Microphysics for neutron star oscillations

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Microphysics for neutron star oscillations: static properties

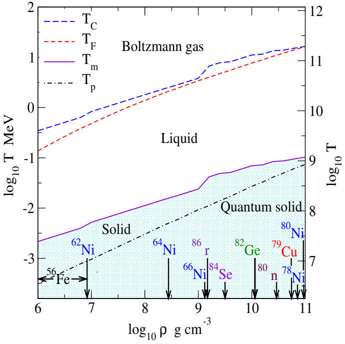
- Equation of state
- Composition
- Pairing properties (superfluidity and superconductivity)

Microphysics for neutron star oscillations: dynamic properties

- Viscosities (first, second), thermal conductivity
- Electrical conductivity in conducting plasma
- Mutual friction for superfluid/superconducting matter

This talk

- Electrical conductivity of a warm crust of a neutron star in magnetic field, following Harutyunyan and Sedrakian, PRC **94**, 025805 (2016).
- Complete transport for 2SC color superconducting matter, following Alford, Nishimura, Sedrakian, PRC **90**, 055205 (2014).



- Plasma parameter

$$\Gamma = \frac{Z^2e^2}{Ta_i}, \quad a_i = \left(\frac{3}{4\pi n_i}\right)^{1/3}$$

- Coulomb and melting temperatures

$$T_c = \frac{Z^2e^2}{a_i}, \quad T_m = \frac{Z^2e^2}{\Gamma_m a_i}$$

- Plasma temperature

$$T_p = \left(\frac{4\pi Z^2e^2n_i}{M}\right)^{1/2}$$

- if $\Gamma \ll 1$ - weakly interacting Boltzmann gas
- if $\Gamma \gg 1$ - strongly coupled plasma
- for $\Gamma > \Gamma_m \simeq 160$ lattice of nuclei - phonons and impurity scattering.
- for $\Gamma < \Gamma_m$ liquid state - electron ion scattering.
- for $T \leq T_p$ quantization of lattice oscillations becomes important

The kinetics of electrons is described by the Boltzmann eq.

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\frac{\partial f}{\partial \boldsymbol{r}} - e(\boldsymbol{E} + [\boldsymbol{v} \times \boldsymbol{H}])\frac{\partial f}{\partial \boldsymbol{p}} = I[f],$$

The collision integral for electron-ion scattering has the form

$$I = -(2\pi)^4 \sum_{234} |\mathcal{M}_{12\to 34}|^2 \delta^{(4)}(p + p_2 - p_3 - p_4)[f(1-f_3)g_2 - f_3(1-f)g_4],$$

For small perturbation

$$f = f^0 + \delta f, \quad \delta f = -\phi \frac{\partial f^0}{\partial \varepsilon}, \quad g(p) = n_i \left(\frac{2\pi}{MT}\right)^{3/2} e^{-\beta\varepsilon}.$$

The most general form of the perturbation is given by

$$\phi = -\frac{e\tau}{1 + (\omega_c\tau)^2} v_i \left[\delta_{ij} - \omega_c\tau\varepsilon_{ijk}h_k + (\omega_c\tau)^2 h_i h_j\right] E_j$$

where the relaxation time

$$\boxed{\tau^{-1}(\varepsilon) = \int \frac{d\omega d\boldsymbol{q}}{(2\pi)^5} \int d\boldsymbol{p}_2 |\mathcal{M}_{12\to 34}|^2 \frac{\boldsymbol{q}\cdot\boldsymbol{p}}{p^2} \delta(\varepsilon - \varepsilon_3 - \omega)\delta(\varepsilon_2 - \varepsilon_4 + \omega) g_2 \frac{1 - f_3^0}{1 - f^0}.}$$

Defintion of the electrical conduction

$$j_i = 2\int \frac{d\boldsymbol{p}}{(2\pi)^3} e v_i \phi \frac{\partial f^0}{\partial \varepsilon} = \sigma_{ij} E_j.$$

If B-field is in the direction of z axis

$$\hat{\sigma} = \begin{pmatrix} \sigma_0 & -\sigma_1 & 0 \\ \sigma_1 & \sigma_0 & 0 \\ 0 & 0 & \sigma \end{pmatrix}.$$

