

Introduction

Demagnetization field, \mathbf{H}_d , is related to the magnetic moment (actually: magnetization) that is the source of this field, \mathbf{M} , by following equation:

$$\mathbf{H}_d = -N_d \mathbf{M},$$

where N_d (or N_{demag}) is the demagnetization tensor.

In following we will evaluate N_d for the case of a rectangular prism. Exact analytical formulas for this case are derived e.g. by Newell (A. J. Newell, W. Williams, and D. J. Dunlop, J. Geophys. Res., [Atmos.] 98, 9551, 1993). Here, we will focus on their approximations: simple dipolar and higher-orders. This last is based on two sources:

- Explanation given by Michael Donahue during his talk at the 6th International Symposium on Hysteresis Modeling and Micromagnetics HMM-2007, Naples, Italy, 2007.

- OOMMF code, version oommf12a4pre-20100909, file demagcoef.cc, functions inside DemagNxxAsymptotic, DemagNxyAsymptotic, and so on.

(Mikes comment "See Notes IV, 26-Feb-2007, p102."). One should call it reverse engineering, I suppose.

We will also evaluate the MFM signal that is related to the derivative of the (demagnetization) field, thus to the derivative of the demagnetization tensor:

$$d(-N_d(\mathbf{r}) \mathbf{M})/d\mathbf{r} = \mathbf{M} d(-N_d(\mathbf{r}))/d\mathbf{r}.$$

Actually, what matters is the derivative along the orthogonal axis, say: z , and only the z -component of this derivative (because the tip is usually magnetized in the z -direction):

$$\left[d(-N_d(\mathbf{r}) \mathbf{M})/dz \right]_z = \mathbf{M} d(-\{N_{zx}, N_{zy}, N_{zz}\})/dz.$$

Remark: N_d is symmetric, e.g. $N_{zx} = N_{xz}$.

Everywhere in the following n_{XX} , n_{XY} mean the xx - or xy -element of following tensor: $-N_{\text{demag}}$. I stress: these are negative values of the demagnetization tensor elements.

Simple dipolar approximation (for debugging mainly) Introduced functions have names: nXXdipol, nXXdipol, etc.

```
Clear[nXXdipol];
nXXdipol[x_, y_, z_, dx_, dy_, dz_] :=
  Block[{rR2, rR},
    rR2 = x * x + y * y + z * z;
    rR = Sqrt[rR2];
    - (mu0 * dx * dy * dz / (4 * pi * rR^5)) * (3 * x^2 - rR^2)
  ];

Clear[nZZdipol];
nZZdipol[x_, y_, z_, dx_, dy_, dz_] := nXXdipol[z, y, x, dz, dy, dx];

Clear[nXYdipol];
nXYdipol[x_, y_, z_, dx_, dy_, dz_] :=
  Block[{rR2, rR},
    rR2 = x * x + y * y + z * z;
    rR = Sqrt[rR2];
    - (mu0 * dx * dy * dz / (4 * pi * rR^5)) * (3 * x * y)
  ];
```

```
Clear[nXZdipol, nYZdipol];  
nXZdipol[x_, y_, z_, dx_, dy_, dz_] := nXYdipol[x, z, y, dx, dz, dy];  
nYZdipol[x_, y_, z_, dx_, dy_, dz_] := nXYdipol[y, z, x, dy, dz, dx];
```

Definitions of “asymptotic (dipolar + higher) approximation for far field”, taken from the OOMMF code.

Introduced functions have names: nXXfunNoncubic, nXXfunCubic, etc.

Renamed variables :

