10 Quantum States and Superposition

A sub-microscopic system (a sub-atomic particle or set of sub-atomic particles moving under a force field or exerting a force on each other) is described by a **"quantum state"** (or just **"state"**) which is a list of physical properties of the system that can be measured simultaneously. Thus, for a free point-particle the state could be specified in terms of the momentum of the particle or in terms of its position but not both, since the simultaneous determination of momentum and position is forbidden by Heisenberg's uncertainty principle.

For a more complicated system such as an electron bound in an atom, we can specify the energy of an electron , but also the **angular momentum** due to its rotation about the nucleus (although it is quite wrong to "picture" an electron rotating around a nucleus in a fixed orbit). Angular momentum is a vector quantity – as well as its magnitude it has a direction determined by the direction of the axis about which the object is rotating. In classical mechanics, this vector can have three well-specified components - one for each dimension - but in Quantum Physics it turns out that only one component of angular momentum can be determined at any one time (in the same way that either the momentum or the position of a point particle can be determined). Thus if we know the component of angular momentum in the z-direction, the components in the x-direction or y-direction are undetermined - that means that the system simply does not possess well-defined values for these quantities. The state of an electron in an atom can therefore be specified by its energy, the magnitude of its angular momentum and one component of its angular momentum (usually chosen by convention to be the z-component). We shall see below that an electron also possesses an additional angular momentum, called spin, whose z-component can take one of two values - this value is also used to specify the state of the electron.

Very often the quantities which are used to specify a quantum state can only take certain specified values and can therefore be denoted by an integer (whole number) called a **"quantum number"**. In the case, considered in section 8, of a free particle in a one dimensional box of size L, whose energy levels are given by

$$E_n \;=\; \frac{n^2 h^2}{8\,m\,L^2}. \tag{10.1}$$

The integer, n, is the energy quantum number - it labels the energy level of the particle (the same is, of course, also true for the other examples of discrete energy levels considered in section 8.)

The wave associated with a system of particles in a given state is described mathematically by a wavefunction, $\Psi_{\{n\}}$, which depends on the set of quantum numbers $\{n\}$.[11] However, a system need not be in one of these quantum states but in a superposition of states described by different sets of quantum numbers $\{n\}_1$, $\{n\}_2$, etc. whose wavefunction is given by a superposition

$$\Psi \;=\; a\Psi_{\{n\}_1} + b\Psi_{\{n\}_2} + \cdots \tag{10.2}$$

---

[11]The notation $\{n\}$ is used to denote a set of numbers $n_1, n_2, \cdots$ which label all the quantities whose values determine the quantum state.

The meaning of such a superposition state in terms of the system of particles is that the system does not possess well defined values for the sets of physical quantities labelled by the quantum numbers $\{n\}$, but the probability that a measurement of these variables will yield the values labelled by $\{n\}_1$, $\{n\}_2$, etc. is given by a^2, b^2, etc. respectively.

As an example, if we return to the problem of a free particle of mass m, in a (one-dimensional) box of size L, the wavefunctions, Ψ_n with wavelength $\lambda_n = 2L/n$ are the wavefunctions representing the particles with energy

$$E_n = \frac{n^2 h^2}{8mL^2}.$$

However, we could have a particle whose wavefunction is given by (for example)

$$\Psi = \frac{1}{2}\Psi_1 + \frac{\sqrt{3}}{2}\Psi_2 \tag{10.3}$$

where Ψ_1 is the wavefunction with wavelength $2L$ and Ψ_2 is the wavefunction with wavelength L. A particle whose wavefunction is given by the above superposition does not have a well-defined energy, but there is a probability $\left(\frac{1}{2}\right)^2$, i.e. $\frac{1}{4}$, that its energy is

$$\frac{h^2}{8mL^2} \quad \text{(corresponding to } n = 1\text{)},$$

and a probability $\left(\frac{\sqrt{3}}{2}\right)^2$, i.e. $\frac{3}{4}$, that its energy is

$$\frac{h^2}{2mL^2} \quad \text{(corresponding to } n = 2.\text{)}$$