Numerical simulations of gravitational singularities

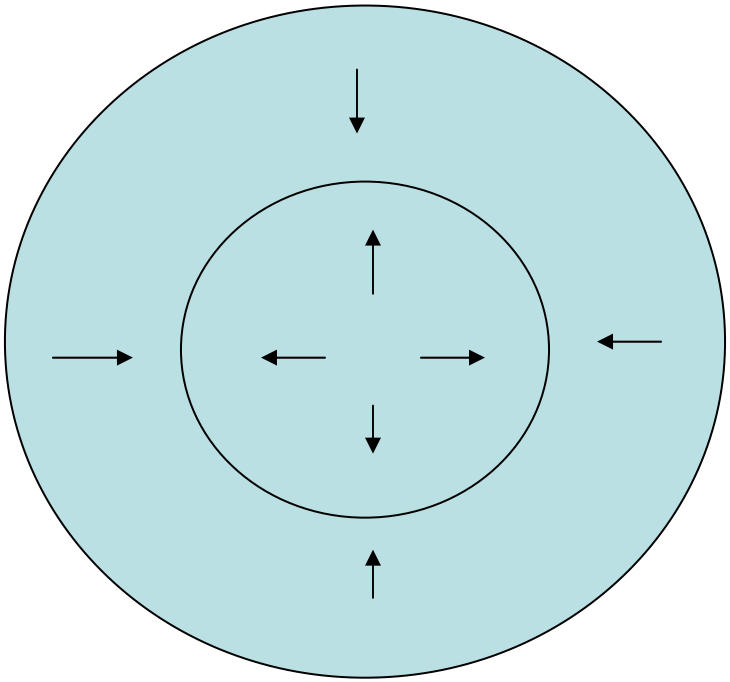
D. Garfinkle, PRL 93, 161101 (2004)

C. Uggla, H. Elst, J. Wainwright, G. Ellis, PRD 68, 103502 (2003)

J. Curtis and D. Garfinkle, gr-qc/0506107 PRD in press

- Gravitational collapse
- Singularity theorems
- BKL conjecture
- Gowdy spacetimes
- General spacetimes

Star in equilibrium between gravity and pressure. Gravity can win and the star can collapse



Singularity theorems

Once a trapped surface forms
(given energy and causality conditions)
Some observer or light ray ends in a
Finite amount of time

Very general circumstances for
Singularity formation

Very little information about
The nature of singularities

Approach to the singularity

As the singularity is approached, some terms in the equations are blowing up

Other terms might be negligible in comparison

Therefore the approach to the singularity might be simple

BKL Conjecture

As the singularity is approached time derivatives become more important than spatial derivatives. At each spatial point the dynamics approaches that of a homogeneous solution.

Is the BKL conjecture correct? Perform numerical simulations and see

Gowdy spacetimes

$$ds^2 = e^{(\lambda+t)/2} (- e^{-2t} dt^2 + dx^2) +$$
$$e^{-t} [e^P (dy+Qdz)^2 +e^{-P} dz^2]$$

P, Q and λ depend only on t and x

The singularity is approached as t goes to infinity

x,y and z are periodic, space is a 3-torus

Einstein field equations

$$P_{tt} - e^{2P} Q_t^2 - e^{-2t} P_{xx} + e^{2(P-t)} Q_x^2 = 0$$

$$Q_{tt} + 2 P_t Q_t - e^{-2t} (Q_{xx} + 2 P_x Q_x) = 0$$

(subscript means coordinate derivative)