- The longitudinal conductivity does not depend on the field

$$\sigma = \frac{e^2}{3\pi^2}\int_0^\infty dp p^2 v^2 \tau \frac{\partial f_0}{\partial \varepsilon}$$

- The transversal (σ_0) and Hall (σ_1) conductivities depend on the B-field

$$\sigma_0 = \frac{e^2}{3\pi^2}\int_0^\infty dp p^2 v^2 \frac{\tau}{1+(\omega_c\tau)^2}\frac{\partial f_0}{\partial \varepsilon}, \quad \sigma_1 = \frac{e^2}{3\pi^2}\int_0^\infty dp p^2 v^2 \frac{\tau^2\omega_c}{1+(\omega_c\tau)^2}\frac{\partial f_0}{\partial \varepsilon}.$$

For two limiting cases of strongly degenerate and non-degenerate electrons the conductivities are given analytically.

- Low temperature limit corresponds to the Drude formula

$$\sigma = \frac{n_e e^2 \tau_F}{\varepsilon_F}, \quad \sigma_0 = \frac{\sigma}{1+(\omega_{cF}\tau_F)^2}, \quad \sigma_1 = (\omega_{cF}\tau_F)\sigma_0, \tag{1}$$

- High temperature limit $T \gg T_F$ similar formula is good to 20% with $\bar{\varepsilon} \simeq 3T$

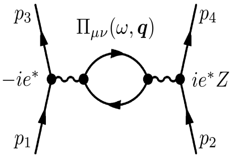
$$\sigma = \frac{n_e e^2 \bar{\tau}}{\bar{\varepsilon}}, \quad \sigma_0 = \frac{\sigma}{1+(\bar{\omega}_c\bar{\tau})^2}, \quad \sigma_1 = (\bar{\omega}_c\bar{\tau})\sigma_0,$$

The anisotropy clear depends on the parameter $\omega_c\tau$

- if $\omega_c\tau \ll 1$ (weak magnetic fields), $\sigma_0 \simeq \sigma$, $\sigma_1 \simeq \omega_c\tau\sigma \ll \sigma$, therefore the tensor is approximately isotropic

$$\sigma_{ik} \simeq \delta_{ik}\sigma.$$

- if $\omega_c\tau \gg 1$ (strong magnetic fields), $\sigma_0 \simeq \sigma(\omega_c\tau)^{-2} \ll \sigma$, $\sigma_1 \simeq \sigma(\omega_c\tau)^{-1} \ll \sigma$, i.e., the transverse conductivities are strongly suppressed.



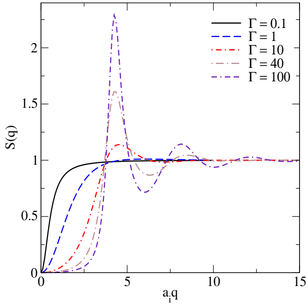
$$\boxed{\mathcal{M}_{12\to 34} = -\frac{J_0 J_0'}{q^2 + \Pi_L'} + \frac{\boldsymbol{J}_t \boldsymbol{J}_t'}{q^2 - \omega^2 + \Pi_T}}$$

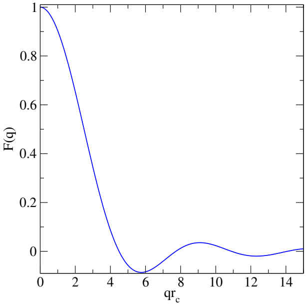
$$\begin{aligned} J^\mu &= -e^* \bar{u}^{s_3}(p_3)\gamma^\mu u^s(p), \\ J'^\mu &= Ze^* v'^\mu = Ze^*(1, \boldsymbol{p}'/M), \end{aligned}$$

