Estimation

-- order-of-magnitude estimates

-- quick-and-dirty calculation

-- approximation

-- sanity checks

-- dimensional analysis

-- guesstimates (or even plain guessing)

-- simplification

When to estimate?

-- ALWAYS (any detailed calculation should really be backed up by an order of magnitude estimate, if only as a sanity check)

-- often only a rough answer is needed (e.g. no point doing a 1% integration in a problem where some parameter may only be known to 10%)

-- often a *quick* answer is desirable (at least initially)

-- is a particular effect worth worrying about in detail at all? (don't agonize until you've established it'll be worth it!)

Why estimate?

The idea is generally to concentrate on (and understand) the underlying concepts, rather than the detailed maths. i.e. distill the important bits and filter out the irritating minutiae...

===> ESTIMATION IS PROBABLY MORE IMPORTANT DAY TO DAY THAN DETAILED MATHS!

A rough estimate is pretty much guaranteed to be correct with a factor of 10 (and when it's not, this is usually telling you something important -- e.g. a key physical effect has been ignored!)

Detailed estimates are designed to be highly precise, but are complex enough so it's easy for them to be completely wrong (it's common to get answers to exam question given to 10 decimal places but off by 10 orders of magnitudes!)

GENERAL RULES:

-- Divide and conquer!

-- Guess! Both initially and whenever you need to! Use your gut to check if guesses are sensible.

-- Be an optimist:

-- Assume whatever you need to get going and hope for the best

-- i.e. assume wildly (but not blindly!). e.g. consider a spherical cow...

-- Punt! In a first go, ignore any physical effect that would make life too difficult; related to assume wildly -- just make sure to remember what you've punted on and see if you can/need to return to it later. But the key point is to explore the problem, and if you've got to ignore something that would stop you cold, ignore it (typical example: might ignore air resistance in a problem initially and then see if we can really get away with that)

-- Don't be perfectionist -- lower your standards

-- if you can't solve the whole problem, solve whatever bit of it you can --> you might end up seeing the way to the full solution

-- don't be precise -- the whole point of estimation is to get a rough, but reliable answer, so don't waste your time doing maths to 1% if you've made approximations that are wrong by factors of 10!

-- Cross-check: try getting to the solution in several ways and make sure they agree; use related facts to help you make an estimate (e.g. insurance in van example).

-- Sanity check: Often the point of estimation is to provide a sanity check for a more detailed calculation. But don't forget to sanity check the sanity check - if your rough estimate is out by x1000 from your gut-instinct initial guess, don't blindly assume that your gut was wrong. The beauty of the estimate is that it's so simply that you should be able to see where/why your gut was wrong (*if* it was wrong -- sometimes you'll find you screwed up your estimate, i.e. your gut was right!)

-- Use symbols: freely define symbols for quantities that arise; sometimes even useful to allow for dimensionless fudge factors (e.g. efficiencies) that account for physical effects you've punted on. These can act as convenient reminders that you did punt...

-- Work in symbols until the very end; if you've got to change some parameter in the problem, you don't want to have to repeat tons of algebra, you just want to have a final equation that shows how things depend on the parameter. Also, some things you might think would matter might cancel out (so no point wasting time trying to estimate those things).

-- Make sure you know and keep track of units!

EXAMPLES

[Below, I'll sometimes be a little sloppy, but that's because I know where I'm going and I'm trying to illustrate *thought processes*, not algebra -- don't use this as an excuse to be sloppy in your own work]

How much money is in a Securicor van?
=====================================

First guess: 1 pound? 10? 100? 1000?
1e9? 1e8? 1e7?

Probably O(1e6) [i.e. on the order of 1e6]

Volume of Storage Area: 2m x 2m x 2m = 10 m^3 = 1e7 cm^3

Volume of a typical bank note: 5cm x 15 cm x 1e-2 cm = 1 cm^3

(Width of bill is guess; can be estimated from stack of bills, or by
folding a bill)

Paper is made from wood, which just floats. Water has density

1000 kg/m^3 = 1 g / cc

So each note weighs ~ 1 g.

So can fit about 1e7 notes into van if completely full. Mass would be 1e7 g = 1e4 kg = 10 tons.

Probably too much for a van. Also have to have guard in the back, and space for coins, equipment etc, so actual volume devoted to bills is probably 0.1 of storage. Then ~ 1 ton (reasonableish) and 1e6 bills. Typical denomination is ~ 10 pounds (most common). So 1e7 pounds = 10 million. This is with car completely packed (i.e. max specification), so typically probably have a lot less than that, perhaps another 0.1, so sth like 1 million.

Sanity check: By googling I found a report in a paper talking about a theft worth 5e5 pounds. So not too bad...

