# 12 Wavefunction Collapse

We have mentioned a number of times that a system can be in a state in which it does not possess a well defined value of a quantity that we would normally expect a classical system to possess - such a exact position, and exact momentum, or in the case of particles with spin, a well-defined component of spin in *any* given direction. We have also pointed out that when a measurement of a given quantity is made, the state of the system is converted into one in which a given quantity is indeed well-defined. What happens to the wavefunction describing the system when that happens is called "wavefunction collapse" and was first proposed by Heisenberg [24]. Wavefunction collapse takes a system from being a superposition of states, such as a superposition of a spin up state and spin down state to a wavefunction which either describes spin up ($z$-component of spin equal to $+\frac{1}{2}\hbar$) or spin down ($z$-component of spin equal to $-\frac{1}{2}\hbar$) but no longer a superposition of the two.

Wavefunction collapse is not well understood, although a great deal of work has gone into this problem. There is no quantum interaction acting on a system, which effects a transition from a quantum superposition in which a system does not possess well-defined values for certain properties, to a **classical probabilistic state** in which the system does indeed possess well-defined values for these properties albeit with different probabilities. As an example, such a classical probabilistic state describes the situation in which you spin a coin and cover it with your hand without looking at it. We know that the coin is either "heads up" or "tails up" but we do not know which. This is fundamentally different from a quantum superposition of "heads up" and "tails up" in which the coin would not be either "heads up" or "tails up" but both at the same time.

As we have explained above, one can choose to measure one property or another. For example one can choose to measure accurately the position of a particle, in which case the environment includes an apparatus for the accurate measurement of position and the measurement induces wavefunction collapse into a state of well-defined position, or one can choose to measure accurately the momentum of a particle in which case the environment includes an apparatus for the accurate measurement of momentum and the measurement induces wavefunction collapse into a state of well defined momentum (but we cannot do both simultaneously). For an electron (or other particle with spin) one can measure the component of spin in the $z$-direction, in which case the environment includes a Stern-Gerlach apparatus with its magnetic field in the $z$-direction and after passing through the apparatus the wavefunction for the particle collapses either into one describing a spin up particle or into one describing a particle spin down particle. Alternatively we may choose to measure the component of spin in the $x$-direction in which case the environment includes a Stern-Gerlach apparatus with its magnetic field in the $x$-direction and after passing through the apparatus the wavefunction for the particle collapses either into one describing a particle whose $x$-component of spin is $+\frac{1}{2}\hbar$ or into one describing a particle whose component of spin in the $x$-direction is $-\frac{1}{2}\hbar$. However, we cannot perform these two experiments simultaneously. At the end of the last section, we explained what happens when these two experiments are performed sequentially - the interaction of the spin-one half particle with the first apparatus affects is spin state and consequently alters the results of the second experiment.

It is tempting and comforting to interpret this very strange behaviour of sub-microscopic systems in terms of so-called “hidden variables”. This is the proposition that a particle does possess both position and momentum or a component of spin both in the $z$-direction and the $x$-direction, but that the experimentalist does not know what the values of any of these quantities are until (s)he performs an experiment to measure it. As we shall see later, this is not the case - it was shown by John Bell that this postulate gives results which are not consistent with the predictions of Quantum Mechanics. We have to accept the fact that at the sub-microscopic level a system simply does not possess these quantities and that the act of measurement - or the interaction of the system with a macroscopic environment - changes the wavefunction. Quantum Mechanics permits one to determine the probability that a measurement will yield a given result (e.g. an electron which has passed through a Stern-Gerlach apparatus with its magnetic field in the $x$-direction and subsequently through a Stern-Gerlach apparatus with its magnetic field in the z-direction has a probability of $\frac{1}{2}$ to give a result $+\frac{1}{2}\hbar$ and a probability of $\frac{1}{2}$ to give a result $-\frac{1}{2}\hbar$). This means that if we perform the experiment one hundred times, we expect that within statistical errors, we will get on average $+\frac{1}{2}\hbar$ fifty times, and $-\frac{1}{2}\hbar$ fifty times. However we cannot predict what the result will be for any one measurement - very simply because before the measurement is effected, the particle does not possess a component of spin in the $z$-direction.

