7 REGULARIZATION

How should we do book keeping for divergences?

CUT OFF

$$\int_0^{\Lambda} \frac{d^4k}{k^2} \sim \Lambda^2 \qquad \Lambda \to \infty$$

PAULI VILLARS

$$\frac{1}{k^2-m^2} \to \sum_{i=0}^{\infty} a_i \frac{1}{k^2-m_i^2} \qquad a_0=1\,,\ m_0=m$$

Expand as series in k^2:

$$\sum_{i=0}^{\infty} a_i \frac{1}{k^2} + \sum_{i=0}^{\infty} a_i \frac{m_i^2}{k^4} + \Theta(1/k^6)$$

these terms give you convergent pieces in sensible theories

So require

$$\sum_{i=0}^{\infty} a_i = 0$$

$$\sum_{i=0}^{\infty} a_i m_i^2 = 0$$

But neither of these work for gauge theories

- gauge transforms shift p^μ so what about Λ?
- gauge bosons must be massless.

DIMENSIONAL REGULARIZATION

$$\int_0^{\Lambda} \frac{d^4k}{k^4} \sim \ln \Lambda^2 \qquad\qquad \int \frac{d^3k}{k^4} \sim \text{finite}$$

The idea is to do integrals in $d=4-2\epsilon$ dimensions & take $\epsilon \to 0$. Divergences show up as $1/\epsilon$ poles in the answer. Changing the dimension leaves symmetries unchanged

The main integral result we need is (in Euclidean space)

$$I_0(\alpha) = \frac{1}{(2\pi)^d} \int \frac{d^dk}{(k^2+A^2)^\alpha}$$

$$= \frac{\Gamma(\alpha - d/2)}{(4\pi)^{d/2}\,\Gamma(\alpha)} A^{d-2\alpha}$$

$$\Gamma(t) \equiv \int_0^\infty x^{t-1} e^{-x}\, dx$$

$\Gamma(1) = 1$

$\Gamma(t+1) = t\,\Gamma(t)$ (show by integrating by parts)

so $\Gamma(n) = (n-1)!$ for integers

Away from integer values $\Gamma(1/2) = \sqrt{\pi}$, $\Gamma(3/2) = \frac{\sqrt{\pi}}{2}$

$\Gamma(5/2) = 3\sqrt{\pi}/4 \ \ldots$

If you want to prove the result above start on page 249 of Peskin & find the proof of (7.82) on wikipedia!

eg $\alpha=2$: $\quad \frac{1}{16\pi^2}\,\Gamma(\epsilon)\left(\frac{4\pi}{A^2}\right)^{\epsilon}$

$$\underset{\substack{\text{Laurent}\\ \text{expansion}}}{=} \frac{1}{16\pi^2}\left[\frac{1}{\epsilon} + \ln\frac{4\pi\mu^2}{A^2} - \gamma_E + O(\epsilon) + \dots\right]$$

↑ Eulers const 0.577...

The appearance of a scale μ here is peculiar – it's sort of part of the ∞... but needed in the log on dim. grounds. Physically it will become the scale at which you fix the coupling exptly.

We also need the integral

$$\int \frac{d^d k}{(2\pi)^d}\,\frac{k^\mu k^\nu}{(k^2+A^2)^\alpha} \equiv I_2(\alpha)\,g^{\mu\nu}$$

Note $\int d^d l\,\frac{l^\mu}{D^\alpha} = 0$ since summing over vector in all directions

$\int d^d l\,\frac{l^\mu l^\nu}{D^\alpha}$ will be zero for the same reason is $\mu\neq\nu$

