QUANTUM FIELD THEORY 1

Problem sheet 1

1. Verify that the commutation relations between the creation and annihilation operators

$$\left[a(\mathbf{p}), a^\dagger(\mathbf{p}')\right] = (2\pi)^3 2E_p \delta^3(\mathbf{p}-\mathbf{p}').$$

   leads to the equal time commutation relation between the real scalar field, $\phi(y)$ and its canonical conjugate momentum, $\pi(x)$

$$[\pi(x), \phi(y)]_{x_0=y_0} = -i\delta^3(\mathbf{x}-\mathbf{y}).$$

2. Verify that the expansion of the field $\phi(x)$ in terms of creation and annihilation operators is consistent with the Euler-Lagrange equations of motion for $\phi(x)$.

3. Show that the real scalar field may be expressed in terms of creation and annihilation operators in a manifestly Lorentz invariant form as

$$\phi(x) = \int \frac{d^4 p}{(2\pi)^2} \delta(p^2 - m^2) e^{-ip\cdot x} \left(a(p)\theta(p_0) + a^\dagger(p)\theta(-p_0) \right)$$

4. Calculate the matrix-element

$$\langle 0|\phi(x)|p\rangle,$$

   of the field operator between a vacuum and a single particle state with momentum, **p**.

5. Verify that the Feynman propagator, $\Delta_F(x,y)$ obeys the Green function equation

$$(\Box_x + m^2)\Delta_F(x,y) = -\delta^4(x-y)$$

6. From the definition of the space-like component of the momentum operator, P_i, in terms of the energy-momentum tensor, show that it may be written as

$$P_i = \int d^3\mathbf{x}\pi(x)\partial_i\phi(x).$$

   Hence show, using the equal time canonical commutation relations between $\pi(x)$ and $\phi(x)$ that this operator generates translations, i.e.

$$[P_i(t), \phi(x)]_{t=x_0} = -i\partial_i\phi(x)$$

   Would you expect this commutation relation to hold even when $t \neq x_0$ (give your reasons) ?

Note:

$$\int d^3\mathbf{p} e^{i\mathbf{p}\cdot\mathbf{x}} = (2\pi)^3\delta^3(\mathbf{x})$$

and similarly in 4-dimensions. and

$$\lim_{\varepsilon\to 0} \frac{1}{x+i\varepsilon} = \mathcal{P}\left(\frac{1}{x}\right) - i\pi\delta(x),$$

where $\mathcal{P}$ means the principal part.