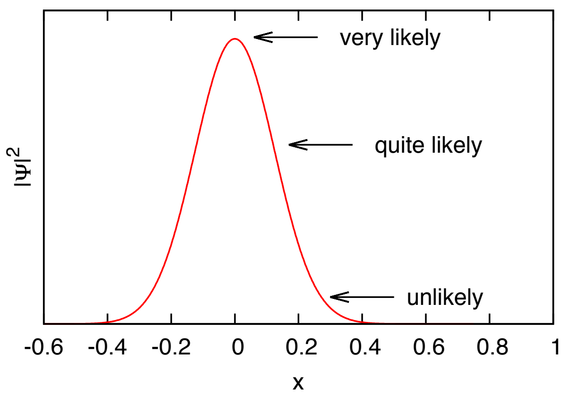

Figure 30: Graph of $|\Psi|^2$ against x, showing the relative probability of finding the particle at different points

5 What is a Matter (de Broglie) Wave?

We have discussed the wavelength of "matter waves" or "de Broglie waves", but we haven't yet discussed what these are waves of i.e. what is the disturbance that is oscillating in analogy with a wave on a violin string where the disturbance is the displacement of the string from its equilibrium position that is oscillating and travelling along the string.

The interpretation of these matter waves was given by Max Born [23]. The actual disturbance does not have a direct physical interpretation, but it does have an indirect meaning. The matter wave disturbance at a position $\mathbf{x}$, and time t, is described by a function of $\mathbf{x}$ and t called the "wavefunction" and is usually written as $\Psi(\mathbf{x}, t)$.[7] The amplitude of the wave oscillation as that point and time is written as $|\Psi(\mathbf{x}, t)|$. The square of this amplitude is the probability density for finding the particle at the point, $\mathbf{x}$. This means that $|\Psi(\mathbf{x}, t)|^2 \, dV(\mathbf{x})$ is the probability that at time t the particle is in the small volume, $dV(\mathbf{x})$ centred at the point $\mathbf{x}$ (i.e. $|\Psi(\mathbf{x}, t)|^2$ is the probability density - probability per unit volume - for the position of the particles at the point $\mathbf{x}$). This is easier to understand if we restrict ourselves to one dimension so that a point in space is denoted by a single variable, x. In this case $|\Psi(x, t)|^2 \, dx$ is the probability that the particle is somewhere between x and $x + dx$, where dx is a small interval in x.

A plot of the square of the amplitude of the oscillations at a given time, t, is shown as a function of (one-dimensional) x in Fig. 30. The peak (arbitrarily set at $x = 0$) is at the position where the particle is very likely to be. If we move out either side of this to a distance 0.2 units we find that it is quite likely that the particle will be found at either of these points, whereas if we go out further than 0.4 units either side of $x = 0$ then it is very unlikely that the particle will be there. For the rest of this section we confine our consideration to motion

[7] $\mathbf{x}$ is written in bold-face since it stands for a vector in three-dimensional space, whose components are the three coordinates of the point at which the amplitude is being considered.

in one direction only.

Thus we see that for a matter wave which is a wavepacket of a given width, the position of the particle is not well defined - the particle does not actually possess a single position - it can be *anywhere* within the wavepacket. This is very difficult for us to "understand" in the sense of visualization, although we have no difficulty when we think of the particle in terms of its matter wave structure. The particle's position is only defined *after* its position has been measured. Before such a measurement has been effected, we can only predict the probability that such a measurement will yield a given result. With this interpretation of a wavefunction, we see the analogy between a wavepacket, which exists everywhere but has an amplitude which is negligible far away from the point of maximum amplitude, and a particle which exists everywhere, having a maximum probability to be at a certain point but having a probability to be found at a different point, which decreases as that point moves away from the location of the maximum and becoming negligible very far away from it. The act of measuring the position of the particle, changes the wavepacket in such a way that its position is then determined up to the accuracy of the experimental measurement. The process of the change in the wavefunction resulting from a measurement is called **"wavefunction collapse"** . It is not well-understood - we will discuss this in more detail later.