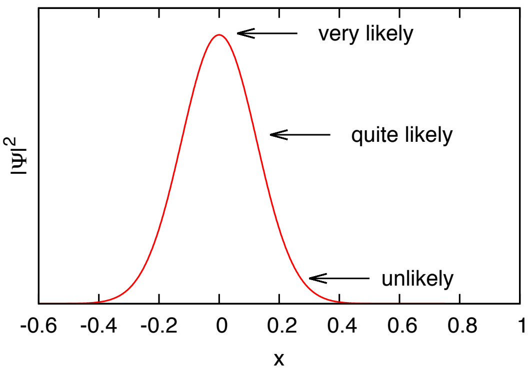

Figure 30: Graph of $|\Psi|^2$ against x, showing the relative probability of finding the particle at different points

5 What is a Matter (de Broglie) Wave?

We have discussed the wavelength of "matter waves" or "de Broglie waves", but we haven't yet discussed what these are waves of – i.e. what is the disturbance that is oscillating in analogy with a wave on a violin string where the disturbance is the displacement of the string from its equilibrium position that is oscillating and travelling along the string.

The interpretation of these matter waves was given by Max Born [23]. The actual "disturbance" does not have a direct physical interpretation, but it does have an indirect meaning. The matter wave disturbance at a position $\mathbf{x}$, and time t, is described by a function of $\mathbf{x}$ and t called the "wavefunction" and is usually written as $\Psi(\mathbf{x}, t)$.[7] The amplitude of the wave oscillation as that point and time is written as $|\Psi(\mathbf{x}, t)|$. The square of this amplitude is the probability density for finding the particle at the point, $\mathbf{x}$. This means that $|\Psi(\mathbf{x}, t)|^2 \, dV(\mathbf{x})$ is the probability that at time t the particle is in the small volume, $dV(\mathbf{x})$ centred at the point $\mathbf{x}$ (i.e. $|\Psi(\mathbf{x}, t)|^2$ is the probability density - probability per unit volume - for the position of the particles at the point $\mathbf{x}$). This is easier to understand if we restrict ourselves to one dimension so that a point in space is denoted by a single variable, x. In this case $|\Psi(x, t)|^2 \, dx$ is the probability that the particle is somewhere between x and $x + dx$, where dx is a small interval in x.

A plot of the square of the amplitude of the oscillations at a given time, t, is shown as a function of (one-dimensional) x in Fig. 30. The peak (arbitrarily set at $x = 0$) is at the position where the particle is very likely to be. If we move out either side of this to a distance 0.2 units we find that it is quite likely that the particle will be found at either of these points, whereas if we go out further than 0.4 units either side of $x = 0$ then it is very unlikely that the particle will be there. For the rest of this section we confine our consideration to motion

[7] $\mathbf{x}$ is written in bold-face since it stands for a vector in three-dimensional space, whose components are the three coordinates of the point at which the amplitude is being considered.