The relaxation time is then found as

$$\begin{aligned} \tau^{-1}(\varepsilon) &= \frac{\pi Z^2 e^4 n_i}{\varepsilon p^3} \int_{-\infty}^{\varepsilon - m} d\omega e^{-\omega/2T} \frac{f^0(\varepsilon-\omega)}{f^0(\varepsilon)} \\ &\times \int_{q_-}^{q_+} dq (q^2 - \omega^2 + 2\varepsilon\omega) S(q) F^2(q) \frac{1}{\sqrt{2\pi}\theta} e^{-\omega^2/2q^2\theta^2} e^{-q^2/8MT} \\ &\times \left\{ \frac{(2\varepsilon-\omega)^2 - q^2}{|q^2 + \Pi_L'|^2} + \theta^2 \frac{(q^2-\omega^2)[(2\varepsilon-\omega)^2 + q^2] - 4m^2 q^2}{q^2 |q^2 - \omega^2 + \Pi_T|^2} \right\}. \end{aligned}$$

The polarization tensors are evaluated in the Hard-Thermal-Loop approximation to QED plasma.

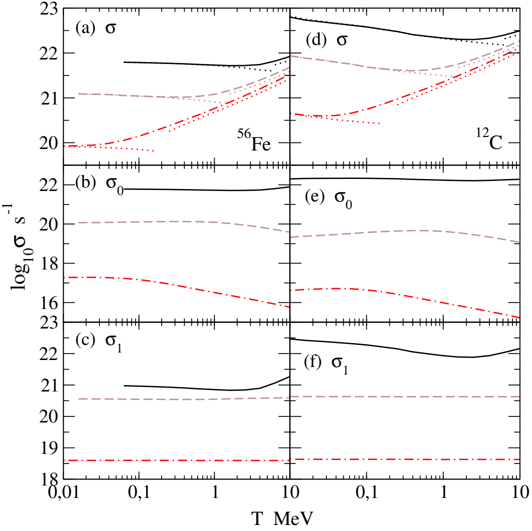




Left: ion structure factor suppresses the scattering with small q; for $a_i q \gg 1$ we find $S(q) \to 1$. Right: Nuclear formfactor where $r_c = 1.15A^{1/3}$ is the nuclear charge.

The structure factor depends on the value of the plasma parameter Γ and is derived in the case fo one sort of ions. It originates from the calculations of Monte Carlo calculations of Galam and Hansen for $\Gamma > 2$ and analytical expressions by Tamashiro for $\Gamma < 2$.

The nuclear structure factor assumes spherical nuclei, which is a good approximation in the warm and dilute crust (below neutron drip).



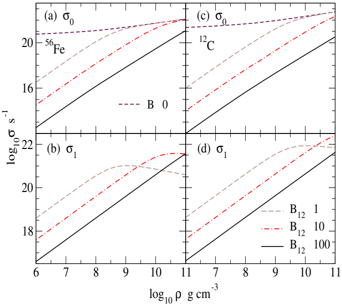
The temperature dependence shows a minimum at approximately $T/T_F = 0.3$ (transition from degenerate to the non-degenerate regime). Dots show the degenerate regime.

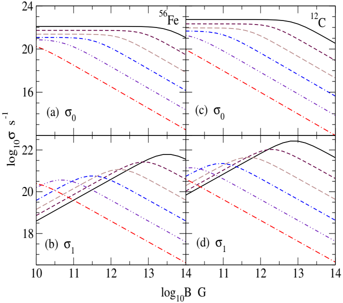
- At high densities or small magnetic fields $\omega_c\tau \ll 1$ (isotropic region) and

$$\sigma_0 \simeq \sigma, \quad \sigma_1 \simeq \sigma\omega_c\tau \simeq \frac{B}{n_e e}\sigma^2 \tag{2}$$

- At low densities or strong magnetic fields $\omega_c\tau \gg 1$ (anisotropic region) and

$$\sigma_0 \simeq \frac{\sigma}{(\omega_c\tau)^2} \simeq \left(\frac{n_e e}{B}\right)^2 \sigma^{-1}, \quad \sigma_1 \simeq \frac{\sigma}{\omega_c\tau} = \frac{n_e e}{B}. \tag{3}$$





Fit formulae are obtained for components of conductivity tensor

$$\sigma^{\rm fit} = CZ^{-1}T_F^a\left(\frac{T}{T_F}\right)^{-b}\left(\frac{T}{T_F}+d\right)^{b+c},$$

The other components of the tensor are given by

$$\sigma_0^{\rm fit} = \frac{\sigma'}{1+\delta^2\sigma'^2}, \quad \sigma' = \sigma^{\rm fit}\left(\frac{T_F}{\varepsilon_F}\right)^g,$$
$$\sigma_1^{\rm fit} = \frac{\delta\sigma''^2}{1+\delta^2\sigma''^2}, \quad \sigma'' = \sigma^{\rm fit}\left(1+\frac{T}{T_F}\right)^h,$$

where $\delta = B(n_e ec)^{-1}$ in c g s .

