Peak effect in vortex systems with strong pinning

X. B. Xu,[1,*] H. Fangohr,[2] X. N. Xu,[1] M. Gu,[1] Z . H. Wang,[1] S. M. Ji,[1] S . Y. Ding,[1] D. Q. Shi,[3] and S. X. Dou[3]

[1]*National Laboratory of Solid State Microstructures, Department of physics, Nanjing University, Nanjing 210093, P. R. China*

[2]*School of Engineering Sciences, University of Southampton, Southampton SO17 1BJ, United Kingdom*

[3] *Institute for Superconducting and Electronic Materials, University of Wollongong, NSW 2522, Australia*

(Dated: December 7, 2008)

We have performed 2D-Langevin simulations studying the peak effect (PE) of the critical current taking into account the temperature dependence of the competing forces. We observe and report that the occurrence of the PE results from the competition of vortex-vortex interactions and vortex-pin interactions and thermal fluctuations which have different temperature dependencies. The simulations reveal that the PE can only take place for certain pinning strengths, densities of pinning centers and driving forces, which is in good agreement with experiments. No apparent vortex order-disorder transition is observed across the PE regime. Besides, the PE is a dynamical phenomenon and thermal fluctuations can speed up the process for the formation of the PE.

PACS numbers: 74.25.Fy, 74.25.Qt

One pronounced phenomenon in type-II superconductors is the so called peak effect (PE), which is the appearance of a peak in the critical current density J_c before decreasing to 0 with increasing temperature or field[1]. The PE has widely been observed in a variety of low and high temperature superconductors by different experimental techniques[2], such as transport[3–5], magnetization[6, 7], and ac-susceptibility[8–10]. It was proposed long ago that the PE originated from softening of the elastic moduli of vortex lattice[1], which caused by the competitions between elastic energy E_{el}, pinning energy E_{pin} of vortex lattice and the energy of thermal fluctuations E_{th}[11]. Considering the competition between the strength of vortex lattice and the pinning force of isolated pinning center, Labusch showed that $J_c > 0$ only if the pinning force dominates over the strength of vortex lattice, otherwise $J_c = 0$[12]. According to the Labusch criterion the weak pinning centers in superconductors cannot pin the vortex lattice. Larkin and Ovchinnikov showed that weak pinning centers can act collectively in the correlation volume V_c ($=L_c R_c^2$, where L_c and R_c are the longitudinal and transverse correlation lengths respectively), where the vortices are of long-range order, reducing much the Labusch criterion[13]. In the collective pinning theory the PE was interpreted as resulting from the abrupt decrease of V_c due to the reduction in elastic interaction[14]. The PE was recently shown to appear naturally at the crossover from weak collective to strong pinning [15]. Further calculations showed that the "Bragg glass" phases exist to be a quasi-long-range ordered vortex lattice on a length scale $r \gg R_c$[2]. Furthermore, the occurrence of PE was explained as evidence for a Bragg glass transition or an order-disorder (OD) transition[2, 5, 9, 16]. The OD transition was suggested to be a thermodynamic phase transition induced by thermal fluctuations or pinning centers[2], which has been confirmed experimentally by the direct structural observation of the vortex lattice, such as small angle neutron scattering (SANS)[9], and muon spin relaxation[17]. Also, the investigation of the reversibility of the OD transition provided strong support for its thermodynamical nature[18].

However, the exact nature of the PE phenomenon remains controversial[6, 7, 10, 19–21]. Recently, across the PE regime no noticeable change in the order of the vortex lattice was observed by SANS[19], and by Bitter decoration[20]. It was also reported that the PE is situated on a boundary separating the strong pinning regime from the thermal fluctuations dominated regime, while the weak-strong pinning crossover is located far from the PE regime[7]. On the other hand, for some superconductors with strong pinning centers, it has been found that the occurrence of the PE depends strongly on the strength and amount of the pinning centers[3, 6, 22, 23]. Especially, the PE can be adjusted by changing the pinning strength of the twin boundaries (changing the angle between field and twin boundaries), which is difficult to explain through the OD mechanism[3].