## 12.1 Decoherence

Wavefunction collapse is the mechanism in which a system, by interacting with its environment (which includes a measuring apparatus), is transformed from a superposition of states into a definite (classical) state with a well-defined value of a given measurable quantity. This mechanism is not understood, although significant progress has been made in recent years. It is the property of a wave that leads to interference which gives rise to the strange phenomenon that permits a quantum system to simultaneously possess several (often an infinite number) of values for a particular measurable property. This property is called coherence and is related to the fact that the phases of two waves that interfere have to be “locked” so that there is a constant phase difference between them which generates this interference. The process of unlocking this phase so that the phase difference of the two waves is randomized is known as **“decoherence”** and this is now believed to be the mechanism that leads to wavefunction collapse. When decoherence occurs, the quantum property of superposition of states, with many different possible values for a given quantity, is lost - the wavefunction has collapsed into the wavefunction for a state in which that quantity is indeed well-defined and has a unique value.

Monochromatic (single wavelength) light, or other monochromatic electromagnetic radiation, or particles with the same momentum, used in the Young’s double slit experiment, can interfere (giving rise to interference patterns) because the incident wave is coherent, that is to say the wavefront at the double slits has peaks and troughs at the same time (or at least with a fixed time interval between them) - they are phase locked. If something happens to randomize the phases of the wave incident on the two slits, the interference is lost (decoherence has occurred). The attempt to determine through which slit a photon or electron has passed is an example of a disturbance of the system which leads to decoherence and consequently loss of the interference pattern.

Decoherence was introduced in Quantum Physics by Dieter Zeh in 1970 [28] and developed in the 1980s. The idea is that when a sub-microscopic system which is in a superposition of states interacts with a macroscopic environment, the evolution in time, determined by Quantum Mechanics, will change the entire system (the original sub-microscopic system plus the environment) into a different superposition of states of the entire system, i.e the sub-microscopic system plus the environment. Subsequently a measurement which selects a particular state of the environment automatically selects a particular state of the sub-microscopic system. As an example, suppose the sub-microscopic system is the spin of an electron which is in a state which is a superposition of spin-up and spin-down,

$$a\Psi_{\uparrow} + b\Psi_{\downarrow},$$

and the environment is a detector which has two states which we call “red” and “green”. If the environment is in state “red” then a light shines red and if it is the state “green” the light shines green. Now suppose that the sub-microscopic system (the electron spin) interacts with the detector in such a way that if the electron has spin-up the detector is transformed into the state “red” and if the electron is spin-down the detector is transformed into the state “green”. After this interaction the combined wavefunction for the electron-spin and the detector is

$$a\Psi_{\uparrow}\Psi_{\mathrm{red}} + b\Psi_{\downarrow}\Psi_{\mathrm{green}}.$$

This is still a superposition of states, but it is a superposition of states of a system consisting of the electron-spin plus the detector.

Observation of the detector (observing whether the light is red or green) then determines if the electron is spin-up or spin-down. The probability that the result is found to be spin-up, i.e the probability that the light is red is $a^2$ - and similarly the probability that the result is found to be spin-down, i.e the probability that the light is green is $b^2$.

The rate at which this decoherence takes place depends on a number of properties of the environment - such as its temperature - but estimates of the decoherence time have suggested that for macroscopic systems, the time-scale for such a decoherence process is too short for any interference to occur (two waves can only interfere if they remain coherent for a period which is much longer than the period of the waves) which is why we do not observe quantum behaviour in macroscopic objects.

Decoherence does not lead directly to wavefunction collapse. It does not directly explain why a measurement returns a particular value of a property of a quantum system, despite the fac that the system was initially in a state for which that value was not defined. It does, however, provide us with a mechanism which could explain how wavefunction collapse is observed.