The relative error in σ is $\gamma \simeq 11\%$ for ^{12}C and $\gamma \simeq 13\%$ for ^{56}Fe and β-equilibrium composition. The relative error in σ_0 and σ_1 is $\gamma \simeq 12\%$ for ^{12}C and $\gamma \simeq 15\%$ for ^{56}Fe and β-equilibrium composition at temperatures $T > 0.15$ MeV.

Text only tables are available in the arxiv source (free access) and as supplemental material http://link.aps.org/supplemental/ 10.1103/PhysRevC.94.025805

General form of order parameter

$$\Delta \propto \langle 0|\psi^a_{\alpha\sigma}\psi^b_{\beta\tau}|0\rangle$$

- Antisymmetry in spin σ, τ for the BCS mechanism to work
- Antisymmetry in color a, b for attraction
- Antisymmetry in flavor α, β to avoid Pauli blocking

At low densities 2SC phase (Bailin and Love '84)

$$\boxed{\Delta(2SC) \propto \Delta\epsilon^{ab3}\epsilon_{\alpha\beta}}$$

Important variations on 2SC phase (crystalline-color-superconductor)

$$\boxed{\Delta(CSC) \propto \Delta\epsilon^{ab3}\epsilon_{\alpha\beta}, \qquad \delta\mu \neq 0, \qquad m_s \neq 0.}$$

At high densities we expect 3 flavors of u, d, s massless quarks. The ground state is the color-flavor-locked phase

$$\boxed{\Delta(CFL) \propto \langle 0|\psi^a_{\alpha L}\psi^b_{\beta L}|0\rangle = -\langle 0|\psi^a_{\alpha R}\psi^b_{\beta R}|0\rangle = \Delta\epsilon^{abC}\Delta\epsilon_{\alpha\beta C}}$$

Phase diagram in NJL see Buballa-Shovkovy-Rischke, Sandin-Blaschke, ... (2006+)

Boltzmann equation for fermions:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_1 \cdot \nabla_{\mathbf{x}}\right) f_1 = -(2\pi)^4 \sum_j \nu_j \sum_{234} |M_{ij}|^2 \times \left[f_1 f_2 (1-f_3)(1-f_4) - f_3 f_4 (1-f_1)(1-f_2)\right] \delta^4(p_{\rm in} - p_{\rm out})$$

f - fermion distribution function, M_{ij} scattering matrix element.
ν_j - the degeneracy factors (spin, flavor, color)

Fermions in the basis:

$$\Psi_i = \{\Psi_{bu}, \Psi_{bd}, \Psi_e\} = \{\text{blue up quark } (bu), \text{ blue down quark } (bd), \text{ electron } (e)\}.$$

the indices i and j specify the species of the ungapped fermions in this basis.

Further assumptions:

- Red and green colors are gapped and do not contribute to the transport
- No strangeness (number of s-quarks too small)
- High-density, low-temperature regime $T, m \ll \mu_q$
- Light flavor (isospin) asymmetry typical for neutron stars $\mu_u \ll \mu_d$ (β-equilibrium)

Gauge bosons: write the covariant derivative as

$$\boxed{D_\mu \Psi = \left(\partial_\mu - i\sum_a A^a_\mu Q^a\right)\Psi} \tag{4}$$

Two basis for gauge bosons - <u>standard</u> $(T_8, Q)$and <u>rotated</u> $(X, \tilde{Q})$

$$A_\mu = A^{T_8}_\mu T_8 + A^Q_\mu Q = A^X_\mu X + A^{\tilde{Q}}_\mu \tilde{Q}. \tag{5}$$

related by rotations via mixing angle φ

$$A^X_\mu = \cos\varphi\, A^{T_8}_\mu + \sin\varphi\, A^Q_\mu \tag{6}$$

$$A^{\tilde{Q}}_\mu = -\sin\varphi\, A^{T_8}_\mu + \cos\varphi\, A^Q_\mu \qquad \cos\varphi = \frac{\sqrt{3}g}{\sqrt{e^2+3g^2}}. \tag{7}$$