- R --> rR

- R2 --> rR2

- Nxx --> nNxx

Non-cube case means: $dx^2 \neq dy^2 \parallel dx^2 \neq dz^2 \parallel dy^2 \neq dz^2$

```

Clear[nXXfunNoncubic];
nXXfunNoncubic[x_, y_, z_, dx_, dy_, dz_] :=
Block[{a1, a2, a3, a4, a5, a6, b10, b1, b2, b3, b4, b5, b6, b7, b8, b9, dx2, dx4, dy2, dy4,
  dz2, dz4, rR2, rR, sx2, sx4, sx6, sy2, sy4, sy6, sz2, sz4, sz6, term3, term5, term7},
  rR2 = x * x + y * y + z * z;
  rR = Sqrt[rR2];
  sx2 = x * x / rR2;
  sy2 = y * y / rR2;
  sz2 = z * z / rR2;
  sx4 = sx2 * sx2;
  sy4 = sy2 * sy2;
  sz4 = sz2 * sz2;
  sx6 = sx4 * sx2;
  sy6 = sy4 * sy2;
  sz6 = sz4 * sz2;
  dx2 = dx * dx;
  dy2 = dy * dy;
  dz2 = dz * dz;
  dx4 = dx2 * dx2;
  dy4 = dy2 * dy2;
  dz4 = dz2 * dz2;
  term3 = 2 * sx2 - sy2 - sz2;
  a1 = 8 * dx2 - 4 * dy2 - 4 * dz2;
  a2 = -24 * dx2 + 27 * dy2 - 3 * dz2;
  a3 = -24 * dx2 - 3 * dy2 + 27 * dz2;
  a4 = 3 * dx2 - 4 * dy2 + 1 * dz2;
  a5 = 6 * dx2 - 3 * dy2 - 3 * dz2;
  a6 = 3 * dx2 + 1 * dy2 - 4 * dz2;
  term5 = a1 * sx4 + a2 * sx2 * sy2 + a3 * sx2 * sz2 + a4 * sy4 + a5 * sy2 * sz2 + a6 * sz4;
  term5 *= 1 / 4;
  term7 = 0;
  b1 = 32 * dx4 - 40 * dx2 * dy2 - 40 * dx2 * dz2 + 12 * dy4 + 10 * dy2 * dz2 + 12 * dz4;
  b2 = -240 * dx4 + 580 * dx2 * dy2 + 20 * dx2 * dz2 - 202 * dy4 - 75 * dy2 * dz2 + 22 * dz4;
  b3 = -240 * dx4 + 20 * dx2 * dy2 + 580 * dx2 * dz2 + 22 * dy4 - 75 * dy2 * dz2 - 202 * dz4;
  b4 = 180 * dx4 - 505 * dx2 * dy2 + 55 * dx2 * dz2 + 232 * dy4 - 75 * dy2 * dz2 + 8 * dz4;
  b5 = 360 * dx4 - 450 * dx2 * dy2 - 450 * dx2 * dz2 - 180 * dy4 + 900 * dy2 * dz2 - 180 * dz4;
  b6 = 180 * dx4 + 55 * dx2 * dy2 - 505 * dx2 * dz2 + 8 * dy4 - 75 * dy2 * dz2 + 232 * dz4;
  b7 = -10 * dx4 + 30 * dx2 * dy2 - 5 * dx2 * dz2 - 16 * dy4 + 10 * dy2 * dz2 - 2 * dz4;
  b8 = -30 * dx4 + 55 * dx2 * dy2 + 20 * dx2 * dz2 + 8 * dy4 - 75 * dy2 * dz2 + 22 * dz4;
  b9 = -30 * dx4 + 20 * dx2 * dy2 + 55 * dx2 * dz2 + 22 * dy4 - 75 * dy2 * dz2 + 8 * dz4;
  b10 = -10 * dx4 - 5 * dx2 * dy2 + 30 * dx2 * dz2 - 2 * dy4 + 10 * dy2 * dz2 - 16 * dz4;
  term7 = b1 * sx6 + b2 * sx4 * sy2 + b3 * sx4 * sz2 + b4 * sx2 * sy4 +
    b5 * sx2 * sy2 * sz2 + b6 * sx2 * sz4 + b7 * sy6 + b8 * sy4 * sz2 + b9 * sy2 * sz4 + b10 * sz6;
  term7 *= 1 / 16;
  (-dx * dy * dz * mu0 / (4 * pi)) * (((term7 / rR2 + term5) / rR2 + term3) / (rR2 * rR))
];

```

```

Clear[nXXfunCubic];
nXXfunCubic[x_, y_, z_, dx_, dy_, dz_] :=
Block[{a1, a2, a3, a4, a5, a6, b10, b1, b2, b3, b4, b5, b6, b7, b8, b9, dx2, dx4, dy2, dy4,
  dz2, dz4, rR2, rR, sx2, sx4, sx6, sy2, sy4, sy6, sz2, sz4, sz6, term3, term5, term7},
  rR2 = x * x + y * y + z * z;
  rR = Sqrt[rR2];
  sx2 = x * x / rR2;
  sy2 = y * y / rR2;
  sz2 = z * z / rR2;
  sx4 = sx2 * sx2;
  sy4 = sy2 * sy2;
  sz4 = sz2 * sz2;
  sx6 = sx4 * sx2;
  sy6 = sy4 * sy2;
  sz6 = sz4 * sz2;
  dx2 = dx * dx;
  dy2 = dy * dy;
  dz2 = dz * dz;
  dx4 = dx2 * dx2;
  dy4 = dy2 * dy2;
  dz4 = dz2 * dz2;
  term3 = 2 * sx2 - sy2 - sz2;
  (* a1=8*dx2-4*dy2-4*dz2;
  a2=-24*dx2+27*dy2-3*dz2;
  a3=-24*dx2-3*dy2+27*dz2;
  a4=3*dx2-4*dy2+1*dz2;
  a5=6*dx2-3*dy2-3*dz2;
  a6=3*dx2+1*dy2-4*dz2;
  term5=a1*sx4+a2*sx2*sy2+a3*sx2*sz2+a4*sy4+a5*sy2*sz2+a6*sz4;
  term5*=0.25; *)
  term5 = 0;
  term7 = 0;
  b1 = 32 * dx4 - 40 * dx2 * dy2 - 40 * dx2 * dz2 + 12 * dy4 + 10 * dy2 * dz2 + 12 * dz4;
  b2 = -240 * dx4 + 580 * dx2 * dy2 + 20 * dx2 * dz2 - 202 * dy4 - 75 * dy2 * dz2 + 22 * dz4;
  b3 = -240 * dx4 + 20 * dx2 * dy2 + 580 * dx2 * dz2 + 22 * dy4 - 75 * dy2 * dz2 - 202 * dz4;
  b4 = 180 * dx4 - 505 * dx2 * dy2 + 55 * dx2 * dz2 + 232 * dy4 - 75 * dy2 * dz2 + 8 * dz4;
  b5 = 360 * dx4 - 450 * dx2 * dy2 - 450 * dx2 * dz2 - 180 * dy4 + 900 * dy2 * dz2 - 180 * dz4;
  b6 = 180 * dx4 + 55 * dx2 * dy2 - 505 * dx2 * dz2 + 8 * dy4 - 75 * dy2 * dz2 + 232 * dz4;
  b7 = -10 * dx4 + 30 * dx2 * dy2 - 5 * dx2 * dz2 - 16 * dy4 + 10 * dy2 * dz2 - 2 * dz4;
  b8 = -30 * dx4 + 55 * dx2 * dy2 + 20 * dx2 * dz2 + 8 * dy4 - 75 * dy2 * dz2 + 22 * dz4;
  b9 = -30 * dx4 + 20 * dx2 * dy2 + 55 * dx2 * dz2 + 22 * dy4 - 75 * dy2 * dz2 + 8 * dz4;
  b10 = -10 * dx4 - 5 * dx2 * dy2 + 30 * dx2 * dz2 - 2 * dy4 + 10 * dy2 * dz2 - 16 * dz4;
  term7 = b1 * sx6 + b2 * sx4 * sy2 + b3 * sx4 * sz2 + b4 * sx2 * sy4 +
    b5 * sx2 * sy2 * sz2 + b6 * sx2 * sz4 + b7 * sy6 + b8 * sy4 * sz2 + b9 * sy2 * sz4 + b10 * sz6;
  term7 *= 1 / 16;
  (-dx * dy * dz * mu0 / (4 * pi)) * (((term7 / rR2 + term5) / rR2 + term3) / (rR2 * rR))
];

```

```

Clear[nZZfunCubic, nZZfunNoncubic];
nZZfunCubic[x_, y_, z_, dx_, dy_, dz_] := nXXfunCubic[z, y, x, dz, dy, dx];
nZZfunNoncubic[x_, y_, z_, dx_, dy_, dz_] := nXXfunNoncubic[z, y, x, dz, dy, dx];

Clear[nXYfunNoncubic];
nXYfunNoncubic[x_, y_, z_, dx_, dy_, dz_] :=
Block[{a1, a2, a3, b1, b2, b3, b4, b5, b6, dx2, dx4, dy2,
  dy4, dz2, dz4, rR2, rR, sx2, sx4, sy2, sy4, sz2, sz4, term3, term5, term7},
  rR2 = x * x + y * y + z * z;
  rR = Sqrt[rR2];
  sx2 = x * x / rR2;
  sy2 = y * y / rR2;
  sz2 = z * z / rR2;
  sx4 = sx2 * sx2;
  sy4 = sy2 * sy2;
  sz4 = sz2 * sz2;
  dx2 = dx * dx;
  dy2 = dy * dy;
  dz2 = dz * dz;
  dx4 = dx2 * dx2;
  dy4 = dy2 * dy2;
  dz4 = dz2 * dz2;
  term3 = 3;
  a1 = 4 * dx2 - 3 * dy2 - 1 * dz2;
  a2 = -3 * dx2 + 4 * dy2 - 1 * dz2;
  a3 = -3 * dx2 - 3 * dy2 + 6 * dz2;
  term5 = a1 * sx2 + a2 * sy2 + a3 * sz2;
  term5 *= 5 / 4;
  b1 = 16 * dx4 - 30 * dx2 * dy2 - 10 * dx2 * dz2 + 10 * dy4 + 5 * dy2 * dz2 + 2 * dz4;
  b2 = -40 * dx4 + 105 * dx2 * dy2 - 5 * dx2 * dz2 - 40 * dy4 - 5 * dy2 * dz2 + 4 * dz4;
  b3 = -40 * dx4 - 15 * dx2 * dy2 + 115 * dx2 * dz2 + 20 * dy4 - 35 * dy2 * dz2 - 32 * dz4;
  b4 = 10 * dx4 - 30 * dx2 * dy2 + 5 * dx2 * dz2 + 16 * dy4 - 10 * dy2 * dz2 + 2 * dz4;
  b5 = 20 * dx4 - 15 * dx2 * dy2 - 35 * dx2 * dz2 - 40 * dy4 + 115 * dy2 * dz2 - 32 * dz4;
  b6 = 10 * dx4 + 15 * dx2 * dy2 - 40 * dx2 * dz2 + 10 * dy4 - 40 * dy2 * dz2 + 32 * dz4;
  term7 = b1 * sx4 + b2 * sx2 * sy2 + b3 * sx2 * sz2 + b4 * sy4 + b5 * sy2 * sz2 + b6 * sz4;
  term7 *= 7 / 16;
  (-dx * dy * dz * x * y * mu0 / (4 * pi * rR2)) * (((term7 / rR2 + term5) / rR2 + term3) / (rR2 * rR))
];

```

```

Clear[nXYfunCubic];
nXYfunCubic[x_, y_, z_, dx_, dy_, dz_] :=
  Block[{a1, a2, a3, b1, b2, b3, b4, b5, b6, dx2, dx4, dy2,
    dy4, dz2, dz4, rR2, rR, sx2, sx4, sy2, sy4, sz2, sz4, term3, term5, term7},
    rR2 = x * x + y * y + z * z;
    rR = Sqrt[rR2];
    sx2 = x * x / rR2;
    sy2 = y * y / rR2;
    sz2 = z * z / rR2;
    sx4 = sx2 * sx2;
    sy4 = sy2 * sy2;
    sz4 = sz2 * sz2;
    dx2 = dx * dx;
    dy2 = dy * dy;
    dz2 = dz * dz;
    dx4 = dx2 * dx2;
    dy4 = dy2 * dy2;
    dz4 = dz2 * dz2;
    term3 = 3;
    (*a1=4*dx2-3*dy2-1*dz2;
    a2=-3*dx2+4*dy2-1*dz2;
    a3=-3*dx2-3*dy2+6*dz2;
    term5=a1*sx2+a2*sy2+a3*sz2;
    term5*=5/4;*)
    term5 = 0;
    b1 = 16 * dx4 - 30 * dx2 * dy2 - 10 * dx2 * dz2 + 10 * dy4 + 5 * dy2 * dz2 + 2 * dz4;
    b2 = -40 * dx4 + 105 * dx2 * dy2 - 5 * dx2 * dz2 - 40 * dy4 - 5 * dy2 * dz2 + 4 * dz4;
    b3 = -40 * dx4 - 15 * dx2 * dy2 + 115 * dx2 * dz2 + 20 * dy4 - 35 * dy2 * dz2 - 32 * dz4;
    b4 = 10 * dx4 - 30 * dx2 * dy2 + 5 * dx2 * dz2 + 16 * dy4 - 10 * dy2 * dz2 + 2 * dz4;
    b5 = 20 * dx4 - 15 * dx2 * dy2 - 35 * dx2 * dz2 - 40 * dy4 + 115 * dy2 * dz2 - 32 * dz4;
    b6 = 10 * dx4 + 15 * dx2 * dy2 - 40 * dx2 * dz2 + 10 * dy4 - 40 * dy2 * dz2 + 32 * dz4;
    term7 = b1 * sx4 + b2 * sx2 * sy2 + b3 * sx2 * sz2 + b4 * sy4 + b5 * sy2 * sz2 + b6 * sz4;
    term7 *= 7 / 16;
    (-dx * dy * dz * x * y * mu0 / (4 * pi * rR2)) * (((term7 / rR2 + term5) / rR2 + term3) / (rR2 * rR))
  ];

Clear[nXZfunCubic, nXZfunNoncubic, nYZfunCubic, nYZfunNoncubic];
nXZfunCubic[x_, y_, z_, dx_, dy_, dz_] := nXYfunCubic[x, z, y, dx, dz, dy];
nXZfunNoncubic[x_, y_, z_, dx_, dy_, dz_] := nXYfunNoncubic[x, z, y, dx, dz, dy];
nYZfunCubic[x_, y_, z_, dx_, dy_, dz_] := nXYfunCubic[y, z, x, dy, dz, dx];
nYZfunNoncubic[x_, y_, z_, dx_, dy_, dz_] := nXYfunNoncubic[y, z, x, dy, dz, dx];

```

Calculating the MFM signal, i.e. $D[\{nXZ, nYZ, nZZ\}, z]$

First-order dipolar approximation

```

tmp = FullSimplify[{nXZdipol[x, y, z, dx, dy, dz],
  nYZdipol[x, y, z, dx, dy, dz], nZZdipol[x, y, z, dx, dy, dz]}, dx > 0 && dy > 0 && dz > 0];

```

```
dN = D[tmp, z];
```

```
dNdip = FullSimplify[dN, dx > 0 && dy > 0 && dz > 0]
```

$$\left\{ -\frac{3 \, dx \, dy \, dz \, \mu_0 \, x \, (x^2 + y^2 - 4 \, z^2)}{4 \, \pi \, (x^2 + y^2 + z^2)^{7/2}}, -\frac{3 \, dx \, dy \, dz \, \mu_0 \, y \, (x^2 + y^2 - 4 \, z^2)}{4 \, \pi \, (x^2 + y^2 + z^2)^{7/2}}, \frac{3 \, dx \, dy \, dz \, \mu_0 \, z \, (-3 \, (x^2 + y^2) + 2 \, z^2)}{4 \, \pi \, (x^2 + y^2 + z^2)^{7/2}} \right\}$$

Higher-order approximation, cubic case

```
tmp = FullSimplify[{nXZfunCubic[x, y, z, dx, dy, dz],
```

```
  nYZfunCubic[x, y, z, dx, dy, dz], nZZfunCubic[x, y, z, dx, dy, dz]}, dx > 0 && dy > 0 && dz > 0];
```

```
dN = D[tmp, z];
```

```
dNcc = FullSimplify[dN /. {dy -> dx, dz -> dx}, dx > 0]
```

$$\left\{ \left(dx^3 \mu_0 x \left(-48 \left(x^2 + y^2 - 4 \, z^2 \right) \left(x^2 + y^2 + z^2 \right)^4 + 7 \, dx^4 \left(7 \, x^6 - 6 \, x^4 \, y^2 + 13 \, y^6 - 3 \left(47 \, x^4 - 20 \, x^2 \, y^2 + 65 \, y^4 \right) z^2 + 15 \left(15 \, x^2 + 11 \, y^2 \right) z^4 - 56 \, z^6 \right) \right) / \left(64 \, \pi \left(x^2 + y^2 + z^2 \right)^{15/2} \right), \left(dx^3 \mu_0 y \left(-48 \left(x^2 + y^2 - 4 \, z^2 \right) \left(x^2 + y^2 + z^2 \right)^4 + 7 \, dx^4 \left(13 \, x^6 - 6 \, x^2 \, y^4 + 7 \, y^6 - 3 \left(65 \, x^4 - 20 \, x^2 \, y^2 + 47 \, y^4 \right) z^2 + 15 \left(11 \, x^2 + 15 \, y^2 \right) z^4 - 56 \, z^6 \right) \right) / \left(64 \, \pi \left(x^2 + y^2 + z^2 \right)^{15/2} \right), - \left(dx^3 \mu_0 z \left(48 \left(3 \left(x^2 + y^2 \right) - 2 \, z^2 \right) \left(x^2 + y^2 + z^2 \right)^4 - 7 \, dx^4 \left(43 \, x^6 + 30 \, x^4 \, y^2 + 30 \, x^2 \, y^4 + 43 \, y^6 - 15 \left(15 \, x^4 + 8 \, x^2 \, y^2 + 15 \, y^4 \right) z^2 + 147 \left(x^2 + y^2 \right) z^4 - 14 \, z^6 \right) \right) / \left(64 \, \pi \left(x^2 + y^2 + z^2 \right)^{15/2} \right) \right\}$$

Higher-order approximation, non-cubic case

```
tmp = FullSimplify[{nXZfunNoncubic[x, y, z, dx, dy, dz], nYZfunNoncubic[x, y, z, dx, dy, dz],
```

```
  nZZfunNoncubic[x, y, z, dx, dy, dz]}, dx > 0 && dy > 0 && dz > 0];
```

```
dN = D[tmp, z];
```

