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Workshop problem sheet : Special Relativity

This workshop sheet will walk you through all the significant steps in the relativity section of PHYS1015/16, and can be used to form a basis for revision of this section of the course. It therefore includes a significant amount of bookwork (just as the exam questions will). At the same time, the idea is to emphasise important elements of answering physics problems in general and in Relativity in particular.

In general it is important to define quantities, draw a diagram, state your assumptions, and work with algebra until the end of the problem. (*N.B.* You can then and always should check your answer is dimensionally correct. If it is not you can go backwards and readily find the point where you made the error.)

It seems a number of you find using extended algebra difficult. In particular, please note that it is not always possible to see your way to the answer using physical insight alone. Often you just have to do the algebra and see where it leads, first. In real problems you often need to let the mathematics answer the problem for you, this way. A physics problem always divides into a statement of the physics of the situation (the particular conditions and the laws that apply) and then after that, a degree of manipulation (which is just mathematics). Such problems are more advanced than the ones met at school but are at the heart of what physics is really about: "the language of physics is mathematics".

This approach –letting the algebra answer the question for you– is particularly prevalent in Relativity, and the following questions are designed to emphasise this in particular.

Finally, these questions are designed to remind you about the particular 'rules' that are useful for solving relativity problems. Firstly, identify all the *events* (t,x) and be sure to be clear in what frame they take these values. For energy/momentum problems it can often be important to consider their values in a useful frame (at rest with respect to one of the particles for example). Secondly, suspend your intuition and instead apply the rules carefully (see the comments above). After getting the answer you can ponder what it means!

A. Consider an inertial reference frame S' travelling at velocity v along the x direction, with respect to an inertial reference frame S. The inertial reference frames are set so that the event $t = x = y = z = 0$ in S is described in S' also as $t' = x' = y' = z' = 0$.

1. If an event takes place at a general (t,x,y,z) in S, at what time and coordinates does the event take place in S'?
2. Consider an event in S' that takes place at (t',x',y',z'). Using the symmetry of the situation compared to part 1, write down the formula that relates the time and position in S for this event to the values (t',x',y',z').

B. 1. What is meant by a proper time interval?

2. Define two events separated by a proper time interval τ in S, and show that in S' the time separation is $\tau' = \gamma\tau$. (*N.B.* It is always possible to make life easy for yourself and define one of the events to happen at $x = t = 0$.)

C. 1. What is meant by a proper length?

2. A rod has proper length L. By defining two events, show that when it is moving in the direction of its length, it has length $L' = L/\gamma$.

D. A rocket travels from station A to station B at constant velocity v. According to the station-masters, the rocket takes a time T to make the trip.

1. How far apart is B from A, according to the station-masters?
2. How far apart is B from A, according to the passengers?
3. How long does the rocket take as far as the passengers are concerned? Is this time longer, shorter or the same as T?
4. Using your last two answers, how fast is the rocket (relative to the stations) according to the passengers?

E. Show that two events that are simultaneous in frame S', are separated in time in frame S by an amount proportional to Δx. Which event happens first in frame S?

F. This is a question taken from Tipler (question 19, p1302) about the full Lorentz transformations and simultaneity. Define t to be zero at the first explosion and redefine the x coordinate so that $x = 0$ at the first explosion. Then, by ensuring that you leave all quantities related to the second explosion algebraic until the end, and interpreting your answer, you should be able to see that there is actually a one line way of answering this problem. If you can see how to answer it in one line, provide a derivation based on the full Lorentz transformations anyway!

Observers in a reference frame S see an explosion located at $x_1 = 480$ m. A second explosion occurs 5 μs later at $x_2 = 1200$ m. In reference frame S', which is moving along the $+x$ axis at speed v, the explosions occur at the same point in space. What is the separation in time between the two explosions as measured in S'?

G. 1. What is the relativistic formula relating the frequency f' of light from a receding source to its frequency f_0 when at rest? Is the frequency f' larger or smaller than f_0?

2. By a symmetry argument write down the corresponding formula for f' when the source is approaching.
3. What is the relation between the frequencies f' and f_0 and the corresponding wavelengths λ' and λ_0?
4. A relativistic source is moving away from an experimenter directly towards a mirror. The experimenter forms the geometric average (*i.e.* the square-root of the product) of the frequencies received from the source and the mirror. What value is obtained?

H. 1. Define the relativistic energy E of a particle and explain how this is related to the particle's rest energy and kinetic energy.

2. Define the relativistic momentum p.

3. Derive the following useful formulae: $pc = Ev/c$ and $E^2 - p^2c^2 = m^2c^4$.

I. Reconsider problem B of sheet 10. Show that the resonance (or threshold) kinetic energy of the electron is given by

$$K_e = \frac{c^2}{2m_e}(m_\pi^2 - 4m_e^2),$$

where m_e and m_π are the mass of an electron and positron respectively. For what value of m_e, does this expression vanish. Explain the result physically.