-In the rotated basis the $\tilde{Q}$ charge is massless, i.e., $\tilde{Q}$ color magnetic field penetrates the 2SC phase
-In the rotated basis the X charge is massive, i.e., there is a Meissner effect (more precisely color magnetic flux tubes)

The charges Q^a are defined to be the product of the coupling constant and the charge matrix for the ungapped fermions:

$$
\begin{aligned}
Q^{T_8} &= g \cdot \mathrm{diag}\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, 0\right) \\
Q^{Q} &= e \cdot \mathrm{diag}\left(+\frac{2}{3}, -\frac{1}{3}, -1\right)
\end{aligned} \tag{8}
$$

in the standard (T_8, Q) basis and

$$
\begin{aligned}
Q^{X} &= g\cos\varphi \cdot \mathrm{diag}\left(-\frac{1-2\tan^2\varphi}{\sqrt{3}}, -\frac{1+\tan^2\varphi}{\sqrt{3}}, -\sqrt{3}\tan^2\varphi\right) \\
Q^{\tilde{Q}} &= e\cos\varphi \cdot \mathrm{diag}\,(1, 0, -1)
\end{aligned} \tag{9}
$$

in the rotated $(X, \tilde{Q})$ basis.

- The longitudinal part of the screening is evaluated in the standard basis
- The transverse part of the screening is evaluated in the rotated basis

Computing the matrix element for scattering: $p_{1i} + p_{2j} \to p_{3i} + p_{4j}$ (flavor i, j)

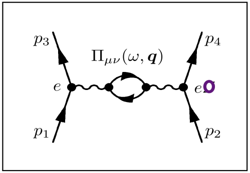
Standard Feynman rules give:

$$M_{ij} = J^{\mu}_{a,i}\left(D^{ab}_{\mu\nu}\right)J^{\nu}_{b,j} \tag{10}$$

$$J^{\mu}_{a,i} = Q^a_i \bar{u}\left(\mathbf{p}_3\right)\gamma^{\mu}u\left(\mathbf{p}_1\right)/2p_1 \qquad J^{\nu}_{b,j} = Q^b_j \bar{u}\left(\mathbf{p}_4\right)\gamma^{\nu}u\left(\mathbf{p}_2\right)/2p_2 \tag{11}$$

where the most general form of the propagator is given by

$$\left(D^{ab}_{\mu\nu}\right)^{-1} = g_{\mu\nu}\left(\omega^2 - q^2\right)\delta^{ab} + \Pi^{ab}_{\mu\nu} \tag{12}$$



Screening in a plasma is taken into account via self-energies $\Pi_{\mu\nu}$

Decomposition all the quantities (matrix elements, gauge propagators) into longitudinal and transverse parts:

$$M_{ij} = \sum_{a=\{T_8,Q\}} \frac{J^0_{a,i}J^0_{a,j}}{q^2 + \Pi^{aa}_l} - \sum_{a=\{X,\tilde{Q}\}} \frac{\mathbf{J}^t_{a,i}\cdot\mathbf{J}^t_{a,j}}{q^2 - \omega^2 + \Pi^{aa}_t} \tag{13}$$

$$\Pi^{aa}_l = \sum_i \left(q^a_{D,i}\right)^2 \chi_l + 4\left(q^a_{D,C}\right)^2 \chi_l \qquad \text{in the } (T_8, Q) \text{ basis}$$

$$\Pi^{aa}_t = \sum_i \left(q^a_{D,i}\right)^2 \chi_t + 4\left(q^a_{D,C}\right)^2 \chi_t + 4\left(q^a_{D,C}\right)^2 \chi_{sc} \qquad \text{in the } (X, \tilde{Q}) \text{ basis} \tag{14}$$

The screening functions, χ_l and χ_t in the static limit (Hard Thermal Loop approximation)

$$\chi_l = 1, \qquad \chi_t = i\frac{\pi}{4}\frac{\omega}{q}, \qquad \chi_{sc} = \frac{1}{3}. \tag{15}$$