What would be another good way of estimating typical/max contents?
Insurance!

[Turns out vans are apparently not insured for cargos worth *less* 1e5!
Could not find max amount (don't want to publicise!), so probably 1e6 - 1e7 is in right ballpark!]

===> Think laterally, and always try to cross-check your answers.

Rubber Tyre
===========

How much rubber is lost each year from the tyres of cars in Southampton?

Mass of rubber lost = M_tot [kg/yr]

Total mileage of cars in Southampton = L_tot [km/yr]

Mass lost per km = m [kg/km]

Then:

M_tot = L_tot * m

Need to estimate L_tot and m

Number of people ~ 3e5

Number of cars ~ 1 per family ~ 0.3 * N_people ~ 1e5

Typical car does 1e4 km/yr

So all the cars together do 1e9 km/yr. (Not all of that in Soton, but assume equal flux in/out.)

Now each car has 4 tyres, and each tyre loses about 2 cm of tread over its life, which is sth like 5e4km. A tyre has radius 30 cm and width 10 cm, so the total volume of rubber lost over 5e4 km is about

2 * pi * 0.3 * 0.1 * 0.02 = 6e0 * 3e-1 * 2e-2 = 4e-3 m^3

Now density of rubber is about 1000 kg / m^3 = 1e3 kg/m^3, so mass lost per 5e4 km is 4 kg, i.e. mass lost per km is about 1e-4 kg = 0.1 g. So total mass lost is 1e-4 kg/km * 1e9 km/yr = 1e5 kg/yr = 100 tons!

NOTE: Easier way to do this estimate is just by asking how *often* one typically changes tyres (simpler than estimating mileage and tyre life in miles)

Sediment!!

Maximum Height of Mountains
==========================

What is the maximum height of a mountain on Earth? How about Mars?

First guess: Earth should be about 10 km or so (i.e. Everest). Mars???

What limits the size of mountains?

When pressure at based gets so large as to break chemical bonds of rock, then adding more material at top destroys the base, so mountain can't grow...

Pressure --> Units --> Force / Area = Energy / Length * 1 / Area = Energy / Volume ==> Pressure is a measure of energy density!

OK, so let's think of breaking chemical bonds in terms of energy density: what happens when you break lots of bonds: melting

So probably sth to do with latent heat of melting

For volcanic rock, L = 5e5 J/kg

The pressure (=energy density) at the bottom of the mountain is M*g /A (where M is the mass of the mountain and A is the area). Here,

M = A * H * rho

E [J/m^3] = H * rho * g

Now when this energy density (energy / unit volume) becomes comparable to the energy required to melt the mass of rock per unit volume, the mountain will sink. Now the energy require to melt 1m^3 of rock is L [J/kg] * rho [kg/m^3]. So the critical height is determined by

H * rho * g = L * rho

H = L / g

Does this make sense? Well, g is acceleration [L/T^2] but can convert units --> Energy = mgh --> J = kg*acceleration*m, so acceleration = J/kg/m. No L = J/kg, so L/g is indeed length. Does it work? Well,

H = 5e5 / 10 = 5e4 m = 50 km.

Too big, but right ballpark...

Why overestimate? Probably because we don't really have to *melt* all this rock -- a few stress fractures would do it...

Pole Vaulting
=============

How high can a pole vaulter possible jump?

Convert all of KE in to height!

$KE = 1/2\ m\ v^2$

$PE = m\ g\ h$

--> $h = v^2 / (2\ g)$

But this ignores center of mass of vaulter being above ground. So it's really delta $h = v^2/2\ g$. Say plus 1 m for CoM.

What have we neglected?

-- Some energy for going forward over bar
-- Fosbury flop means CoM can actually go *under* bar (by a bit)
-- dissipation in pole and between pole and ground
-- air drag

Perfume
=======

How long for perfume to diffuse across room?

Diffusion coefficients have units of m^2/s -- why?

What matters? Mean free path and speed

D ~ lambda * v

For random walk, number of walk segments is

N = R^2 / lambda^2

Total distance travelled is L = N * lambda

This takes time

T = L / v = R^2 / (v * lambda) = R^2 / D

v from ===>   m v^2 ~ kT

so v = root(kT/m)

This gives sth like 500 m/s

Air: lambda ~ 1e-7m = 0.1 microns.

(get this from size of molecules [a few Angstrom] and number density of molecules at STP] --> consider cross section of molecule)

We then get D ~ 5e2 m/s * 1e-7 m = 5e-5 m^2 /s = 5e-1 cm^2/s (cf.book)

Then T ~ long!

Counterintuitive (seems much too long) -- what's going on?

Is diffusion really what's transporting the perfume?

Written by Christian Knigge, reformatted and edited by BSL, 2007.