Note that spatial derivatives are multiplied by decaying exponentials

Numerical simulations

Centered differences for spatial derivatives

$$f_x=(f_{i+1}-f_{i-1})/(2dx)$$

$$f_{xx}=(f_{i+1}+f_{i-1}-2f_i)/(dx^2)$$

Evolution through ICN

$d_t S=W$

is implemented as

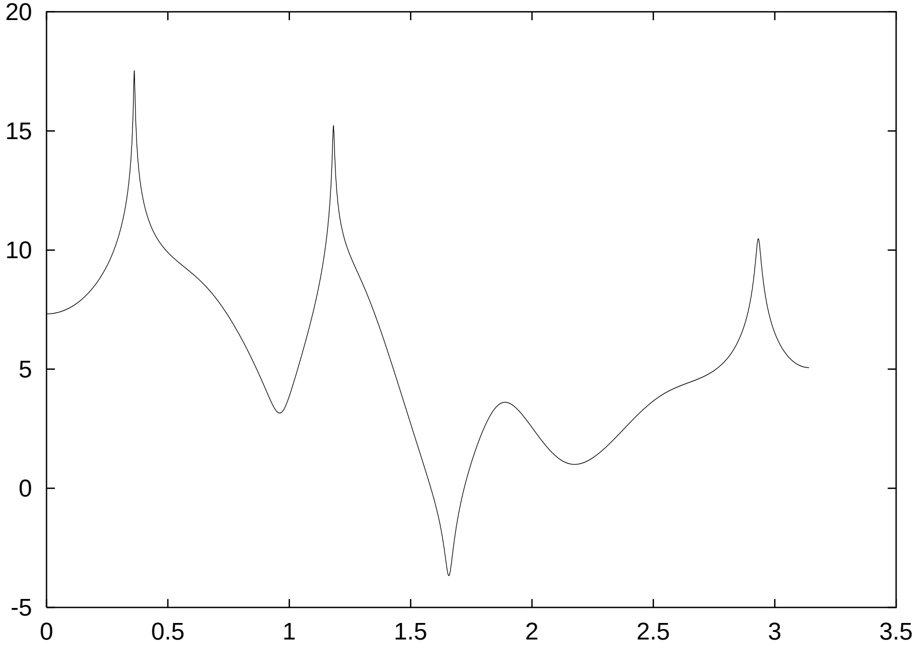
$$S^{n+1}=S^n+(dt/2)(W(S^n)+W(S^{n+1}))$$

As t goes to infinity

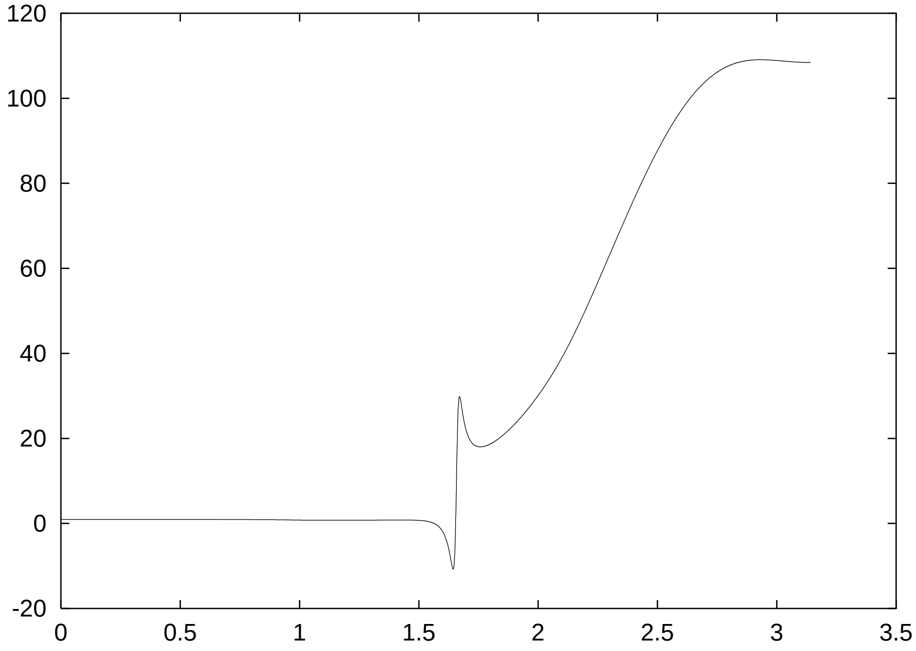
$P = P_0(x)+tv_0(x)$
$Q = Q_0(x)$

But there are spikes

P



Q



General case

Variables are scale invariant commutators of tetrad

$e_0 = N^{-1}\, d_t \qquad e_\alpha = e_\alpha{}^i\, d_i$

$[e_0, e_\alpha] = u_\alpha\, e_0 - (H\delta_\alpha{}^\beta + \sigma_\alpha{}^\beta)\, e_\beta$

$[e_\alpha, e_\beta] = (2\, a_{[\alpha}\delta_{\beta]}{}^\gamma + \varepsilon_{\alpha\beta\delta}\, n^{\delta\gamma})\, e_\gamma$