(better done by Rischke and co-workers). To leading order in ω/q, we thus have

$$\Pi_l^{T_8T_8} = \sum_i (Q_i^{T_8})^2 \frac{\mu_i^2}{\pi^2} + 4(Q_C^{T_8})^2 \frac{\mu_C^2}{\pi^2} \tag{16}$$

$$\Pi_l^{QQ} = \sum_i (Q_i^{Q})^2 \frac{\mu_i^2}{\pi^2} + 4(Q_C^{Q})^2 \frac{\mu_C^2}{\pi^2} \tag{17}$$

$$\Pi_t^{XX} = \frac{4}{3}(Q_C^{X})^2 \frac{\mu_C^2}{\pi^2} \tag{18}$$

$$\Pi_t^{\tilde{Q}\tilde{Q}} = i\frac{\omega}{q}\Lambda^2 \qquad \text{where} \quad \Lambda^2 \equiv \sum_i (Q_i^{\tilde{Q}})^2 \frac{\mu_i^2}{4\pi} \tag{19}$$

The Q's can be found in the paper.

The squared matrix element summed over the final spins and averaged over the initial spins is

$$
\begin{aligned}
|M_{ij}|^2 \quad = \quad & L_l \left| \sum_{a=\{T_8,Q\}} \frac{Q_i^a\, Q_j^a}{q^2 + \Pi_l^{aa}} \right|^2 + L_t \left| \sum_{a=\{X,\tilde{Q}\}} \frac{Q_i^a\, Q_j^a}{q^2 - \omega^2 + \Pi_t^{aa}} \right|^2 \\
& -2L_{lt}\, \Re \left[\left(\sum_{a=\{T_8,Q\}} \frac{Q_i^a\, Q_j^a}{q^2 + \Pi_l^{aa}} \right) \left(\sum_{a=\{X,\tilde{Q}\}} \frac{Q_i^a\, Q_j^a}{q^2 - \omega^2 + \Pi_t^{aa}} \right)^* \right] + \delta_{ij}\gamma_{\rm int}
\end{aligned} \tag{20}
$$

where

$$
\begin{aligned}
L_l &= \left(1 - \frac{q^2}{4p_1^2}\right)\left(1 - \frac{q^2}{4p_2^2}\right) \\
L_{lt} &= \left(1 - \frac{q^2}{4p_1^2}\right)^{1/2}\left(1 - \frac{q^2}{4p_2^2}\right)^{1/2} \cos\theta \\
L_t &= \left(1 - \frac{q^2}{4p_1^2}\right)\left(1 - \frac{q^2}{4p_2^2}\right)\cos^2\theta + \frac{q^2}{4p_1^2} + \frac{q^2}{4p_2^2}
\end{aligned} \tag{21}
$$

The interference γ_{int} term is small and is neglected.

Transport coefficients - definitions of electrical and thermal conductivities and shear viscosity

$$j_\alpha = -\sigma\partial_\alpha U = \int \frac{d^3p}{(2\pi)^3} e v_\alpha\, \delta f \tag{22}$$

$$h_\alpha = -\kappa\partial_\alpha T = \int \frac{d^3p}{(2\pi)^3} (\epsilon - \mu)\, v_\alpha\, \delta f \tag{23}$$

$$\sigma_{\alpha\beta} = -\eta V_{\alpha\beta} = \int \frac{d^3p}{(2\pi)^3} p_\alpha v_\beta\, \delta f \tag{24}$$

where $V_{\alpha\beta}$ is the traceless part of the spatial derivative of fluid velocity $\mathbf{V}$,

$$V_{\alpha\beta} = \partial_\alpha V_\beta + \partial_\beta V_\alpha - \frac{2}{3}\delta_{\alpha\beta}\nabla\cdot\mathbf{V}. \tag{25}$$

Comparing the left-hand-sides we obtain a universal relation

$$\boxed{\xi Y = \sum_i \nu_i \int \frac{d^3p}{(2\pi)^3}\phi_i\, \delta f_i} \tag{26}$$

where ν_i is a spin factor for a particle flavor i, ξ stands σ, κ, or η, - Y stands $-\partial_\alpha U$, $-\partial_\alpha T$, or $-V_{\alpha\beta}$