`dNncc = FullSimplify[dN, dx > 0 && dy > 0 && dz > 0]`

$$\left\{ \frac{1}{64 \pi (x^2 + y^2 + z^2)^{15/2}} \right. \\
\begin{aligned}
& dx \, dy \, dz \, \mu_0 x \left(-14 \, dy^4 (x^6 - 15 x^4 y^2 + 16 y^6 - 6 (x^4 - 25 x^2 y^2 + 40 y^4) z^2 - 15 (x^2 - 11 y^2) z^4 - 8 z^6) - \right. \\
& 14 \, dx^4 (8 x^6 - 15 x^2 (y^2 - 15 z^2) (y^2 + z^2) + 5 (y^2 - 8 z^2) (y^2 + z^2)^2 - 12 x^4 (y^2 + 13 z^2)) + \\
& 2 (-24 (x^2 + y^2 - 4 z^2) (x^2 + y^2 + z^2)^4 + 30 \, dz^2 (x^2 + y^2 + z^2)^2 ((x^2 + y^2)^2 - 12 (x^2 + y^2) z^2 + 8 z^4) + \\
& 7 \, dz^4 (-5 (x^2 + y^2)^3 + 120 (x^2 + y^2)^2 z^2 - 240 (x^2 + y^2) z^4 + 64 z^6) + \\
& 5 \, dy^2 (7 \, dz^2 (- (x^2 - 8 y^2) (x^2 + y^2)^2 + 15 (x^2 - 11 y^2) (x^2 + y^2) z^2 + 240 y^2 z^4 - 16 z^6) + \\
& 4 (x^2 + y^2 + z^2)^2 (x^4 - 6 y^4 + 51 y^2 z^2 - 6 z^4 - 5 x^2 (y^2 + z^2))) + \\
& 5 \, dx^2 (21 \, dz^2 (2 x^6 + 3 x^4 y^2 - y^6 + 15 (-3 x^4 - 2 x^2 y^2 + y^4) z^2 + 80 x^2 z^4 - 16 z^6) - \\
& 4 (x^2 + y^2 + z^2)^2 (4 x^4 + x^2 (y^2 - 41 z^2) - 3 (y^2 - 6 z^2) (y^2 + z^2)) + 7 \, dy^2 (2 x^6 - 21 x^4 (y^2 + z^2) - \\
& 15 x^2 (y^4 - 20 y^2 z^2 + z^4) + (y^2 + z^2) (8 y^4 - 83 y^2 z^2 + 8 z^4))) \left. \right), \frac{1}{64 \pi (x^2 + y^2 + z^2)^{15/2}} \\
& dx \, dy \, dz \, \mu_0 y \left(-14 \, dx^4 (16 x^6 - 240 x^4 z^2 - 15 x^2 (y^2 - 11 z^2) (y^2 + z^2) + (y^2 - 8 z^2) (y^2 + z^2)^2) - \right. \\
& 14 \, dy^4 (5 x^6 + 8 y^6 - 156 y^4 z^2 + 225 y^2 z^4 - 40 z^6 - 15 x^4 (y^2 + 2 z^2) - 3 x^2 (4 y^4 - 70 y^2 z^2 + 25 z^4)) + \\
& 2 (-24 (x^2 + y^2 - 4 z^2) (x^2 + y^2 + z^2)^4 + 30 \, dz^2 (x^2 + y^2 + z^2)^2 ((x^2 + y^2)^2 - 12 (x^2 + y^2) z^2 + 8 z^4) + \\
& 7 \, dz^4 (-5 (x^2 + y^2)^3 + 120 (x^2 + y^2)^2 z^2 - 240 (x^2 + y^2) z^4 + 64 z^6) + \\
& 5 \, dy^2 (-21 \, dz^2 (x^6 - 3 x^2 y^4 - 2 y^6 - 15 (x^2 - 3 y^2) (x^2 + y^2) z^2 - 80 y^2 z^4 + 16 z^6) + \\
& 4 (x^2 + y^2 + z^2)^2 (3 x^4 - 4 y^4 + 41 y^2 z^2 - 18 z^4 - x^2 (y^2 + 15 z^2))) + \\
& 5 \, dx^2 (-4 (x^2 + y^2 + z^2)^2 (6 x^4 + 5 x^2 y^2 - y^4 - 51 x^2 z^2 + 5 y^2 z^2 + 6 z^4) + \\
& 7 \, dz^2 ((8 x^2 - y^2) (x^2 + y^2)^2 + 15 (-11 x^4 - 10 x^2 y^2 + y^4) z^2 + 240 x^2 z^4 - 16 z^6) + \\
& 7 \, dy^2 (8 x^6 - 15 x^4 (y^2 + 5 z^2) + (y^2 + z^2) (2 y^4 - 23 y^2 z^2 + 8 z^4) - 3 x^2 (7 y^4 - 100 y^2 z^2 + 25 z^4))) \left. \right), \\
& - \frac{1}{64 \pi (x^2 + y^2 + z^2)^{15/2}} dx \, dy \, dz \, \mu_0 z \left(42 \, dy^4 (x^6 - 15 x^4 y^2 + 16 y^6 + 10 y^2 (3 x^2 - 8 y^2) z^2 - \right. \\
& 3 (x^2 - 15 y^2) z^4 - 2 z^6) + 42 \, dx^4 (16 x^6 + y^6 - 80 x^4 z^2 - 3 y^2 z^4 - 2 z^6 - 15 x^2 (y^2 - 3 z^2) (y^2 + z^2)) + \\
& 2 (24 (3 (x^2 + y^2) - 2 z^2) (x^2 + y^2 + z^2)^4 - 10 \, dz^2 (x^2 + y^2 + z^2)^2 (15 (x^2 + y^2)^2 - 40 (x^2 + y^2) z^2 + 8 z^4) + \\
& 7 \, dz^4 (35 (x^2 + y^2)^3 - 210 (x^2 + y^2)^2 z^2 + 168 (x^2 + y^2) z^4 - 16 z^6) + \\
& 5 \, dy^2 (-4 (x^2 + y^2 + z^2)^2 (3 (x^2 - 6 y^2) (x^2 + y^2) - (x^2 - 41 y^2) z^2 - 4 z^4) + \\
& 7 \, dz^2 (5 (x^2 - 8 y^2) (x^2 + y^2)^2 - 15 (x^2 - 15 y^2) (x^2 + y^2) z^2 - 12 (x^2 + 13 y^2) z^4 + 8 z^6) - \\
& 5 \, dx^2 (7 \, dz^2 (5 (8 x^2 - y^2) (x^2 + y^2)^2 + 15 (-15 x^4 - 14 x^2 y^2 + y^4) z^2 + 12 (13 x^2 + y^2) z^4 - 8 z^6) + \\
& 7 \, dy^2 (8 x^6 - 75 x^4 y^2 - 75 x^2 y^4 + 8 y^6 - 15 (x^4 - 20 x^2 y^2 + y^4) z^2 - 21 (x^2 + y^2) z^4 + 2 z^6) - \\
& 4 (x^2 + y^2 + z^2)^2 (18 x^4 - 3 y^4 + y^2 z^2 + 4 z^4 + x^2 (15 y^2 - 41 z^2))) \left. \right) \left. \right\}
\end{aligned}$$