In this work we investigated the PE of a vortex system with strong pinning by using Langevin dynamics simulations. The random pinning centers act independently. The temperature dependence of pinning force $f_{pv}(T)$ is different from that for the elastic forces between vortices $f_{vv}(T)$. Thus, the competition between $f_{pv}(T)$ and $f_{vv}(T)$ takes place as temperature changes. The PE was clearly seen for certain pinning strengths and densities of pinning. No marked change in the order of the vortices was observed. In addition, the PE is a dynamical phenomenon, and thermal fluctuations are not crucial for the occurrence of the PE but can speed up its dynamical process.

We use the overdamped Langevin equation of motion for a vertex at position $\mathbf{r}_i$ is[24]

$$\mathbf{F}_i = \sum_{j\neq i}^{N_\mathrm{v}} \mathbf{F}^{\mathrm{vv}}(\mathbf{r}_i - \mathbf{r}_j) + \sum_{k}^{N_\mathrm{p}} \mathbf{F}^{\mathrm{vp}}(\mathbf{r}_i - \mathbf{r}_k^{\mathrm{p}}) + \mathbf{F}^{\mathrm{L}} + \mathbf{F}_i^{\mathrm{T}} = \eta \frac{d\mathbf{r}_i}{dt}$$

where $\mathbf{F}_i$ is the total force acting on vortex i, $\mathbf{F}^{\mathrm{vv}}$ and $\mathbf{F}^{\mathrm{vp}}$ are the forces due to vortex-vortex and vortex-pin interactions, respectively, $\mathbf{F}^{\mathrm{L}}$ is the driving force due to the current $\mathbf{J}$ ($\mathbf{F}^{\mathrm{L}} \propto \mathbf{J} \times \hat{\mathbf{z}}$) and $\mathbf{F}^{\mathrm{T}}$ is the thermal stochastic force, η is the Bardeen-Stephen friction coefficient: $\eta \propto \phi_0 B_{\mathrm{c2}}/\rho_{\mathrm{n}}$, N_v the number of vortices, N_p the number of pinning centers and $\mathbf{r}_k^{\mathrm{p}}$ the position of the kth pinning center. We choose $\mathbf{F}^{\mathrm{vv}}(\mathbf{r}_i - \mathbf{r}_j) = (\phi_0^2 s)(2\pi\mu_0\lambda^2)^{-1}(\mathbf{r}_i - \mathbf{r}_j)(|\mathbf{r}_i - \mathbf{r}_j|)^{-2} = f_{\mathrm{vv}}(\mathbf{r}_i - \mathbf{r}_j)(|\mathbf{r}_i - \mathbf{r}_j|)^{-2}$, where ϕ_0 is the flux quantum, s the length of the vortex, μ_0 the vacuum permeability. We employ periodic boundary conditions and cut off the logarithmic vortex-vortex repulsion potential smoothly[25]. Pinning centers exert an attractive force on the vortices: $\mathbf{F}^{\mathrm{vp}}(\mathbf{r}_i - \mathbf{r}_k^{\mathrm{p}}) = -f_{\mathrm{pv}}(r_{ik}/r_{\mathrm{p}})\exp(-(r_{ik}/r_{\mathrm{p}})^2)\hat{\mathbf{r}}_{ik}$, where f_{pv} tunes the strength of this force and r_{p} determines its range[26]. We assume $r_{\mathrm{p}} = 0.2\lambda$ and $f_{\mathrm{pv}} \propto B_{\mathrm{c2}}^2(1 - B/B_{\mathrm{c2}})\xi^2/\kappa^2$ as core pinning is considered[27], where $\kappa = \lambda/\xi$, B_{c2} depends on the temperature via $B_{\mathrm{c2}} = B_{\mathrm{c2}}(0)(1 - (T/T_{\mathrm{c}})^2)$ (The form for $B_{\mathrm{c2}}(T)$ (also for $\lambda(T)$ and $\xi(T)$ in the followed context) is correct in the Ginzburg-Landau theory when T approaches T_{c}, while providing a good fit to the BCS form over the whole temperature range[28]). The thermal force is taken as $\mathbf{F}_i^{\mathrm{T}} = f_{\mathrm{th}} \sum_{\mathrm{j}} \delta(t - t_{\mathrm{j}})\Gamma(t_{\mathrm{j}})\Theta(p - q_{\mathrm{j}})$, where $\mathrm{f_{th}}$ represents the intensity of thermal force[29], $\Gamma(t_{\mathrm{j}})$ is a random number chosen from a Gaussian distribution of mean 0 and width 1, p represents the frequency of the action to the vortex by thermal noise, and q_{j} is a random number uniformly distributed between 0 and 1. To represent the temperature dependence of $\mathbf{F}^{\mathrm{vv}}$, $\mathbf{F}^{\mathrm{vp}}$, κ and $\mathbf{F}^{\mathrm{T}}$, we use $\lambda(T)/\lambda(0) = (1 - T/T_{\mathrm{c}})^{-1/2}$ and $\xi(T)/\xi(0) = (1 - T/T_{\mathrm{c}})^{-1/2}$ [28]. The average x component of the velocities of the vortices is $\langle V_{\mathrm{x}} \rangle = \frac{1}{N_{\mathrm{v}}} \sum_i^{N_\mathrm{v}} v_{\mathrm{x}i}$ which is proportional to the resulting voltage. We normalize lengths by $\lambda_0 = \lambda(0)$, forces by $f_0 = (\phi_0^2 s)(2\pi\mu_0\lambda_0^3)^{-1}$ and time by $\tau_0 = \lambda_0\eta(0)/f_0$. All quantities shown in the following figures are expressed in simulation units.

