DIM REG $\quad d = 4 - 2\varepsilon$

The main integral result is (in Euclidean space)

$$I_0(\alpha) = \frac{1}{(2\pi)^d} \int \frac{d^d k}{(k^2 + A^2)^\alpha}$$

$$= \frac{\Gamma(\alpha - d/2)}{(4\pi)^{d/2}\, \Gamma(\alpha)} A^{d - 2\alpha}$$

pg 249 Peskin & schroedinger eq (7.82) isn't proved

$$\Gamma(t) \equiv \int_0^\infty x^{t-1} e^{-x}\, dx$$

$\Gamma(1) = 1$

$\Gamma(t+1) = t\,\Gamma(t)$ integrate by parts!

so $\Gamma(n) = (n-1)!$ for integers

away from integer values $\Gamma(1/2) = \sqrt{\pi}$

$\Gamma(3/2) = \sqrt{\pi}/2$

$\Gamma(5/2) = \frac{3\sqrt{\pi}}{4}$

eg $\alpha = 2$ $\quad I_0(2) = \frac{1}{16\pi^2}\Gamma(\varepsilon)\left(\frac{4\pi}{A^2}\right)^{\varepsilon}$

$d = 4 - 2\varepsilon$

Euler's const 0.577

$$\underset{\text{Laurent expansion}}{=} \frac{1}{16\pi^2}\left[\frac{1}{\varepsilon} + \ln\frac{4\pi\mu^2}{A^2} - \gamma_E + O(\varepsilon) + \ldots\right]$$

μ contains the dimension of A, k...

We also need

$$I_2(\alpha)\, g^{\mu\nu} \equiv \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu}{(k^2+A^2)^\alpha}$$

Note: $\int d^dk \frac{k^\mu}{D^\alpha} = 0$ since summing over all vectors k^μ gives zero.

$\int d^dk \frac{k^\mu k^\nu}{D^\alpha}$ is zero when $\mu \neq \nu$ for the same reason

$\longrightarrow$ hence diagonal.

Hit with $g_{\mu\nu}$ & use $g^{\mu\nu} g_{\mu\nu} = d$

$k^2 + A^2 - A^2$

$$I_2(\alpha) = \frac{1}{d} \frac{1}{(2\pi)^d} \int \frac{k^2\, d^dk}{(k^2+A^2)^\alpha}$$

$$I_2(\alpha) = \frac{1}{d}\left[I_0(\alpha-1) - A^2 I_0(\alpha) \right]$$

$$= \frac{1}{d}\left[\frac{\Gamma(\alpha-1-d/2)}{(4\pi)^{d/2}\,\Gamma(\alpha-1)} A^{d-2\alpha+2} - \frac{A^{2+d-2\alpha}\,\Gamma(\alpha-d/2)}{(4\pi)^{d/2}\,\Gamma(\alpha)} \right]$$

use $\Gamma(t) = (t-1)\Gamma(t-1)$

$$= \frac{1}{d}\,\frac{A^{2+d-2\alpha}}{(4\pi)^{d/2}}\left[\frac{\Gamma(\alpha-1-d/2)\,(\alpha-1)}{\Gamma(\alpha)} - \frac{(\alpha-1-d/2)\,\Gamma(\alpha-1-d/2)}{\Gamma(\alpha)} \right]$$

$$= \frac{1}{(4\pi)^{d/2}} A^{2+d-2\alpha}\,\frac{1}{2}\,\frac{\Gamma(\alpha-1-d/2)}{\Gamma(\alpha)}$$

eg $\alpha = 3$ $\quad I_2(3) = \frac{1}{2}\,\frac{\Gamma(\varepsilon)}{2}\,\frac{1}{16\pi^2}\left(\frac{4\pi}{A^2}\right)^{\varepsilon}$

DIMENSIONS

$$S = \int d^{4-2\varepsilon}x \; \mathcal{L}$$

$$\bar{\Psi} \not{\partial} \Psi \rightarrow D[\Psi] = 3/2 - \varepsilon$$

$$(\partial \phi)^2, (\partial A^\mu)^2 \rightarrow D[\phi, A^\mu] = 1 - \varepsilon$$

$$S_{int} = \int d^{4-2\varepsilon}x \;\; q \bar{\Psi} \not{A} \Psi \rightarrow q \text{ dim } \varepsilon$$

$$q = \mu^\varepsilon \tilde{q}$$

DIRAC ALGEBRA

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$$g^{\mu\nu} g_{\mu\nu} = d = 4 - 2\varepsilon$$

$$\gamma^\mu \gamma_\mu = (4 - 2\varepsilon) \mathbb{1}$$

- $\gamma^\mu \gamma^\nu \gamma_\mu = -\gamma^\mu \gamma_\mu \gamma^\nu + \gamma^\mu 2 g_{\mu\nu}$

$$= -(4-2\varepsilon)\gamma^\nu + 2\gamma^\nu = (-2+2\varepsilon)\gamma^\nu$$

- $\gamma^\mu \gamma^\nu \gamma^\rho \gamma_\mu = 4 g^{\nu\rho} - 2\varepsilon\, \gamma^\nu \gamma^\rho$
- $\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_\mu = -2\gamma^\sigma \gamma^\rho \gamma^\nu + 2\varepsilon\, \gamma^\nu \gamma^\rho \gamma^\sigma$

Strictly $\mathrm{Tr}\,\mathbb{1} = 2^{d/2}$ but you can use $\mathrm{Tr}\,\mathbb{1} = 4$ & absorb extra $2^{d/2-2}$ into coupling renormalization

<u>Trace ID</u>: $\mathrm{Tr}\,\gamma^\mu = \mathrm{Tr}\,\gamma^\mu \gamma^\nu \ldots_{\text{odd}} = 0$ $\qquad \mathrm{Tr}\,\gamma^\mu \gamma^\nu = 4\eta^{\mu\nu}$

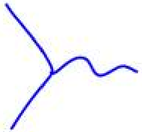
$$\mathrm{Tr}\,\gamma^\mu \gamma^\rho \gamma^\nu \gamma^\sigma = 4(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho})$$

QED

Here QED is the Feynman rules:

 $\dfrac{i}{\not{p} - m}$

 $\dfrac{-i g_{\mu\nu}}{p^2}$ (Feynman Gauge)

 $-ie\gamma_\mu$

$\uparrow$ $\mu^{\epsilon}\tilde{e}$

$$\partial_\mu J^\mu = 0$$

1-loop diagrams: