7 Heisenberg's Uncertainty Principle

In this section we again consider a particle moving in one dimension only. We have seen that a wavepacket of matter waves describes a particle which is not located at a particular point. Note that this is not the same as saying that its position is unknown, which implies - falsely - that its exact position exists but that the experimentalist does not know what it is. The particle simply does not have a well-defined position until a measurement is made at which point the wavefunction of the particle is changed into one representing a particle of well-defined position (a very narrow wavepacket). Instead we only know what the probability is that the result of such a measurement of its position will yield a given result. This probability is very small, except in a region of width x, and so we identify this range, Δx, as the "uncertainty" in the position of the particle.

We have seen when discussing wavepackets, that a wavepacket of a given width is constructed from a superposition of waves with wavelength, λ, and amplitude $A(\lambda)$. The amplitude $A(\lambda)$ has an interpretation which is analogous to the interpretation of the amplitude, $|\Psi(x,t)|$, of the wave in the wavepacket, namely that its square, $|A(\lambda)|^2\, d\lambda$, is the probability that a measurement of the wavelength of the particle will yield a value between λ and $\lambda + d\lambda$. By the de Broglie wave relation, the wavelength measurement is equivalent to a measurement of the momentum. Since the momentum is actually inversely proportional to λ (i.e. proportional to $1/\lambda$),[9] it is more convenient to plot $A(\lambda)$ as a function of $1/\lambda$. We have a distribution whose square tells us how likely it is that a measurement of $1/\lambda$, will yield a given result and we can see that such a probability is small except over a small range of $1/\lambda$. The particle does not possess a single wavelength, but is a superposition of waves of different wavelengths with different amplitudes. The range over which this amplitude is not small represents the uncertainty in $1/\lambda$.

We recall that the width of the wavepacket, Δx, is inverse to the width of the distribution of $A(\lambda)$ whose width is $\Delta\,(1/\lambda)$, i.e for a wavepacket with large width, Δx, representing a particle with a large uncertainty in its position, the distribution $A(\lambda)$ is narrow implying a small uncertainty in the quantity $1/\lambda$. On the other hand for a wavepacket with small width, Δx,, representing a particle with a small uncertainty in its position, the distribution $A(\lambda)$ is wide implying a large uncertainty in the quantity $1/\lambda$.

This is demonstrated in Fig. 32. The graphs on the left are wavepackets. The upper wavepacket has a width of about 0.8 nm whereas the lower graph is a narrower wavepacket with a width of 0.4 nm. The graphs on the right are the distributions of the amplitude at wavelength, λ, plotted against $1/\lambda$. The upper distribution has a width of about 1.2 per nm whereas the lower distribution has a width of about 2.4 per nm. In both cases the product of the width in position, x, and the width in the inverse wavelength, $\Delta\,(1/\lambda)$ is approximately one.

From the de Broglie wave relation, the uncertainty, Δp, in the momentum is h (Planck's

[9] The inverse of the wavelength of a wave is called its **wavenumber** and is often denoted by k but we will maintain the notation $1/\lambda$.