Linearization of the Boltzmann equation is given by

$$f_i = f_i^0 + \delta f_i = \frac{1}{e^{(\epsilon-\mu_i)/T}+1} - \frac{\partial f_i^0}{\partial \epsilon}\Phi_i \tag{27}$$

Relaxation time approximation

$$\boxed{\Phi_i = 3\tau_i \psi_i \cdot Y} \tag{28}$$

$$\boxed{\xi_i = -\frac{3\tau_i \nu_i}{\gamma}\int \frac{d^3p}{(2\pi)^3}\left(\phi_i \cdot \psi_i\right)\frac{\partial f_i^0}{\partial \epsilon}} \tag{29}$$

$\gamma = \delta^\alpha_\alpha = 3$ for the electrical and thermal conductivities
$\gamma = \left(\delta^\alpha_\alpha \delta^\beta_\beta + \delta^\alpha_\alpha - 2\delta^\alpha_\alpha/3\right)/2 = 5$ for the shear viscosity. From Eq. (26), we can now define transport coefficient of each component ξ_i as

$$\xi = \sum_i \xi_i = \xi_{bu} + \xi_{bd} + \xi_e \tag{30}$$

Linearization of collision integral

$$\psi_i \cdot Y \frac{\partial f_1^0}{\partial \epsilon_1} = -\frac{(2\pi)^4}{T} \sum_j \nu_j \sum_{234} |M_{ij}|^2$$
$$f_1^0 f_2^0 (1-f_3^0)(1-f_4^0)\, \delta^4(p_{\rm in} - p_{\rm out})\, (\Phi_1 + \Phi_2 - \Phi_3 - \Phi_4)\,. \quad (31)$$

Using the same procedure as for the drift term

$$\xi_i = \frac{9\tau_i}{\gamma} \frac{(2\pi)^4}{T} \sum_j \nu_i \nu_j \sum_{1234} |M_{ij}|^2$$
$$f_1^0 f_2^0 (1-f_3^0)(1-f_4^0) \delta^4(p_{\rm in} - p_{\rm out}) \phi_1 \cdot [\tau_i(\psi_1 - \psi_3) + \tau_j(\psi_2 - \psi_4)]\,. (32)$$

In the limit $\omega, T \ll \mu_q$

$$\xi_i = \frac{\tau_i}{\gamma} \sum_j \nu_i \nu_j \frac{36 T \mu_i^2 \mu_j^2}{(2\pi)^5} \int_0^\infty d\omega \left(\frac{\omega/2T}{\sinh(\omega/2T)}\right)^2 \int_0^{q_M} dq \int_0^{2\pi} \frac{d\theta}{2\pi} |M_{ij}|^2$$
$$\phi_1 \cdot [\tau_i(\psi_1 - \psi_3) + \tau_j(\psi_2 - \psi_4)] \quad (33)$$

$q_M = \min\,[2p_1, 2p_2] = \min\,[2\mu_i, 2\mu_j]$ is the maximum momentum transfer, and θ is again the angle between $\mathbf{p}_1 + \mathbf{p}_3$ and $\mathbf{p}_2 + \mathbf{p}_4$. In the limit $T/\mu_q \ll 1$ $p_1, p_2 \to \mu_i, \mu_j$.

Comparing Eqs. (29) and (33) we obtain relaxation times τ_i for the three gapless fermion species.

Qualitative understanding

- Transport in the 2SC phase occurs via the ungapped fermions: the blue up quark, the blue down quark, and the electron.