Scale invariant variables

$\{d_t\ ,\ E_\alpha{}^i\ d_i\} = \{e_0, e_\alpha\}/H$

$\{\Sigma_{\alpha\beta}, A^\alpha, N_{\alpha\beta}\} = \{\sigma_{\alpha\beta}, a^\alpha, n_{\alpha\beta}\}/H$

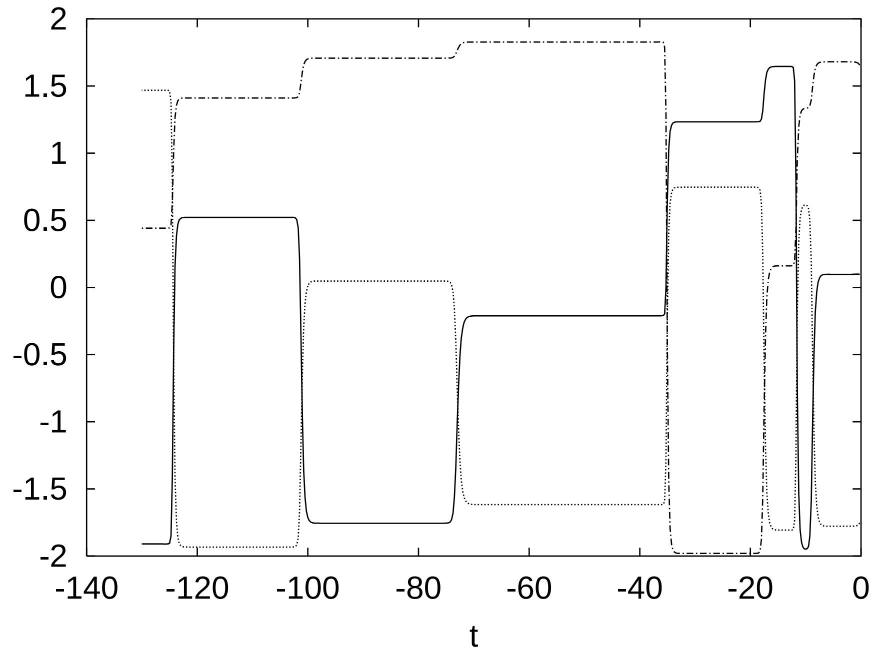
The vacuum Einstein equations become evolution equations and constraint equations for the scale invariant variables.

Results of simulations

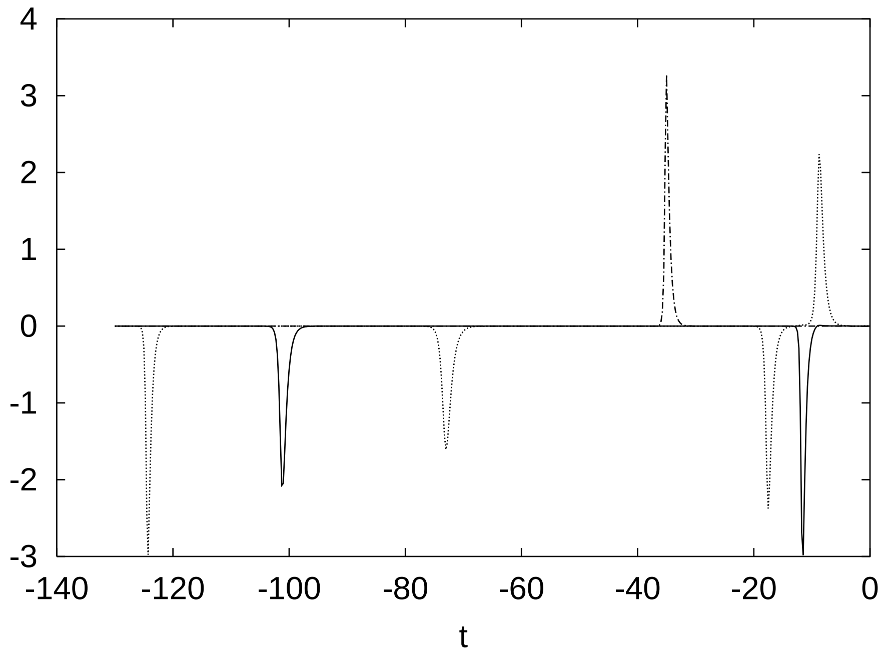
Spatial derivatives become negligible

At each spatial point the dynamics of the scale invariant variables becomes a sequence of “epochs” where the variables are constant, punctuated by short “bounces” where the variables change rapidly

