Optimal Fiscal Policy over the Business Cycle Revisited

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ESEWM 2009
The Question

- How should fiscal policy be set over the business cycle?

- Can we rationalize a ‘fiscal stimulus’ in a stochastic neoclassical growth model?
The Question

- How should fiscal policy be set over the business cycle?
- Can we rationalize a ‘fiscal stimulus’ in a stochastic neoclassical growth model?
- What is the optimal amount of debt in the long run?
American Recovery and Reinvestment Act 2009:

- Increase in spending (3.5% of GDP)
  - 1/4 of which for infrastructures (.85% of GDP)

- Tax cuts (2.0% of GDP)
  - Largely labor income taxes

- Financed by debt
Prescription from Chari et al. ’94 (CCK)

In the period of a negative productivity shock

- Labor income taxes: depends on preferences
- Capital income taxes: *increase substantially*
- Debt: *decreases*
- Gov. Spending: exogenous (in their model)
Main Features of the Model

▶ Business cycle driven by technology shocks

▶ Government Infrastructures are an input of production

▶ The government optimizes fiscal policy (taxes, debt and new infrastructures)

▶ Investment becomes productive within the period
Main Findings

In the period of a negative shock

- Infrastructure spending increases
- Labor income taxes decrease
- Capital income taxes decrease
- State contingent debt *decreases*!
Main Findings

In the period of a negative shock

- Infrastructure spending increases
- Labor income taxes decrease
- Capital income taxes decrease
- State contingent debt decreases!

Without state contingent debt

- Infrastructure spending increases
- Labor income taxes decrease
- Capital income taxes increase slightly
- Debt can increase!
Related Literature

- Complete markets
  - Chari, Christiano, and Kehoe, 1994

- Incomplete markets without $\tau_k$
  - Aiyagari, Marcet, Sargent, and Seppälä, 2002
  - Scott, 2007
  - Marcet and Scott, 2009

- Incomplete markets with $\tau_k$
  - Farhi, 2009
Household Problem

- Preferences

\[ \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(c(s^t), l(s^t)) \]

- Budget Constraint

\[ c(s^t) + k(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t) b(s_{t+1}|s^t) \]

\[ = w(s^t) l(s^t) + r(s^t) k(s^t) + k(s^{t-1}) + b(s_t|s^{t-1}) \]
Microfoundation

- Period $t$ composed by $n$ sub-periods.

- First subperiod budget constraint

\[
\sum_{s_{t+1}} q(s_{t+1}|s^t)b(s_{t+1}|s^t) + c(s^t, 1) + k(s^t, 1)
\]

\[
= w(s^t, 1)l(s^t, 1) + (1 + r(s^t, 1))k(s^{t-1}) + b(s_t|s^{t-1})
\]

- Sub-periods 2 to $n$

\[
c(s^t, i) + k(s^t, i)
\]

\[
= w(s^t, i)l(s^t, i) + (1 + r(s^t, i))k(s^t, i - 1), \quad i = 2, ..., n
\]
Microfoundation

▶ Summing up the subperiod’s budget constraints

\[
\sum_{s_{t+1}} q(s_{t+1}|s^t) b(s_{t+1}|s^t) + \sum_{i=1}^{n} c(s^t, i) + k(s^t, n)
\]

\[
= \sum_{i=1}^{n} \left[ w(s^t, i) l(s^t, i) + (r(s^t, i)) k(s^t, i - 1) \right] + k(s^{t-1}) + b(s_t|s^{t-1})
\]

▶ Usual assumption

\[
\sum_{i=1}^{n} [(r(s^t, i)) k(s^t, i - 1)] = r(s^t) k(s^{t-1})
\]

▶ Opposite extreme

\[
\sum_{i=1}^{n} [(r(s^t, i)) k(s^t, i - 1)] = r(s^t) k(s^t)
\]
Euler Equations

- Conventional timing

\[
U_c(s^t) = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) U_c(s^{t+1})(1 + r(s^{t+1}))
\]

- Our timing

\[
U_c(s^t)(1 - r(s^t)) = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) U_c(s^{t+1})
\]
Technology and Feasibility

- **Production**

\[ y(s^t) = A(s^t)g(s^t)^\gamma k(s^t)^\alpha l(s^t)^{1-\alpha} \]

- **Feasibility**

\[ c(s^t) + c^g(s^t) + k(s^t) + g(s^t) = y(s^t) - \delta k(s^t) - \delta^g g(s^t) + k(s^{t-1}) + g(s^{t-1}) \]
Prices and Taxes