For a fixed magnetic field, we perform simulations with a Lorentz driving force along the x-axis while cooling from $0.99\mathrm{T_c}$ to $0.1\mathrm{T_c}$. The total number of vortices $N_\mathrm{v} = 676$ is used in the calculations presented here. For larger systems, similar results are observed. We employ $B = 0.015B_{\mathrm{c20}}$, $\lambda_0 = 690\mathring{A}$, $s = 12\mathring{A}$, and $\eta_0 = 1.4\times10^{-17}$kg/s and, unless specified otherwise, the pinning strength at zero temperature is $f_{\mathrm{pv0}} = 9f_0$, the rate of change of temperature is $dT/dt = -0.02T_{\mathrm{c}}/t_0$, the driving force $F^L = 2f_0$, the number of pinning centers $N_{\mathrm{p}} = 0.2N_{\mathrm{v}}$, and $f_{\mathrm{th}} = 1$.

Fig. 1(a) shows a typical plot of the average velocity of the vortices in the x-direction, V_{x}, against the reduced temperature $\mathrm{T/T_c}$ (filled triangles). The figure

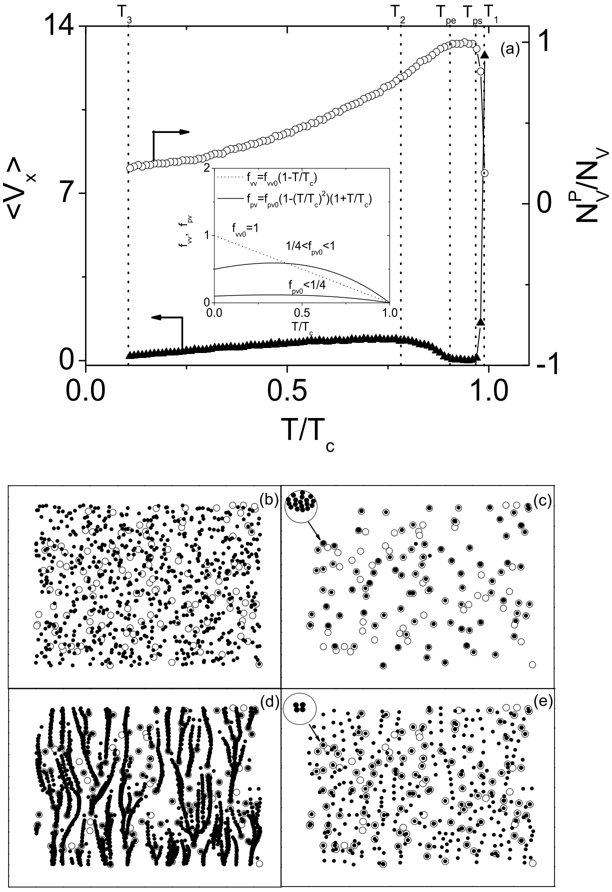

FIG. 1: (a) The dip effect in the velocity $\mathrm{V_x}$ (filled triangles) and number of pinned vortices $N_{\mathrm{v}}^{\mathrm{p}}/N_{\mathrm{v}}$ (open circles) as a function of temperature. Inset: Schematic diagram of elastic force $\mathrm{f_{vv}(T)}$ and pinning force $\mathrm{f_{pv}(T)}$ as a function of temperature. From (b) to (e): Vortex (filled circles) distribution at temperatures marked in Fig. 1(a). (b) $T = T_1$, where almost all vortices are unpinned. (c) $T = T_{\mathrm{ps}}$, where almost all the vortices are pinned, and each pin (open circles) can trap several vortices, shown in the enlarged graph in Fig. 1(c). (d) $T_{\mathrm{pe}} > T \geq T_2$, where part of the pinned vortices are unpinned and the vortex channels are formed. (e) $T = T_3$, where most of vortices are unpinned and only a smaller number of vortices are trapped, shown in the enlarged graph in Fig. 1(e).