Comparison between dipolar- and higher-order approximations. Part I

Forgetting the multiplicative factor $\frac{1}{64 \pi (x^2 + y^2 + z^2)^{15/2}}$ $dx \, dy \, dz \, \mu_0$ we have

tmp =

$$\begin{aligned}
 & \text{Expand} \left[\left\{ x \left(-2 \left(x^2 + y^2 \right)^3 \left(35 dz^4 + 24 \left(x^2 + y^2 \right)^2 \right) + 1680 dz^4 \left(x^2 + y^2 \right)^2 z^2 + 480 \left(x^2 + y^2 \right) \left(-7 dz^4 + \left(x^2 + y^2 \right)^2 \right) z^4 + \right. \right. \\
 & \quad 64 \left(14 dz^4 + 15 \left(x^2 + y^2 \right)^2 \right) z^6 + 720 \left(x^2 + y^2 \right) z^8 + 192 z^{10} - \\
 & \quad 14 dy^4 \left(x^6 - 15 x^4 y^2 + 16 y^6 - 6 \left(x^4 - 25 x^2 y^2 + 40 y^4 \right) z^2 - 15 \left(x^2 - 11 y^2 \right) z^4 - 8 z^6 \right) - \\
 & \quad 35 dy^2 dz^2 \left(\left(x^2 - 8 y^2 \right) \left(x^2 + y^2 \right)^2 - 15 \left(x^2 - 11 y^2 \right) \left(x^2 + y^2 \right) z^2 - 240 y^2 z^4 + 16 z^6 \right) - \\
 & \quad 14 dx^4 \left(8 x^6 - 15 x^2 \left(y^2 - 15 z^2 \right) \left(y^2 + z^2 \right) + 5 \left(y^2 - 8 z^2 \right) \left(y^2 + z^2 \right)^2 - 12 x^4 \left(y^2 + 13 z^2 \right) \right) + \\
 & \quad 35 dx^2 \left(3 dz^2 \left(2 x^6 + 3 x^4 y^2 - y^6 + 15 \left(-3 x^4 - 2 x^2 y^2 + y^4 \right) z^2 + 80 x^2 z^4 - 16 z^6 \right) + \right. \\
 & \quad \left. dy^2 \left(2 x^6 - 21 x^4 \left(y^2 + z^2 \right) - 15 x^2 \left(y^4 - 20 y^2 z^2 + z^4 \right) + \left(y^2 + z^2 \right) \left(8 y^4 - 83 y^2 z^2 + 8 z^4 \right) \right) \right) \right\}, \\
 & y \left(-2 \left(x^2 + y^2 \right)^3 \left(35 dz^4 + 24 \left(x^2 + y^2 \right)^2 \right) + 1680 dz^4 \left(x^2 + y^2 \right)^2 z^2 + 480 \left(x^2 + y^2 \right) \left(-7 dz^4 + \left(x^2 + y^2 \right)^2 \right) z^4 + \right. \\
 & \quad 64 \left(14 dz^4 + 15 \left(x^2 + y^2 \right)^2 \right) z^6 + 720 \left(x^2 + y^2 \right) z^8 + 192 z^{10} - \\
 & \quad 14 dy^4 \left(5 x^6 - 15 x^4 y^2 - 12 x^2 y^4 + 8 y^6 - 6 \left(5 x^4 - 35 x^2 y^2 + 26 y^4 \right) z^2 - 75 \left(x^2 - 3 y^2 \right) z^4 - 40 z^6 \right) - \\
 & \quad 105 dy^2 dz^2 \left(x^6 - 3 x^2 y^4 - 2 y^6 - 15 \left(x^2 - 3 y^2 \right) \left(x^2 + y^2 \right) z^2 - 80 y^2 z^4 + 16 z^6 \right) - \\
 & \quad 14 dx^4 \left(16 x^6 - 240 x^4 z^2 - 15 x^2 \left(y^2 - 11 z^2 \right) \left(y^2 + z^2 \right) + \left(y^2 - 8 z^2 \right) \left(y^2 + z^2 \right)^2 \right) + \\
 & \quad 35 dx^2 \left(dz^2 \left(\left(8 x^2 - y^2 \right) \left(x^2 + y^2 \right)^2 + 15 \left(-11 x^4 - 10 x^2 y^2 + y^4 \right) z^2 + 240 x^2 z^4 - 16 z^6 \right) + \right. \\
 & \quad \left. dy^2 \left(8 x^6 - 15 x^4 \left(y^2 + 5 z^2 \right) + \left(y^2 + z^2 \right) \left(2 y^4 - 23 y^2 z^2 + 8 z^4 \right) - 3 x^2 \left(7 y^4 - 100 y^2 z^2 + 25 z^4 \right) \right) \right) \right) \right\}, \\
 & z \left(-2 \left(x^2 + y^2 \right)^3 \left(245 dz^4 + 72 \left(x^2 + y^2 \right)^2 \right) - 60 \left(x^2 + y^2 \right)^2 \left(-49 dz^4 + 8 \left(x^2 + y^2 \right)^2 \right) z^2 - \right. \\
 & \quad 48 \left(x^2 + y^2 \right) \left(49 dz^4 + 10 \left(x^2 + y^2 \right)^2 \right) z^4 + 224 dz^4 z^6 + 240 \left(x^2 + y^2 \right) z^8 + 96 z^{10} - \\
 & \quad 42 dy^4 \left(x^6 - 15 x^4 y^2 + 16 y^6 + 10 y^2 \left(3 x^2 - 8 y^2 \right) z^2 - 3 \left(x^2 - 15 y^2 \right) z^4 - 2 z^6 \right) - \\
 & \quad 35 dy^2 dz^2 \left(5 \left(x^2 - 8 y^2 \right) \left(x^2 + y^2 \right)^2 - 15 \left(x^2 - 15 y^2 \right) \left(x^2 + y^2 \right) z^2 - 12 \left(x^2 + 13 y^2 \right) z^4 + 8 z^6 \right) - \\
 & \quad 42 dx^4 \left(16 x^6 + y^6 - 80 x^4 z^2 - 3 y^2 z^4 - 2 z^6 - 15 x^2 \left(y^2 - 3 z^2 \right) \left(y^2 + z^2 \right) \right) + \\
 & \quad 35 dx^2 \left(dz^2 \left(5 \left(8 x^2 - y^2 \right) \left(x^2 + y^2 \right)^2 + 15 \left(-15 x^4 - 14 x^2 y^2 + y^4 \right) z^2 + 12 \left(13 x^2 + y^2 \right) z^4 - 8 z^6 \right) + \right. \\
 & \quad \left. dy^2 \left(8 x^6 - 75 x^4 y^2 - 75 x^2 y^4 + 8 y^6 - 15 \left(x^4 - 20 x^2 y^2 + y^4 \right) z^2 - 21 \left(x^2 + y^2 \right) z^4 + 2 z^6 \right) \right) \right) \right\} \right];
 \end{aligned}$$