- Transport is dominated by the fermion that feels the least influence from surrounding particles (i.e. long relaxation time or mean-free-path)

Relevant interactions

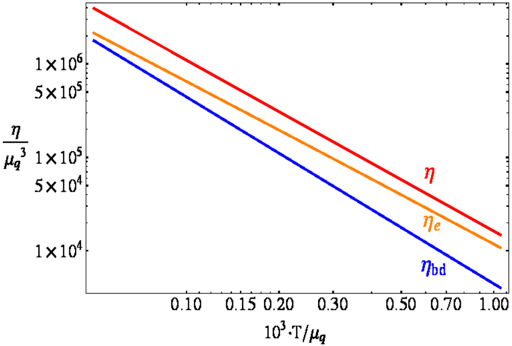
- longitudinal strong interaction (T_8) - Debye screened (short range)
- longitudinal electromagnetic interaction (Q), - Debye screened (short range)
- transverse "rotated" strong interaction (X) - Meissner screening (short ranged)
- transverse "rotated" electromagnetic interaction ($\tilde{Q}$) (not screened, only Landau damped - long ranged at low T)

At low-T the *bu* quark and electron carry $\tilde{Q}$ charge, *bd* does not. Transport is dominated by *bd* quarks (!)

At high T the Landau damping of the $\tilde{Q}$ is more significant. Relaxation times are dominated by the X and T_8 interactions.

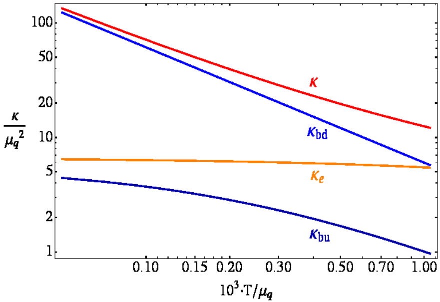
Electron, which has no T_8 charge and only a very small X charge, dominates transport.

A transition from the regime dominated by the *bd* quark to a regime dominated by electrons as the temperature is rised.



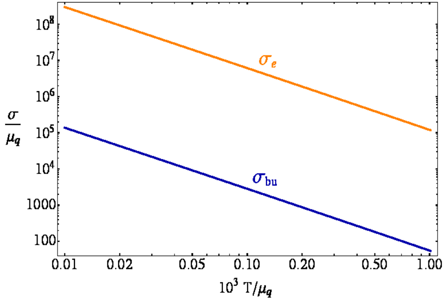
$$\frac{\eta_{bu}}{\mu_q} = \frac{0.150}{(T/\mu_q)^{5/3} + 2490\,(T/\mu_q)^2}, \quad \frac{\eta_e}{\mu_q} = \frac{0.171}{(T/\mu_q)^{5/3} + 2.78\,(T/\mu_q)^2} \tag{34}$$

Numerical calculation of shear viscosity as a function of temperature, taking $\alpha_s = 1$. In this temperature range we see electron and quark contributing equally at high temperature and electron domination at low temperature.



$$\frac{\kappa_{bu}}{\mu_q} = \frac{5.69}{1 + 3720\,(T/\mu_q)}, \quad \frac{\kappa_e}{\mu_q} = \frac{6.70}{1 + 6.92\,(T/\mu_q)^{2/3}} \tag{35}$$

Numerically calculated thermal conductivity in units of quark chemical potential μ_q in the 2SC phase with $\alpha_s = 1$. In this temperature range we see the crossover from electron domination at high temperature to blue down quark domination at low temperature.



$$\frac{\sigma_{bu}}{\mu_q} = \frac{0.000672}{(T/\mu_q)^{5/3} + 2.11\,(T/\mu_q)^2}, \quad \frac{\sigma_e}{\mu_q} = \frac{1.46}{(T/\mu_q)^{5/3} + 2.11\,(T/\mu_q)^2} \tag{36}$$

Numerically calculated electrical ($\tilde{Q}$) conductivity as a function of temperature, both expressed in units of the quark chemical potential μ_q, taking strong interaction coupling $\alpha_s = 1$. The electrons dominate because the *bu* relaxation time is shortened by its strong interaction with the *bd* quarks.

Summary

- Highly accurate results for conductivity of dilute and warn compact star matter including dynamical screening effects. Other transport coefficients will follow.
- Text-only tables and fit formulas are available for application in MHD computations of compact stars
- Complete set of transport coefficients (thermal and electrical conductivity and shear viscosity) for 2SC phase; second viscosity is also in the literature
- Fit formulas for all transport coefficients of 2SC phase and flux-fermion (quasi) mutual friction are available.
- Future: continue systematically computing accurate transport coefficients in regimes relevant for compact stars.