also shows the number of pinned vortices $\mathrm{N_v^p}$ normalized by $\mathrm{N_v}$ as function of reduced temperature $\mathrm{T/T_c}$ (open circles). A dip of $\mathrm{V_x}$, which corresponds to a peak in $\mathrm{J_c}$, can be seen in the temperature regime from $\mathrm{T_{ps}}$ to $\mathrm{T_{pe}}$. The simulated results show three distinct regimes. We discuss the features going from high to low temperature.

At higher temperatures ($T_{\mathrm{ps}} < T < T_{\mathrm{c}}$), the number of pinned vortices increases with decreasing temperature, which results in the average velocity of vortices decreasing accordingly. Close to T_{c}, as comparing with the Lorentz force and thermal fluctuations, the pinning force is too small for pinning centers to pin vortices. This is supported by most of the vortices being unpinned and randomly distributed in the simulation cell at T_1, as shown in Fig. 1(b). When decreasing the temperature, the pinning force becomes more important than both the thermal fluctuations and the elastic force ($f_{\rm pv} > f_{\rm vv}$, see also the inset in Fig. 1(a)). Thus, more and more vortices are trapped by the random pinning centers. Close to $T =_{\rm ps}$, almost all of vortices are pinned.

In the PE region ($T_{\rm pe} \leq T \leq T_{\rm ps}$), the vortices are pinned and disordered. It is clear that the high J_c is due to $f_{\rm pv}(T)$ dominating over either $f_{\rm vv}(T)$ or thermal fluctuations. Besides, because the vortex pinning is strong enough, several vortices are trapped by one pinning site, see for example the insert of Fig. 1(c). This is consistent with the recent experimental and numerical observations[30].

At low temperatures ($T < T_{\rm pe}$), $f_{\rm vv}(T)$ becomes more important and thus more and more vortices are depinned, see for example the insert of Fig. 1(e). The vortex paths form channel flows as shown in Fig. 1(d). These channels will be destroyed by the quickly enhancing elastic force at very low temperatures, shown in Fig. 1(e). Note that no ordered vortex structure is observed even in low temperature $T \sim 0$.