For larger distances the dx,dy,dz (def.: dsomething) terms do not count so much, thus we consider only terms that do not have them, thus we take dsomething=0.

And we get back dipolar form :)

$$\text{FullSimplify} \left[\left(\text{tmp} /. \{ dx \rightarrow 0, dy \rightarrow 0, dz \rightarrow 0 \} \right) \frac{1}{64 \pi \left(x^2 + y^2 + z^2 \right)^{15/2}} dx dy dz \mu_0 \right]$$

$$\left\{ -\frac{3 dx dy dz \mu_0 x \left(x^2 + y^2 - 4 z^2 \right)}{4 \pi \left(x^2 + y^2 + z^2 \right)^{7/2}}, -\frac{3 dx dy dz \mu_0 y \left(x^2 + y^2 - 4 z^2 \right)}{4 \pi \left(x^2 + y^2 + z^2 \right)^{7/2}}, \frac{3 dx dy dz \mu_0 z \left(-3 \left(x^2 + y^2 \right) + 2 z^2 \right)}{4 \pi \left(x^2 + y^2 + z^2 \right)^{7/2}} \right\}$$

{nXZdipol[x, y, z, dx, dy, dz], nYZdipol[x, y, z, dx, dy, dz], nZZdipol[x, y, z, dx, dy, dz]}

$$\left\{ -\frac{3 dx dy dz \mu_0 x z}{4 \pi \left(x^2 + y^2 + z^2 \right)^{5/2}}, -\frac{3 dx dy dz \mu_0 y z}{4 \pi \left(x^2 + y^2 + z^2 \right)^{5/2}}, -\frac{dx dy dz \mu_0 \left(-x^2 - y^2 + 2 z^2 \right)}{4 \pi \left(x^2 + y^2 + z^2 \right)^{5/2}} \right\}$$

```

Simplify[
  % - {nXZdipol[x, y, z, dx, dy, dz], nYZdipol[x, y, z, dx, dy, dz], nZZdipol[x, y, z, dx, dy, dz]},
  Assumptions -> Element[x, Reals]]
{0, 0, 0}