Contract both sides with $g_{\mu\nu}$ $\qquad g^{\mu\nu}g_{\mu\nu} = d$

$$I_2(\alpha) = \frac{1}{d}\frac{1}{(4\pi)^d}\int \frac{\overbrace{k^2}^{k^2+A^2-A^2}\,d^d k}{(k^2+A^2)^\alpha}$$

$$= \frac{1}{d}\left[I_0(\alpha-1) - A^2 I_0(\alpha)\right]$$

$$= \frac{1}{d}\left[\frac{\Gamma(\alpha-1-d/2)}{(4\pi)^{d/2}\,\Gamma(\alpha-1)} A^{d-2\alpha+2} - \frac{A^{2+d-2\alpha}\,\Gamma(\alpha-d/2)}{(4\pi)^{d/2}\,\Gamma(\alpha)} \right]$$

use $\Gamma(t) = (t-1)\Gamma(t-1)$ $\Rightarrow$

$$\frac{1}{(4\pi)^{d/2}} \frac{1}{d} A^{2+d-2\alpha} \left[\frac{\Gamma(\alpha-1-d/2)(\alpha-1)}{\Gamma(\alpha)} - \frac{(\alpha-1-d/2)\Gamma(\alpha-1-d/2)}{\Gamma(\alpha)} \right.$$

$$= \frac{1}{(4\pi)^{d/2}} A^{2+d-2\alpha} \frac{1}{2} \frac{\Gamma(\alpha-1-d/2)}{\Gamma(\alpha)}$$

eg $\alpha = 3$ $\quad I_2(3) = \frac{1}{2} \frac{\Gamma(\epsilon)}{2} \frac{1}{16\pi^2} \left(\frac{4\pi}{A^2}\right)^{\epsilon}$

DIMENSIONS

$$S = \int d^{4-2\epsilon}x \; \mathcal{L} \qquad \bar{\psi}\not{\partial}\psi \qquad (\partial\phi)^2$$

$\Rightarrow \quad D[\psi] = 3/2 - \epsilon \qquad D[\phi, A^\mu] = 1 - \epsilon$

$$S_{INT} = \int d^{4-2\epsilon}x \; e\bar{\psi}\not{A}\psi$$

$\uparrow$ dim ϵ $\rightarrow$ $e = \mu^{\epsilon}\hat{e}(\mu)$

Again μ will be the scale where we fix e's value.

Dirac algebra in $4-2\epsilon$ dimensions

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$$g^{\mu\nu} g_{\mu\nu} = 4-2\epsilon$$

$$\gamma^\mu \gamma_\mu = (4-2\epsilon)\mathbb{1}$$

- $\gamma^\mu \gamma^\nu \gamma_\mu = -\gamma^\mu \gamma_\mu \gamma^\nu + \gamma^\mu 2 g_{\mu\nu}$

  $= -(4-2\epsilon)\gamma^\nu + 2\gamma^\nu \quad = (-2+2\epsilon)\gamma^\nu$

- $\gamma^\mu \gamma^\nu \gamma^\rho \gamma_\mu = 4g^{\nu\rho} - 2\epsilon \gamma^\nu \gamma^\rho$

- $\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_\mu = -2\gamma^\sigma \gamma^\rho \gamma^\nu + 2\epsilon \gamma^\nu \gamma^\rho \gamma^\sigma$

Strictly $\mathrm{Tr}\,\mathbb{1} = 2^{d/2}$ but you can use $\mathrm{Tr}\,\mathbb{1} = 4$ & absorb $2^{d/2-2}$ into coupling renormalization (apparently)

Trace Identities

$$\mathrm{Tr}\,\gamma^\mu = \mathrm{Tr}\,\gamma^\mu\gamma^\nu \ldots_{\text{odd}} = 0$$

$$\mathrm{Tr}\,\gamma^\mu\gamma^\nu = 4\eta^{\mu\nu}$$

$$\mathrm{Tr}\,\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\sigma = 4(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}$$

⑧ QED

Here we will basically think of QED as the Feynman rules

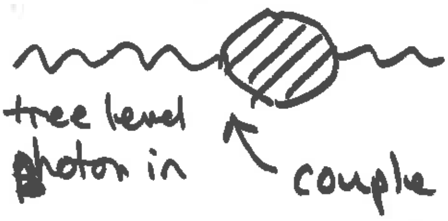
 $\dfrac{i}{\not p - m}$  $\dfrac{-i g_{\mu\nu}}{p^2}$ (Feynman Gauge)  $-ie\gamma_\mu$ ← $\mu^\epsilon e$

