

Units, Dimensions, Dimensional Analysis

Units

Length can be measured in any number of different units, e.g. meters, cm, km, inches, angstroms, astronomical units, spans, yards.

A measurement of length 2 means nothing unless we state what units we are measuring in. A measurement of 2 furlongs means a length which is 2 times as long as a standard furlong defined somewhere and agreed by everyone.

In general physicists should use the standard international units whose definitions are agreed by an international committee of scientists, <http://physics.nist.gov/cuu/Units/units.html>, however as long as you define your units carefully it is always possible to convert between different systems.

Dimensions

We implicitly know that it is not possible to say that a length is bigger or smaller than an area or that 1 second is heavier than a kilogram even though we can say that an ounce is smaller than a kilogram.

This is because that separate from the idea of units a measurement has associated with it a quality called dimension.

Although we can't compare areas and lengths we know that they are related by the fact that the area of a square is given by the length of its side squared. We also know that the volume of a cube is given by a length cubed or an area times a length. However no matter what we do with lengths, areas and volumes on their own we cannot make a time.

There is a basic set of dimensions from which we can define the dimension of all other physically measurable quantities these include the most commonly used ones

Length – L, Mass – M, Time –T, Charge – Q.

However there are other, less used quantities, such as fundamental particle type, e.g. quarks, and the equivalent to charge and mass for the other fundamental forces.

Dimensional Analysis

This is just common sense written down so that we can apply it to mathematical equations.

Whilst in maths it is acceptable to write down any set of symbols in equations e.g. $A=B$ in physics only certain equations are acceptable because for instance a length cannot equal a mass. The basic rules are

- 1) two physical quantities can only be equated if they have the same dimensions
- 2) two physical quantities can only be added if they have the same dimensions
- 3) the dimensions of the multiplication of two quantities is given by the multiplication of the dimensions of the two quantities.
- 4) the dimension of the division of two quantities is given by the division the dimensions of the two quantities.

By analysing an equation in terms of dimensionality we can determine if it could be correct even if we don't know anything else about the physics. This is an extremely powerful technique which has saved many physicists from looking very stupid.

Dimensionless Numbers – not core part of skills session

Often in physics we know the variables which are important to a problem before we understand mechanisms responsible for the behaviour of the system. In that case it is often found that it is possible to produce one or more dimensionless numbers out of the important input parameters and that the behaviour of the system depends on the value of a limited set of these dimensionless numbers independent of the specific values of the much larger set of dimensional input numbers. Famous examples of these dimensionless numbers include the Mach number and Reynolds number.

Examples

A) What are the dimensions of the following common physical quantities? Give two different units for each quantity. e.g. Energy has dimensions ML^2T^{-2} and can be measured in Joules or Calories.

- | | | |
|--------------------------------------|---|-------------------------------------|
| a) Velocity - v , | b) Current - I , | c) Force - \underline{F} , |
| d) Acceleration- \underline{a} , | e) Mass Density - ρ , | f) Momentum - p , |
| g) Pressure - P | h) Magnetic Field - \underline{B} , | g) Electric Field \underline{E} , |
| i) Dipole moment - \underline{d} , | j) Angular Momentum - \underline{L} , | k) Torque - $\underline{\tau}$ |

B) What are the dimensions of the variable K in the following equations. You may need the following q is a charge, t is a time, r is distance, E_n is an energy, f is a frequency.

- i) $\underline{F} = K\underline{L}$
- ii) $\underline{E} = \frac{qr}{Kr^2}$
- iii) $\underline{\tau} = P\underline{K} \times \underline{r}$
- iv) $P = K\rho|\underline{a}|$
- v) $E_n = Kf$

C) Which of the following equations could be true and which cannot? Give a dimensional analysis proof of your answer.

V is a volume, $\underline{\omega}$ is an angular velocity with units radians per sec, r_0 is a distance,

- i) $\frac{\partial^2 \underline{E}}{\partial x^2} = v^2 \frac{\partial^2 \underline{E}}{\partial t^2}$
- ii) $PV = \sum \frac{1}{2}mv^2 + \frac{1}{2}\underline{L}\underline{\omega}$
- iii) $r = r_0 \sin(3t)$

Written by David Smith, edited by BSL, 2007