Our calculations clearly show that there exists the PE for such a 2D disordered vortex system, where pinning is strong and the random pinning centers act independently. Furthermore, the vortices are always in disordered states across the PE regime. In other words, no apparent change in the vortex order can be observed when the PE takes place. The PE can be simply explained by the competition between pinning and elastic forces of the vortices, which results from their different temperature dependencies. It is evident that our interpretation on the PE is different from that based on the OD transition or on the abrupt reduction in the correlation volume. This is the central result of this work.

Fig. 2 demonstrates the V_x-T/T_c curves for different pinning strengths $f_{\rm pv0}$ and densities $n_p (= N_p/N_v)$. It can be seen that the PE appears only for certain $f_{\rm pv0}$ at a given density n_p and Lorentz force. For small $f_{\rm pv0}$, the vortices cannot be effectively pinned, resulting in a very low J_c and no observable PE. While for very large $f_{\rm pv0}$, the vortices are always pinned until to very low temperatures, see the data for $f_{\rm pv0} = 90 f_0$ shown in Fig. 2. Therefore, the PE can be numerically realized by changing pinning strength, which is in good agreement with transport experiment by Kwok and co-workers who successfully observed the PE by adjusted pinning strength of two twin boundaries in YBCO[3].

The insert of Fig. 2 shows V_x-T/T_c curves at several n_p at given $f_{\rm pv0}$. The PE is hardly seen if n_p is either too large or too small. That is, the situation for low density is similar to that for low pinning strength. For the presence of the PE, therefore, these results imply that the vortex-pin interactions depending on the properties of the defects in superconducting materials should

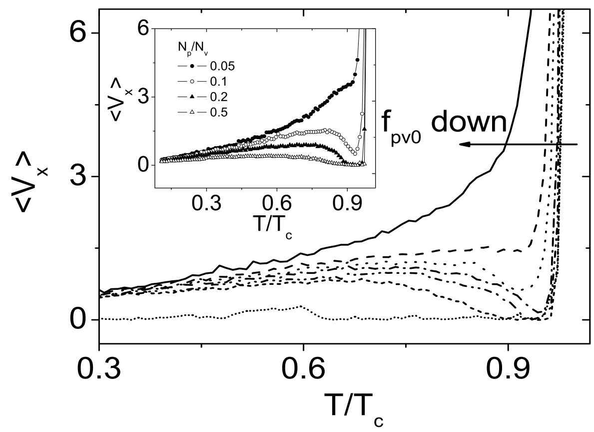

FIG. 2: Effect of pinning strength f_0 on the PE at a fixed density of pinning centers. The Lorentz force $f_{\rm pv0}$ varies from 4.5 to 90 f_0 (4.5, 6, 6.6, 7.5, 9, 10.5, 90) f_0. Inset: Effect of density of pinning center on the PE at fixed pinning strength and Lorentz force.

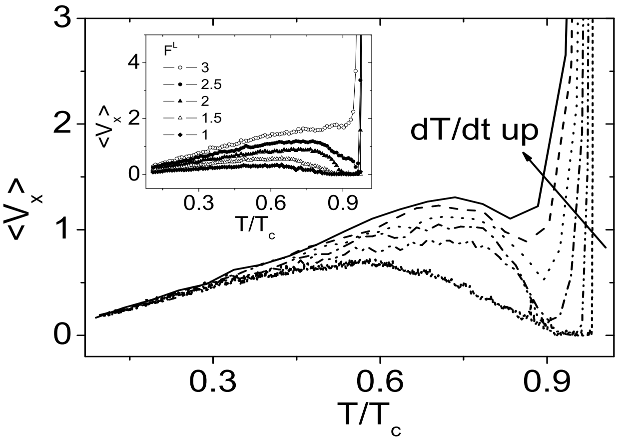

FIG. 3: Effect of temperature ramping rates dT_c/dt_0 on the PE, dT_c/dt_0 ranging from -0.002 to 0.1 T_c/t_0 (-0.002, -0.02, -0.04, -0.06, -0.08, -0.1) T_c/t_0. Inset: Effect of the driving force on the PE.

be comparable with the vortex-vortex interactions being an intrinsic property of vortex matter. Our simulated results are supported by the experimental investigation about the dependence of the PE on the pin density, which was adjusted by radiation or annealing treatments[6, 31].