& the conservation law $\partial_\mu J^\mu = 0$

WARD IDENTITIES

These are identities that hold at all orders in perturbation theory & follow from current conservation $\partial_\mu J^\mu = 0$

eg The full photon propagator is



$$G^{\mu\nu}(q^2) = G_0^{\mu\rho}(q^2)\left[g^{\rho\nu} - i\int d^4x\, e^{-iq\cdot x}\langle 0|T J_\rho(x) A_\nu(0)|0\rangle\right]$$

Now hit with q_ν ....

$$-iq_\nu \int d^4x\, e^{-iq\cdot x} \langle 0|T J_\rho(x) A_\nu(0)|0\rangle = \int d^4x\, \partial_\rho\left[e^{-iq\cdot x}\right] \langle 0|T J_\rho(x) A_\nu(0)|0\rangle$$

$$= -i \int d^4x\, e^{-iq\cdot x}\, \partial_\rho \langle 0|T J_\rho(x) A_\nu(0)|0\rangle$$

Now $\partial_\rho J^\rho = 0$ ... but not so fast ...

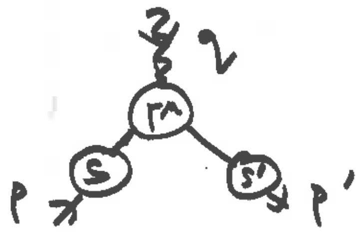
$$\partial_\rho \langle 0|T J_\rho(x) A_\nu(0)|0\rangle = \partial_\rho \Big\{ \langle 0|J_\rho(x) A_\nu(0)|0\rangle\, u(x_0) + \langle 0|A_\nu(0) J_\rho(x)|0\rangle\, \underbrace{u(-x_0)}_{\text{step fns}} \Big\}$$

$$= \langle 0|\left[J_\rho(x), A_\nu(0)\right]|0\rangle\, \delta(x_0)$$

$= 0$ since they commute.

So $$\boxed{q_\nu G^{\mu\nu}(q^2) = q_\nu G_0^{\mu\nu}(q^2)}$$

eg Another one follows from



$$= S(p')\,\Gamma^\mu(p,p')\,S(p)\,(2\pi)^4\delta^4(p+q-p')$$

$$= \int d^4x\, d^4y\, e^{i(p'\cdot z - p\cdot y - q\cdot x)} \langle 0|T J_\mu(x)\psi(y)\bar\psi(z)|0\rangle$$

Again hit both sides with q_μ & use $q_\mu J^\mu = 0$
(lot's of commutators later)

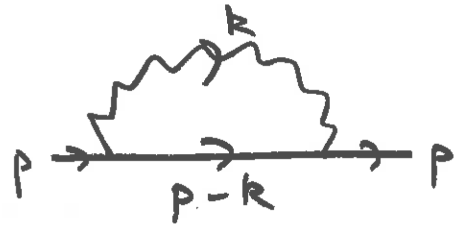
$$\Rightarrow \boxed{q_\mu \Gamma^\mu = i\left(S^{-1}(p') - S^{-1}(p)\right)}$$

eg at leading order: $\quad q^\mu \gamma_\mu = (\not p' - m) - (\not p - m)$

RENORMALIZING QED

For ease of teaching we'll do $m_e = 0$ & only collect divergent pieces of diagrams - the rest is in the printed notes!

ELECTRON SELF ENERGY

$-i\Sigma =$ 

$$\Sigma = i \int \frac{d^d k}{(2\pi)^d} \left(-ie\mu^{\epsilon}\gamma^{\mu}\right) \; i\,\frac{(\not p - \not k)}{(p-k)^2} \left(-ie\mu^{\epsilon}\gamma^{\nu}\right) \frac{-i g_{\mu\nu}}{k^2}$$

FEYNMAN PARAMETERIZE

$$\Sigma = -i\mu^{2\epsilon} e^2 \int \frac{d^d k}{(2\pi)^d} \int_0^1 d\alpha\, d\beta\; \delta(\alpha+\beta-1) \frac{\gamma^{\mu} (\not p - \not k)\gamma_{\mu}}{\left[\alpha(p^2+k^2-2p.k) + \beta k^2\right]^2}$$

