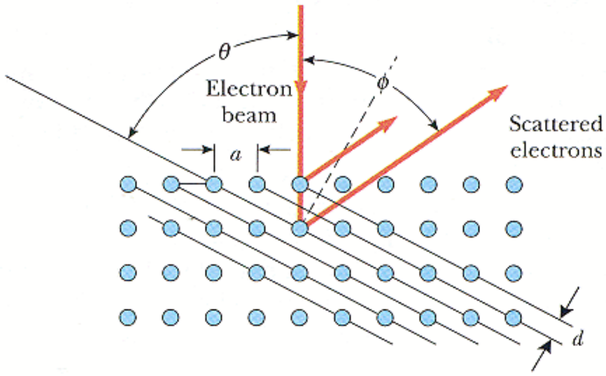

Figure 28: A diagram showing that the relation between the electron scattering angle, ϕ and the angle between the incident electrons and the crystal lattice planes, θ, is given by $\theta = 90^0 - \phi/2$. Therefore the first diffraction maximum occurs at $d \cos\left(\frac{\phi}{2}\right))$ λ, where λ is the de Broglie wavelength of the incident electrons.

given by eq.(3.7), which is known as **de Broglie's wave equation**.

This means that what we think of as "matter" in classical physics would, in Quantum Physics, have associated "matter waves" with the same relation between wavelength and momentum. These matter waves have all the properties of other waves including interference and the existence of wavepackets which can be constructed from a superposition of waves with different wavelengths.

Let us calculate the wavelength of an electron accelerated though a given voltage (skip to eq.(3.10) if you do not wish to follow this):
An electron accelerated through a voltage, V, acquires a kinetic energy, T, where

$$T = \frac{1}{2} m_e v^2 = e V \quad \text{(e is the electric charge of the electron and } m_e \text{ is its mass),} \tag{3.8}$$

and consequently (manipulating eq.(3.8)) momentum, p, where

$$p = m_e v = \sqrt{2 m_e T} = \sqrt{2 m_e e V}. \tag{3.9}$$

By using the de Broglie wave relation, the matter waves have wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2 m_e e V}}. \tag{3.10}$$

As can be seen from the diagram in Fig. 28, and a little geometry, a scattering angle, ϕ, corresponds to an angle θ between the incident electrons and the planes of a crystal where

$$\theta = 90^0 - \frac{\phi}{2}, \tag{3.11}$$

so that by combining the Bragg scattering relation eq.(3.6) and the de Broglie wavelength of the electrons eq.(3.10) the first diffraction maximum occurs if

$$2\,d\,\sin\theta \;=\; 2\,d\,\cos\left(\frac{\phi}{2}\right) \;=\; \lambda \;=\; \frac{h}{\sqrt{2m_e eV}}, \tag{3.12}$$

where d is the spacing between planes in the crystal. Putting in the numbers, we find that for an accelerating potential of $54\,V$, the wavelength of the electron is $\lambda\,=\,0.168$ nm (1 nm is a billionth of a meter). The diffraction maximum for this accelerating potential is 50^0 (bottom left graph of Fig. 27), from which we deduce that the spacing between planes in the nickel crystal, calculated from eq.(3.12) to be $d\,=\,0.093$ nm. This is very close to the value ($d\,=\,0.091$ nm) which is found using Bragg scattering of X-rays off a nickel crystal. This was the first experiment to detect directly the wave properties of electrons and it was the first experimental confirmation of the de Broglie relation between wavelength and momentum.