- **Before-tax prices**
  \[
  \hat{r}(s^t) = f_k(s^t) - \delta \\
  \hat{w}(s^t) = f_l(s^t)
  \]

- **After-tax prices**
  \[
  r(s^t) = \left[1 - \tau^k(s^t)\right] \hat{r}(s^t) \\
  w(s^t) = \left[1 - \tau^w(s^t)\right] \hat{w}(s^t)
  \]
Ramsey Problem

- **Objective function**

\[
\max_{c, l, k, g_{(t,s)}} \sum_{t,s} \beta^t \pi(s^t) U(c(s^t), l(s^t))
\]

- **Subject to feasibility, and implementability**

\[
\sum_{t,s} \beta^t \pi(s^t) [U_c(s^t)c(s^t) + U_l(s^t)l(s^t)] = A_0
\]

Note: GBC holds.
Counter-cyclical Government Spending

If $\delta^g < \delta$, then $g/k$ is counter-cyclical

For any $s^t$, $f_k(s^t) - \delta = f_g(s^t) - \delta^g$
Labor Income Tax Rate

The tax rate on labor income is given by

\[ \tau^w(s^t) = \frac{\lambda(H_l(s^t) - H_c(s^t))}{1 + \lambda + \lambda H_l(s^t)} \]

where

\[ H_c(s^t) = \frac{U_{c,c}(s^t)c(s^t) + U_{c,l}(s^t)l(s^t)}{U_c(s^t)} \]

\[ H_l(s^t) = \frac{U_{l,c}(s^t)c(s^t) + U_{l,l}(s^t)l(s^t)}{U_l(s^t)} \]
Labor Income Tax Rate

- If $U(c, l) = u(c) + v(l)$ and both $u$ and $v$ are CES then $\tau^w(s^t) = \tau^w(\tilde{s}^t)$
Labor Income Tax Rate

- If $U(c, l) = u(c) + v(l)$ and both $u$ and $v$ are CES then $\tau^w(s^t) = \tau^w(\tilde{s}^t)$

- If $U(c, l) = (1 - \sigma)^{-1} c^{1-\sigma} (1 - l)^{\nu(1-\sigma)}$ and $l(s^t) > l(\tilde{s}^t)$, then $\tau^w(s^t) > \tau^w(\tilde{s}^t)$ if and only if

\[1 + \lambda(1 - \sigma)(1 + \nu) > 0\]
Capital Income Tax Rate

If $U(c, l) = u(c) + v(l)$ and $u$ is CES then the capital income tax rate is zero.
If $U(c, l) = u(c) + v(l)$ and $u$ is CES then the capital income tax rate is zero.

If $U(c, l) = (1 - \sigma)^{-1} c^{1-\sigma} (1 - l)^{\nu(1-\sigma)}$, with $\sigma > 1$ and $\nu > 0$ then

Capital tax rate is likely to be pro-cyclical as long as labor is pro-cyclical.
Summarizing Analytical Results

Table: Assumptions & Results

<table>
<thead>
<tr>
<th></th>
<th>Distortionary $\tau_k$</th>
<th>Countercyclical $g/y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Timing</td>
<td>$\sqrt{}$</td>
<td>$x$</td>
</tr>
<tr>
<td>$\delta_g &lt; \delta_k$</td>
<td>$x$</td>
<td>$\sqrt{}$</td>
</tr>
</tbody>
</table>
Calibration

\[ U(c, l) = \frac{c^{1-\sigma} \eta (1 - l)^{\nu(1-\sigma)}}{1 - \sigma} \]

\[ y = Ag^{\gamma} k^{\alpha} l^{(1-\alpha)} \]

\[ A_t = A_0 \mu^t e^{u_t}, \]

\[ u_t = \rho u_{t-1} + \varepsilon_t. \]

**Table: Moments**

<table>
<thead>
<tr>
<th></th>
<th>r</th>
<th>k/y</th>
<th>g/y</th>
<th>gi/y</th>
<th>gi/G</th>
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<th>lab. share</th>
<th>b/y</th>
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<td>.065</td>
<td>.3</td>
<td>.33</td>
<td>.37</td>
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</table>

**Table: Parameter Values**

<table>
<thead>
<tr>
<th></th>
<th>( \beta )</th>
<th>( \alpha )</th>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>( \delta_g )</th>
<th>( \sigma )</th>
<th>( \eta )</th>
<th>( \nu )</th>
<th>( \lambda )</th>
<th>( \rho )</th>
<th>( \sigma^2(\varepsilon) )</th>
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<tbody>
<tr>
<td>.96</td>
<td>.33</td>
<td>.03</td>
<td>.1</td>
<td>.065</td>
<td>2</td>
<td>.7</td>
<td>1</td>
<td>.35</td>
<td>.95</td>
<td>.006</td>
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</table>
Quantitative Results