We now study the effects of the experimental speed on the PE. Fig. 3 shows the V_x-T/T_c curves for several temperature ramping rates dT/dt at fixed pinning and driving forces. For high dT/dt, the moving vortices cannot be pinned due to the (relatively slow) vortex relaxation. Thus the PE will progressively disappear with increasing dT/dt. At low temperature ramping rates, the PE is progressively visible because the vortices have enough time to move into the pinning wells. These simulated results

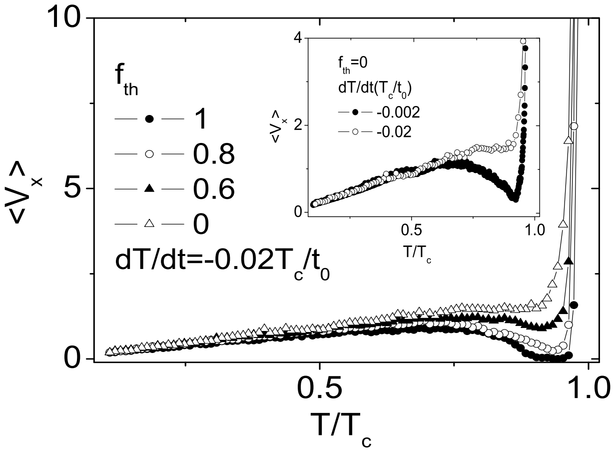

FIG. 4: V_x-T/T_c characteristics for various thermal fluctuation forces. Inset: V_x-T/T_c characteristics without the thermal fluctuations for different temperature ramping rates.

indicate that the PE is a metastable phenomenon.

It is expected that an increment of pinning force at fixed driving current is equivalent to a decrement of applied current at fixed pinning force. We therefore studied the effect of external current on the PE. In the insert of Fig. 3 we show the V_x-T/T_c curves for various driving forces at fixed pinning force. For large F^L $(= 3f_0)$, no PE could be observed, just as the case for small pinning force. Decreasing the driving force F^L $(= 2.5f_0)$ a dip in V_x-T/T_c curve is progressively more pronounced, just as seen here, see Fig. 3 in Ref.[32]. As the driving force is further decreased $(F^L = 2f_0)$, the pinning force becomes dominant, leading to a broadening of the PE[4].

Finally, in order to clarify the role of thermal fluctuations[7], we studied the effects of thermal fluctuations on the PE. Shown in Fig. 4 are V_x-T/T_c characteristics for various f_{th}. It can be seen that the PE is growing with enhanced thermal fluctuations. The reason is that the "shaking" effect of thermal fluctuation can effectively reduce the relaxation time within which the vortices diffuse into the potential wells. To further confirm this we study the effects of the $\mathrm{d}T/\mathrm{d}t$ on the PE under $f_{\mathrm{th}} = 0$, which is shown in the inset of Fig. 4. It is found that the PE occurs for low $\mathrm{d}T/\mathrm{d}t$ $(= -0.002T_c/t_0)$ but disappears for large $\mathrm{d}T/\mathrm{d}t$ $(= -0.02T_c/t_0)$. This is a strong evidence that thermal fluctuations are not crucial for the occurrence of the PE in our 2D vortex system, contrasting with the reported result[7].