Do β integral to set $\alpha + \beta = 1$

$$D = k^2 - 2\alpha\, p.k + \alpha p^2$$

Shift: $k^{\mu} = k'^{\mu} + \alpha p^{\mu}$

$$D = k'^2 + \alpha^2 p^2 - 2\alpha^2 p^2 + \alpha p^2$$

$$= k'^2 + \alpha(1-\alpha)p^2$$

$$= k'^2 - A^2$$

$$\Sigma = -i\mu^{2\epsilon} e^2 \int \frac{d^d k'}{(2\pi)^d} \int_0^1 d\alpha \; \frac{\gamma^\mu \left(\not{p}(1-\alpha) + \overset{\text{from shift}}{\not{k}'} \right) \gamma_\mu}{(k'^2 - A^2)^2}$$

We can drop the $\not{k}'$ term - it's an odd integral.

WICK ROTATE • gain i • $k'^2 - A^2 \to -(k^2 + A^2)$

$\gamma^\mu (1-\alpha) \not{p} \gamma_\mu = -2(1-\epsilon) \not{p} (1-\alpha)$ Gamma IDs in d dim

$$\Sigma = \mu^{2\epsilon} e^2 \int_0^1 d\alpha \; 2(\alpha-1)(1-\epsilon) \not{p} \underbrace{\int \frac{d^d k}{(2\pi)^d} \frac{1}{[k^2 + A^2]^2}}_{\frac{1}{16\pi^2}\left[\frac{1}{\epsilon} + \text{finite pieces} + O(\epsilon)\right]}$$

$$\Sigma_{div} = \frac{e^2}{16\pi^2} \underbrace{\int_0^1 d\alpha \; 2(\alpha-1)}_{2\left[\frac{\alpha^2}{2} - \alpha\right]_0^1 = -1} \not{p} \frac{1}{\epsilon}$$

$$= \frac{-e^2}{16\pi^2} \frac{1}{\epsilon} \not{p} \quad + \text{finite}$$

We now expand in a function of $(\not{p} - m_e)$

$$\Sigma = \delta m + (Z_2 - 1)(\not{p} - m_e) + \Sigma_R(p^2, m_e^2)$$

there's a divergence in here if leave me in computation

our term contributes here

vanishes like $(\not{p} - m)^2$

PHOTON SELF ENERGY

- We need to track back to the path integral to pick up a minus sign

$$e^{iS} \sim e^{iS_0}\left(1+\ldots+\int d^4x\, d^4y\, \bar{\psi}\gamma^\mu\psi(x)\, \bar{\psi}\gamma^\nu\psi(y) + \ldots\right)$$

Need to get two $\psi(x)\bar{\psi}(y)$ propagators in Wick contractions $\Rightarrow$ anticommutation $\Rightarrow$ net $-$ sign

EXTRA FEYNMAN RULE : FERMION (GHOST) LOOPS GET $-$

- We should also think about the Dirac algebra in the loop

same index since return to same vertex

$$\underbrace{(\Gamma)_{ab}\, S(p-k)_{bc}\, (\Gamma)_{cd}\, (S(p))_{da}}_{M_{aa} = \mathrm{Tr}\, M}$$

We should trace over the Dirac matrices in a loop....

- Lorentz symmetry restricts the form of the answer to

$$+i\Pi^{\mu\nu}(q^2) = +iA(q^2)\, g^{\mu\nu} + iB(q^2)\, q^\mu q^\nu$$

& if we use the Ward Identity $q_\mu \Pi^{\mu\nu} = 0$

$$\Rightarrow \quad \Pi^{\mu\nu}(q^2) = (+g^{\mu\nu}q^2 - q^\mu q^\nu)\,\Pi(q^2)$$

• <u>Renormalization</u> of photon propagator. Generically the prop. is

$$-\left(g_{\mu\nu} - \xi \frac{q_\mu q_\nu}{q^2}\right)\frac{i}{q^2} = \frac{-i}{q^2}\left[\underline{\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)} - (\xi-1)\frac{q^\mu q^\nu}{q^2}\right]$$