Σ

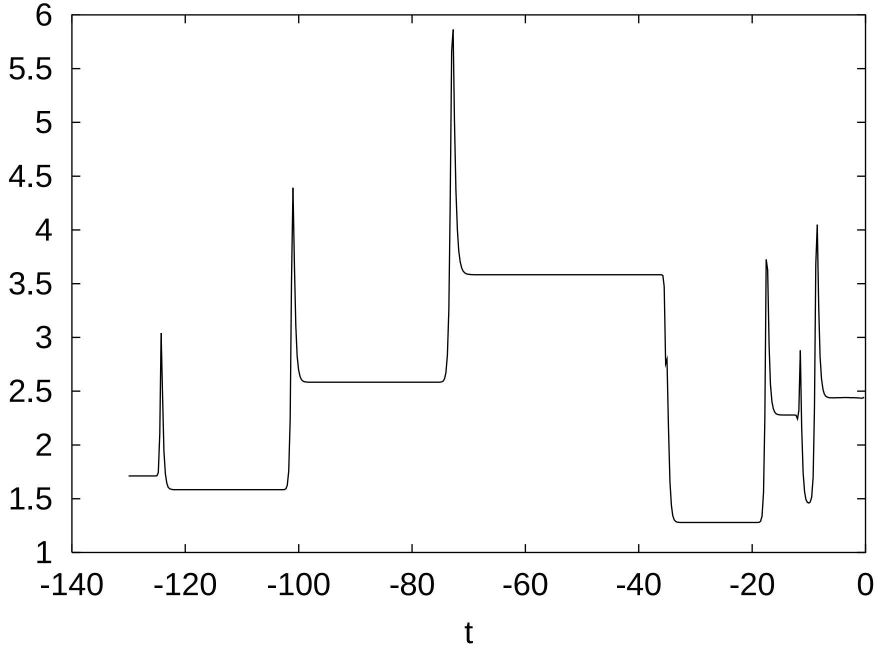


N



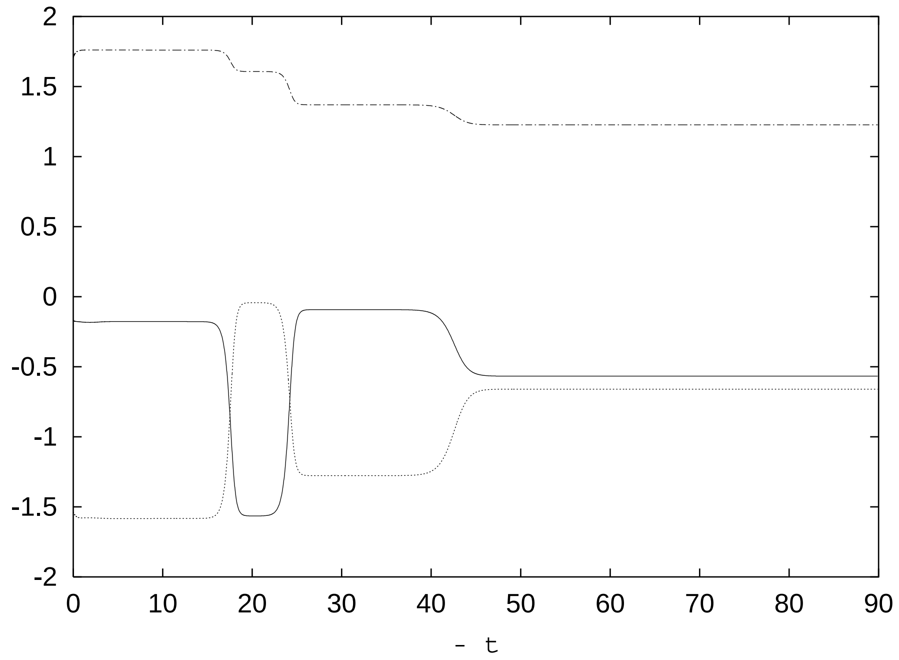
$\Sigma_{\alpha\beta}$ between bounces is characterized by a single number u. BKL conjecture that at a bounce u goes to u-1 if $u>2$ and to $1/(u-1)$ if $u<2$.

u



For stiff fluid the dynamics is similar, but there is a last bounce

Σ for stiff fluid



Conclusion

BKL conjecture appears to be correct

What remains to be done

(1) Non-stiff Matter

(2) Small scale structure

(3) Asymptotically flat case