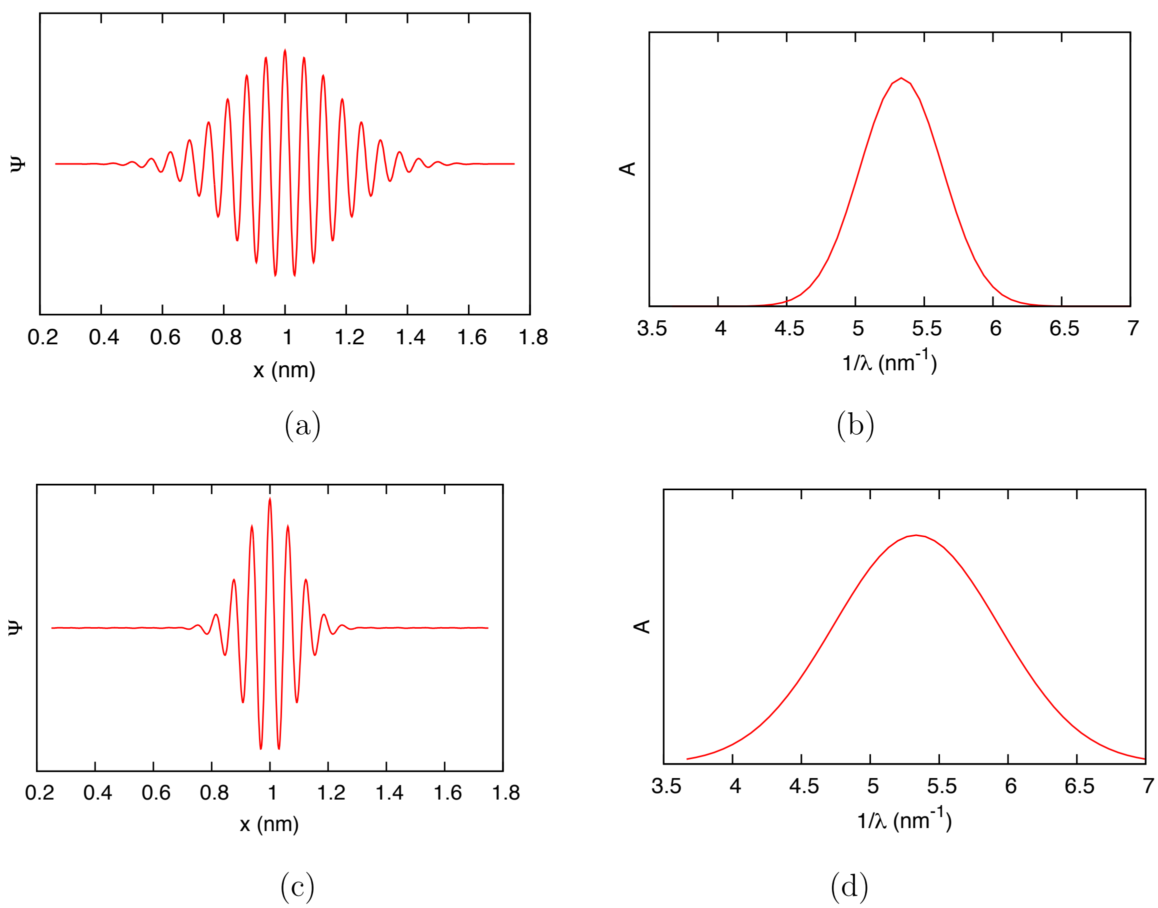

Figure 32: (a): a wavepacket with a width of 0.8 nm; (b): the corresponding distribution in amplitude of wavelength λ (plotted against $1/\lambda$), with a width of 1.2 per nm. (c): a wavepacket with a narrower width of 0.4 nm; (d): the corresponding distribution in amplitude of wavelength λ (plotted against $1/\lambda$), with a broader width of 2.4 per nm.

constant) times $\Delta\,(1/\lambda)$, so that if the product of Δx and $\Delta\,(1/\lambda)$ is approximately one, then the product of Δx and Δp is approximately h and we have the Heisenberg uncertainty relation [24]

$$\Delta x \times \Delta p \ \approx h \tag{7.1}$$

The particle, whose matter wave is a wavepacket, does not have a well-defined (i.e. exact localized) position until its position is measured to within a given accuracy - at which point the wavefunction changes into a narrow wavepacket. Similarly, it is also the case that the particle whose wavefunction is described by that particular wavepacket, does not possess a well-defined momentum (since it is constructed out of a range of wavelengths) until its momentum is measured to a given accuracy,[10] at which point the wavefunction changes to a much broader wavepacket but with a much narrower distribution in wavelength and hence a narrower distribution in momentum. One can measure the position accurately or the momentum accurately but not both. The product of the uncertainty, Δx, in the measurement of position (the width of the wavepacket) and the uncertainty, and Δp, in the measurement of momentum must be larger than Planck's constant, h, in keeping with the uncertainty relation.

To give you some idea of why we do not experience this uncertainty in our everyday observations, suppose we were looking under a microscope at a human blood cell whose mass is of order of a millionth of a microgram and the uncertainty in position is around one micron (a millionth of a meter - about the smallest resolution possible with visible light) then the uncertainty in its velocity due to Heisenberg's uncertainty relation would be of the order of one micron per year !!

[10]The momentum of a particle is proportional to its velocity (neglecting relativistic corrections) so that a measurement of a particles momentum is equivalent to a measurement of its velocity.