As before Taylor expand $\Pi(q^2) = \Pi(0) + q^2\Pi_R(q^2)$

$$\rightarrow \frac{\overset{\leftarrow 1+\Pi(0)}{Z_3}}{q^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)$$

$\Pi_R(q^2)$ is moved to the denominator

$$\Rightarrow \frac{-i}{q^2}\left[Z_3 \frac{\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)}{(1-\Pi_R(q^2))} + i(\xi-1)\frac{q^\mu q^\nu}{q^2}\right]$$

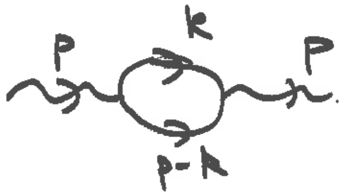
longitudinal piece is left untouched

Note there's no mass renormalization (there's no mass!)

⑨ MORE QED

PHOTON SELF ENERGY

$$+i\,\Pi^{\mu\nu}(q^2) =$$



$$= -\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\,\mathrm{Tr}\left[(-ie\gamma^\mu)\; i\frac{\not k-\not q}{(k-q)^2}\;(-ie\gamma^\nu)\; i\frac{\not k}{k^2}\right]$$

FERMION LOOP.

Feynman Parameterize:

$$\Pi^{\mu\nu} = i\mu^{2\epsilon}e^2\int\frac{d^dk}{(2\pi)^d}\int_0^1 d\alpha\, d\beta\;\delta(\alpha+\beta-1)\;\frac{\mathrm{Tr}\;\gamma^\mu(\not k-\not q)\gamma^\nu\not k}{[\alpha(k-q)^2+\beta k^2]^2}$$

Do β integral to set $\alpha+\beta=1$ $\qquad D = k^2+\alpha q^2-2\alpha k\cdot q$

$$\mathrm{Tr}\;\gamma^\mu(\not k-\not q)\gamma^\nu\not k = (k-q)^\rho k^\sigma\;\mathrm{Tr}\;\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\sigma$$

$$4\left[g^{\mu\rho}g^{\nu\sigma}-g^{\mu\nu}g^{\rho\sigma}+g^{\mu\sigma}g^{\nu\rho}\right]$$

$$= 4\left[-(k-q)\cdot k\; g^{\mu\nu} + (k-q)^\mu k^\nu + (k-q)^\nu k^\mu\right]$$

$$\Pi^{\mu\nu} = -ie^2\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\int d\alpha\; 4\;\frac{g^{\mu\nu}(k^2-k\cdot q)-2k^\mu k^\nu+q^\mu k^\nu+q^\nu k^\mu}{(k^2+\alpha q^2-2\alpha k\cdot q)^2}$$

shift $k^\mu = k'^\mu + \alpha q^\mu$

$$D \Rightarrow k'^2 + \alpha^2 q^2 - 2\alpha^2 q^2 + \alpha q^2 \quad = k'^2 - \alpha^2 q^2 + \alpha q^2$$

$$N \Rightarrow g^{\mu\nu}\left(k'^2 + \alpha^2 q^2 - \alpha q^2\right) - 2k'^\mu k'^\nu - 2\alpha^2 q^\mu q^\nu + 2\alpha q^\mu q^\nu$$

+ terms linear in k that integrate to zero

I can get $\Pi(q^2)$ from either $q^\mu q^\nu$ terms or $g^{\mu\nu}$ so

$$-q^\mu q^\nu \Pi(q^2) = -q^\mu q^\nu \, 4i\, e^2 \mu^{2\epsilon} \int \frac{d^d k'}{(2\pi)^d} \int_0^1 d\alpha \; \frac{2\alpha(1-\alpha)}{(k'^2 - \alpha^2 q^2 + \alpha q^2)^2}$$