In summary, we performed 2D-Langevin simulations studying the PE for a vortex system with random strong pinning at various temperatures. The pinning centers act independently when they pin the disordered vortices. Due to different temperature dependencies the forces of vortex-vortex, vortex-pin and thermal fluctuations compete with each other as temperature changes. The PE is ubiquitously observed for certain strengths and densities of pinning sites and the Lorentz force, which is in good agreement with the experiments. No vortex order-disorder transition or collective-individual vortex pinning cross is observed. The PE can be simply explained by the competition between pinning and elastic forces due to their different temperature dependencies. The relaxation relating to the PE is observed by changing the temperature ramping rate or thermal fluctuation force, demonstrating its dynamical characteristics. In addition, thermal fluctuations are not crucial to the PE in the strongly pinned system but it can speed up the process for the formation of the PE.

* Corresponding author. Email: xxb@nju.edu.cn

[1] A. B. Pippard, Phil. Mag. **19**, 597 (1969).
[2] T. Giamarchi and S. Bhattacharya, cond-mat/0111052 (2001).
[3] W. K. Kwok et al., Phys. Rev. Lett.
[4] W. Henderson et al., Phys. Rev. Lett. **77**, 2077 (1996).
[5] Z. L. Xiao et al., Phys. Rev. Lett. **85**, 3265 (2000).
[6] M. Werner et. al., Phys. Rev. B **61**, 14795 (2000).
[7] S. Mohan et al., Phys. Rev. Lett. **98**, 027003 (2007).
[8] I. K. Dimitrov et al., Phys. Rev. Lett. **99**, 047001 (2007).
[9] X. S. Ling et al., Phys. Rev. Lett. **86**, 712 (2001).
[10] Y. Wang et al., Phys. Rev. B **75**, 134505 (2007).
[11] G. Blatter et al., Rev. Mod. Phys. **66**, 1125 (1994); E. H. Brandt, Rep. Prog. Phys. **58**, 1465 (1995).
[12] R. Labushch, Cryst. Lattice Defects **1**, 1 (1969).
[13] A. I. Larkin and Y. N. Ovchinnikov, J. Low. Temp.Phys. **34**, 409(1979). **73**, 2614 (1994).
[14] P. Koorevaar et al., Phys. Rev. B **42**, 1004 (1990).
[15] G. Blatter et al., Phys. Rev. Lett. **92**, 067009 (2004).
[16] Y. Paltiel et al., Phys. Rev. Lett. **85**, 3712 (2000). which are strong pinning
[17] T. V. C. Rao et al., Physica C **299**, 267 (1998).
[18] M. J. Higgins, and S. Bhattacharya, Physica C **257**, 232 (1996).
[19] A. Pautrat et al., Phys. Rev. B **75**, 224512 (2007).
[20] Y. Fasano et al., Phys. Rev. B **66**, R020512 (2002).
[21] M. Chandran, cond-mat/0407309 (2006).
[22] F. P. Missell et al., Phys. Rev. B **14**, 2255 (1976).
[23] A. M. Campbell and J. E. Evetts, Adv. in Phys **21**, 199 (1972).
[24] X. B. Xu et al., Phys. Rev. B **73**, 214521 (2006); X. B. Xu et al., Phys. Rev. B **75**, 224507 (2007).
[25] H. Fangohr et al., Comput. Phys. **162**, 372 (2000).
[26] B. Y. Zhu et al, Phys. Rev. Lett. **92**, 180602 (2004).
[27] H. J. Jensen et al., Phys. Rev. B **41**, 6394 (1990).
[28] M. Tinkham, Introduction to Superconductivity, McGraw Hill, New York (1975)
[29] B. Y. Zhu et al., Phys. Rev. B **57**, 5063 (1998).
[30] I. V. Grigorieva et al., Phys. Rev. Lett. **99**, 147003 (2007).
[31] R. Meier-Hirmer et al., Phys. Rev. B **31**, 183 (1985).
[32] W. K. Kwok et al., Phys. Rev. B **58**, 14594 (1998).