In the period of a 2% negative shock:

- Government spending: 0.26 % of GDP
- Labor income tax: -0.40 % of GDP
- Capital income tax: -0.04 % of GDP
- Government debt: -4.32 % of GDP
Ruling out State Contingent Debt

- Budget constraint with state contingent debt

\[ c(s^t) + k(s^t) + \sum_{s^t+1} q(s^t) b(s^t) (s^t) = w(s^t) l(s^t) + r(s^t) k(s^t) + k(s^{t-1}) + b(s^t) \]

- Budget constraint without state contingent debt

\[ c(s^t) + k(s^t) + q(s^t) b(s^t) = w(s^t) l(s^t) + r(s^t) k(s^t) + k(s^{t-1}) + b(s^{t-1}) \]
Ruling out State Contingent Debt

- State by state implementability constraint

\[ c(s^t) + (k(s^t) + b(s^t)) \beta \sum_{s_{t+1} | s^t} \pi(s_{t+1} | s^t) \frac{U_c(s^{t+1})}{U_c(s^t)} \]

\[ = -\frac{U_l(s^t)}{U_c(s^t)} l(s^t) + k(s^{t-1}) + b(s^{t-1}) \]
Ruling out State Contingent Debt

Marcet-Marimon

\[ L = \min_{\lambda_{t,s^t}} \max_{c,l,k,g,b_{(t,s^t)}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) [U(c(s^t), l(s^t), s^t) - \lambda(s^t)(c(s^t) + \frac{U_l(s^t)}{U_c(s^t)}l(s^t) - k(s^{t-1}) - b(s^{t-1}))U_c(s^t) - \lambda(s^{t-1})(k(s^{t-1}) + b(s^{t-1}))U_c(s^t)] \]

s.t.

\[ c(s^t) + c^g(s^t) + k(s^t) + g(s^t) = y(s^t) - \delta k(s^t) - \delta g(s^t) + k(s^{t-1}) + g(s^{t-1}) \]

State Variables: A, k, g, b, \lambda
The Multiplier

- Non negative adjusted martingale

\[ \lambda(s^t) = \frac{\sum_{s_{t+1}|s^t} \pi(s_{t+1}|s^t) U_c(s^{t+1}) \lambda(s^{t+1})}{\sum_{s_{t+1}|s^t} \pi(s_{t+1}|s^t) U_c(s^{t+1})} \]

- With natural limits \( \lambda = 0 \) absorbing state
Bond and $\lambda$ Simulation

![Graph of bond and $\lambda$ simulation](image)

Alessandro Mennuni (Southampton and EUI)

Optimal Fiscal Policy
Bond and $\lambda$ Simulation

- with $g/y = 0.18 \Rightarrow b/y = 0.21$

- with $g/y = 0.21 \Rightarrow b/y = 0.37$
Analytical Results

- If $\delta^g < \delta$, then $g/k$ is counter-cyclical
Quantitative Results with Debt ($\rho = .96$)

In the period of a 2% negative shock:

- Government spending: 0.17% of GDP
- Labor income tax: -0.73% of GDP
  - change in labor tax rate -0.005
- Capital income tax: 3.09% of GDP
  - change in capital tax rate +0.26
- Government debt: -2.86% of GDP
Quantitative Results with Debt ($\rho = .4$)

In the period of a 2% negative shock:

- Government spending: 0.17% of GDP
- Labor income tax revenue: -0.71% of GDP
  - change in labor tax rate -.002
- Capital income tax revenue: 0.17% of GDP
  - change in capital tax rate +.06
- Increase in government debt: 0.52% of GDP
Quantitative Results with Debt ($\rho = .4$)

In the period of a 2% negative shock:

<table>
<thead>
<tr>
<th></th>
<th>g.spending</th>
<th>tax revenues</th>
<th>debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>.85%</td>
<td>-2%</td>
<td>20%</td>
</tr>
<tr>
<td>Model</td>
<td>.17%</td>
<td>-.24%</td>
<td>.52%</td>
</tr>
</tbody>
</table>
Conclusion

- Business cycle model with elastic capital-supply and endogenous gov. infrastructures
- Plausible (and implementable) policy implications
- Study complete and incomplete markets
- Provide a rationale for stimulus packages within the neoclassical framework
- Found an optimal amount of debt/gdp in the long run