Physics Skills February 2006

Work, Energy and Forces (1)

In each of the following problems, think (i) whether you should work directly with forces, (ii) whether it is simpler to consider the work done by forces, or (iii) whether energy is conserved and you can work directly in terms of energy. See if you can solve the problems in more than one way.

Example: Car Accelerating A car accelerates from rest to $50\,\mathrm{km\,h^{-1}}$ in $1.5\,\mathrm{s}$. How long would it take to accelerate from rest to $100\,\mathrm{km\,h^{-1}}$, assuming the power of the engine to be independent of speed and ignoring friction?

Example: How Far Does a Box Slide? You are pushing a $10.0\,\mathrm{kg}$ box across a wood-laminate floor at a steady speed of $2.0\,\mathrm{m\,s^{-1}}$. The coefficient of kinetic friction is $\mu_\mathrm{k} = 0.2$.

How much force do you exert on the box?

If you stop pushing, how far will the box slide before coming to rest?

Example: Mass on a String A mass m is revolving in a vertical circle on the end of a string of length a. The string's length remains fixed.

Is the total energy of the mass m conserved during the motion? Justify your answer (think about the tension in the string).

What is the value of the string tension when the mass is at the top of the circle if circular motion is *just* possible?

In the case that circular motion is just possible:

(a) find the speed of the mass at the top of the circle

(b) show that the tension in the string when the mass is at height h above the centre of the circle is $3mg(1-h/a)$

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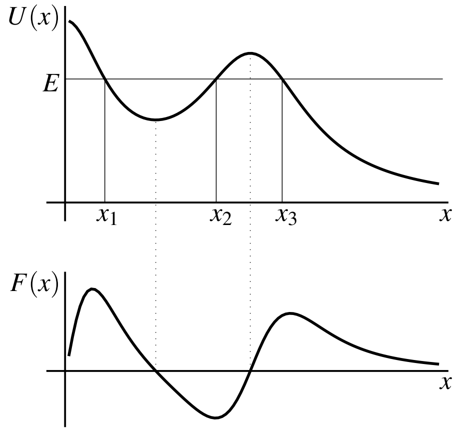
Work, Energy and Forces (2)

Example: Motion in a Given Potential



The figure shows a potential energy $U(x)$ and the corresponding force $F(x)$. A particle of mass m moves with total energy E. Describe the motion of the particle if it is released from rest at points x_1, or x_3 (E, x_1 and x_3 are marked on the figure).

If the particle is released from rest at x_1, show that the period of the subsequent oscillatory motion is given by

$$T = 2\int_{x_1}^{x_2} \frac{dx}{\sqrt{2\big(E - U(x)\big)/m}}.$$

Example: Large Amplitude Pendulum A simple pendulum of length ℓ swings such that it makes a maximum angle of θ_{max} with the vertical. Show that the period of the pendulum is given by

$$T = 2\sqrt{\frac{2\ell}{g}} \int_0^{\theta_{\text{max}}} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_{\text{max}}}}.$$

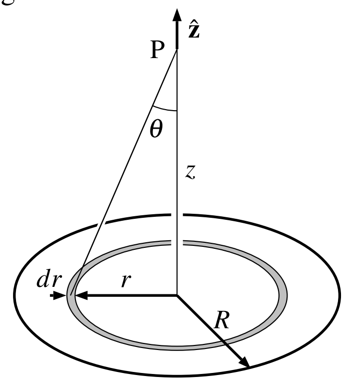
If θ_{max} is small, show that this reduces to the usual result $T = 2\pi\sqrt{\ell/g}$.

Example: Electrostatic Potential A point charge $Q = 4.60\,\mu\text{C}$ is fixed at the origin. A second point charge $q = 1.20\,\mu\text{C}$ with mass $2.80 \times 10^{-4}\,\text{kg}$ is placed at $0.250\,\text{m}\,\hat{\mathbf{x}}$.

(a) What is the potential energy U of the pair of charges? Take U to be zero when the charges have infinite separation.

(b) The second charge, q, is released from rest. What is its speed when its distance from the origin is $0.500\,\text{m}$?

Example: Electric Field Due to a Charged Disk A circular disk of radius R has a uniform surface charge density σ. Find the electric field at a point P, at distance z from the disk along its central axis. You can calculate the field (force per unit test charge) directly, or calculate the potential (potential energy per unit test charge) and differentiate the answer to get the the field.



Example: Is Energy Conserved? A particle slides down a hill of height h without friction. At the top it has potential energy mgh and zero kinetic energy. At the bottom it has kinetic energy $mv^2/2 = mgh$ and no potential energy.

Now consider this in the reference frame of an observer moving with velocity v in the same direction as the particle (at the bottom of the hill). At the top, the particle has *both* potential and kinetic energy, but at the bottom it has neither kinetic nor potential energy. Where did the energy go?