WICK ROTATE • gain i • $k'^2 - A^2 \to -k'^2 - A^2$

$$\Pi(q^2) = -4e^2\mu^{2\epsilon}\int_0^1 d\alpha \; 2\alpha(1-\alpha) \underbrace{\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k'^2 + \alpha^2 q^2 - \alpha q^2)^2}}_{\frac{1}{16\pi^2}\frac{1}{\epsilon} + \text{finite} + O(\epsilon)}$$

$$\Pi(q^2) = \frac{-1}{2\pi^2} e^2 \left[\frac{\alpha^2}{2} - \frac{\alpha^3}{3}\right]_0^1 \frac{1}{\epsilon} + \text{finite}$$

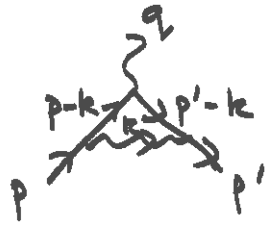
$$= -\frac{1}{12\pi^2} e^2 \frac{1}{\epsilon} + \text{finite}$$

$$Z_3 = 1 + \Pi(0) \quad = \quad 1 - \frac{\alpha}{3\pi}\frac{1}{\epsilon} + \text{finite}$$

$$\& \quad \Pi_R(q^2) = \Pi(q^2) - \Pi(0) \quad = \text{finite}.$$

VERTEX FUNCTION

$-ie\Gamma^\mu(p,p') =$



$$\Gamma^\mu = \mu^{2\epsilon}\int \frac{d^dk}{(2\pi)^d}(-ie\gamma^\nu)\frac{i(\not p'-\not k)}{(p'-k)^2}\gamma^\mu\frac{i(\not p-\not k)}{(p-k)^2}(-ie\gamma^\rho)$$

$$\times \frac{-ig_{\nu\rho}}{k^2}$$

$$= -ie^2\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\frac{\gamma^\nu(\not p'-\not k)\gamma^\mu(\not p-\not k)\gamma_\nu}{(p'-k)^2(p-k)^2k^2}$$

Feynman parameterize with α, β, γ

$$= -2ie^2\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\int d\alpha\, d\beta\, d\gamma\, \delta(\alpha+\beta+\gamma-1)\frac{N}{[\beta(p'-k)^2+\alpha(p-k)^2+\gamma k^2}$$

I'm going to assume external fermions are on shell

$$p^2 = p'^2 = m^2 = 0,\quad q = p'-p$$

$$D = \beta(k^2-2p'\cdot k) + \alpha(k^2-2p\cdot k) + \gamma k^2$$

After γ integral $\quad D = k^2 - 2(\beta p'\cdot k + \alpha p\cdot k)$

Shift $\quad k^\mu = k'^\mu + p\alpha + p'\beta$

$$D = k'^2 + (\alpha p + \beta p')^2$$

$q^2 = (p-p')^2$
$\simeq -2p\cdot p'$

$$= k'^2 + 2\alpha\beta\, p\cdot p' = k'^2 - \alpha\beta q^2$$

Meanwhile in N:

$$N = \gamma^\nu (\not{p}' - \not{k}) \gamma^\mu (\not{p} - \not{k}) \gamma_\nu$$

shifts to

$$N = \gamma^\nu \not{k}' \gamma^\mu \not{k}' \gamma_\nu \quad + \underset{\text{that } \int \to 0}{\text{linear in } k'^\mu} \quad + \text{ convergent terms}$$

Now when we do $\int d^4k' \, k^\alpha k^\beta / (k^2 + \Delta^2)^3 \to \propto g^{\alpha\beta}$

So we want to do the γ-algebra

$$g_{\alpha\beta} \gamma^\nu \gamma^\alpha \gamma^\mu \gamma^\beta \gamma_\nu = \gamma^\nu \gamma^\alpha \gamma^\mu \gamma_\alpha \gamma_\nu$$

$$= -2 \gamma_\alpha^\kappa \gamma^\mu \gamma_\alpha + 2\epsilon \gamma^\alpha \gamma^\mu \gamma_\alpha$$

$$= (4 - 4\epsilon) \gamma^\mu + 4(\epsilon^2 - \epsilon) \gamma^\mu$$

$$= \gamma^\mu (4 - 8\epsilon + 4\epsilon^2)$$

$$= \gamma^\mu 4 (1 - \epsilon)^2$$

Thus

$$\Gamma^\mu = -8i (1-\epsilon)^2 e^2 \gamma^\mu \mu^{2\epsilon} \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu}{(k^2 - \Delta^2)^3}$$

Wick Rotate: $\cdot i \quad \cdot (-1)^3 D \cdot (-1) N$

$$\Gamma^\mu = +8 (1-\epsilon)^2 e^2 \gamma^\mu \mu^{2\epsilon} \underbrace{\int_0^1 d\alpha \int_0^{1-\alpha} d\beta}_{1/2} \underbrace{\int \frac{d^dk}{(2\pi)^4} \frac{k^\mu k^\nu}{(k^2 + \Delta^2)^3}}_{+\frac{1}{(4\pi)^2} \frac{1}{4\epsilon}}$$

$$= + \frac{1}{16\pi^2} e^2 \gamma^\mu \frac{1}{\epsilon} + \text{finite}$$

We absorb the divergence into the definition of q_R

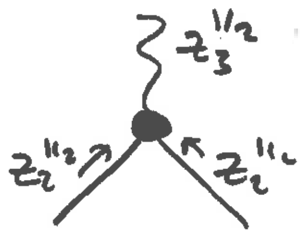
$$-iq\gamma_\mu\left(1 + \frac{e^2}{16\pi^2\epsilon} + \dots\right)$$

$$\equiv \frac{-iq}{Z_1}\gamma^\mu$$

$$\Rightarrow Z_1 = \left(1 - \frac{e^2}{16\pi^2}\frac{1}{\epsilon} + \dots\right)$$

Note this equals Z_2 which follows from the second Ward Identity we saw.

In addition we suck Z_2 & Z_3 factors into the vertex



$$q_R = -iq\,\gamma_\mu\,\frac{Z_2 Z_3^{1/2}}{Z_1}$$

For example we can set this to $q_{\text{measured}}(0)$

One can be lazier though in how one sucks up ∞s.

$$\text{eg}\quad Z_3 = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\epsilon} + \ln 4\pi - \gamma_E - \ln p_e^2/\mu^2\right]$$

external e^- momentum which normally $= m_e^2$

But in $\overline{MS}$ scheme assume external states have $p^2 = \mu^2$ & then absorb only $1/\epsilon + \ln 4\pi - \gamma_E$.

In MS only absorb $1/\epsilon$ .... now q_R is not any observable.....

COUNTER TERM APPROACH

$$\mathcal{L}_{bare} = -\frac{1}{4} F_B^{\mu\nu} F_{B\mu\nu} + \bar{\Psi}_B (i\partial\!\!\!/ - m_B)\Psi_B - e_B \bar{\Psi}_B A\!\!\!/_B \Psi_B$$

$$\Psi_B = Z_2^{1/2} \Psi_R \qquad A_B = Z_3^{1/2} A_R$$

$$m_B = m_R - \delta m \equiv Z_m m_R \qquad e_B = \frac{Z_1}{Z_2 Z_3^{1/2}} e_R$$

OR:
$$\mathcal{L} = -\frac{Z_3}{4} F_R^2 + Z_2 \bar{\Psi}_R i\partial\!\!\!/ \Psi_R - Z_m Z_2 \bar{\Psi}_R m_R \Psi_R$$
$$- Z_1 e_R \bar{\Psi}_R A\!\!\!/_R \Psi_R$$

OR:
$$\mathcal{L} = -\frac{1}{4} F_R^2 + \bar{\Psi}_R (i\partial\!\!\!/ - m_R)\Psi_R - e_R \bar{\Psi}_R A\!\!\!/_R \Psi_R$$
$$- \frac{(Z_3 - 1)}{4} F_R^2 + (Z_2 - 1)\bar{\Psi}_R i\partial\!\!\!/ \Psi_R - (Z_2 + Z_m - 2)\bar{\Psi}_R m_R \Psi_R$$
$$- (Z_1 - 1)\, e \bar{\Psi}_R A\!\!\!/_R \Psi_R$$