```

Comparison between dipolar- and higher-order approximations. Part II

Forgetting the multiplicative factor $\frac{1}{64 \pi (x^2+y^2+z^2)^{15/2}}$ dx dy dz mu0 we have

$$\begin{aligned}
\text{tmp} = \text{Expand} & \left[\left\{ \mathbf{x} \left(-14 \, \text{dy}^4 \left(\mathbf{x}^6 - 15 \, \mathbf{x}^4 \, \mathbf{y}^2 + 16 \, \mathbf{y}^6 - 6 \left(\mathbf{x}^4 - 25 \, \mathbf{x}^2 \, \mathbf{y}^2 + 40 \, \mathbf{y}^4 \right) \mathbf{z}^2 - 15 \left(\mathbf{x}^2 - 11 \, \mathbf{y}^2 \right) \mathbf{z}^4 - 8 \, \mathbf{z}^6 \right) - \right. \right. \\
& 14 \, \text{dx}^4 \left(8 \, \mathbf{x}^6 - 15 \, \mathbf{x}^2 \left(\mathbf{y}^2 - 15 \, \mathbf{z}^2 \right) \left(\mathbf{y}^2 + \mathbf{z}^2 \right) + 5 \left(\mathbf{y}^2 - 8 \, \mathbf{z}^2 \right) \left(\mathbf{y}^2 + \mathbf{z}^2 \right)^2 - 12 \, \mathbf{x}^4 \left(\mathbf{y}^2 + 13 \, \mathbf{z}^2 \right) \right) + \\
& 2 \left(-24 \left(\mathbf{x}^2 + \mathbf{y}^2 - 4 \, \mathbf{z}^2 \right) \left(\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 \right)^4 + 30 \, \text{dz}^2 \left(\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 \right)^2 \left(\left(\mathbf{x}^2 + \mathbf{y}^2 \right)^2 - 12 \left(\mathbf{x}^2 + \mathbf{y}^2 \right) \mathbf{z}^2 + 8 \, \mathbf{z}^4 \right) + \right. \\
& \quad \left. 7 \, \text{dz}^4 \left(-5 \left(\mathbf{x}^2 + \mathbf{y}^2 \right)^3 + 120 \left(\mathbf{x}^2 + \mathbf{y}^2 \right)^2 \mathbf{z}^2 - 240 \left(\mathbf{x}^2 + \mathbf{y}^2 \right) \mathbf{z}^4 + 64 \, \mathbf{z}^6 \right) \right) + \\
& 5 \, \text{dy}^2 \left(7 \, \text{dz}^2 \left(-\left(\mathbf{x}^2 - 8 \, \mathbf{y}^2 \right) \left(\mathbf{x}^2 + \mathbf{y}^2 \right)^2 + 15 \left(\mathbf{x}^2 - 11 \, \mathbf{y}^2 \right) \left(\mathbf{x}^2 + \mathbf{y}^2 \right) \mathbf{z}^2 + 240 \, \mathbf{y}^2 \mathbf{z}^4 - 16 \, \mathbf{z}^6 \right) + \right. \\
& \quad \left. 4 \left(\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 \right)^2 \left(\mathbf{x}^4 - 6 \, \mathbf{y}^4 + 51 \, \mathbf{y}^2 \mathbf{z}^2 - 6 \, \mathbf{z}^4 - 5 \, \mathbf{x}^2 \left(\mathbf{y}^2 + \mathbf{z}^2 \right) \right) \right) + \\
& 5 \, \text{dx}^2 \left(21 \, \text{dz}^2 \left(2 \, \mathbf{x}^6 + 3 \, \mathbf{x}^4 \, \mathbf{y}^2 - \mathbf{y}^6 + 15 \left(-3 \, \mathbf{x}^4 - 2 \, \mathbf{x}^2 \, \mathbf{y}^2 + \mathbf{y}^4 \right) \mathbf{z}^2 + 80 \, \mathbf{x}^2 \mathbf{z}^4 - 16 \, \mathbf{z}^6 \right) - \right. \\
& \quad \left. 4 \left(\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 \right)^2 \left(4 \, \mathbf{x}^4 + \mathbf{x}^2 \left(\mathbf{y}^2 - 41 \, \mathbf{z}^2 \right) - 3 \left(\mathbf{y}^2 - 6 \, \mathbf{z}^2 \right) \left(\mathbf{y}^2 + \mathbf{z}^2 \right) \right) + \right. \\
& \quad \left. 7 \, \text{dy}^2 \left(2 \, \mathbf{x}^6 - 21 \, \mathbf{x}^4 \left(\mathbf{y}^2 + \mathbf{z}^2 \right) - 15 \, \mathbf{x}^2 \left(\mathbf{y}^4 - 20 \, \mathbf{y}^2 \mathbf{z}^2 + \mathbf{z}^4 \right) + \left(\mathbf{y}^2 + \mathbf{z}^2 \right) \left(8 \, \mathbf{y}^4 - 83 \, \mathbf{y}^2 \mathbf{z}^2 + 8 \, \mathbf{z}^4 \right) \right) \right) \right), \\
\mathbf{y} & \left(-14 \, \text{dx}^4 \left(16 \, \mathbf{x}^6 - 240 \, \mathbf{x}^4 \mathbf{z}^2 - 15 \, \mathbf{x}^2 \left(\mathbf{y}^2 - 11 \, \mathbf{z}^2 \right) \left(\mathbf{y}^2 + \mathbf{z}^2 \right) + \left(\mathbf{y}^2 - 8 \, \mathbf{z}^2 \right) \left(\mathbf{y}^2 + \mathbf{z}^2 \right)^2 \right) - \right. \\
& 14 \, \text{dy}^4 \left(5 \, \mathbf{x}^6 + 8 \, \mathbf{y}^6 - 156 \, \mathbf{y}^4 \mathbf{z}^2 + 225 \, \mathbf{y}^2 \mathbf{z}^4 - 40 \, \mathbf{z}^6 - 15 \, \mathbf{x}^4 \left(\mathbf{y}^2 + 2 \, \mathbf{z}^2 \right) - 3 \, \mathbf{x}^2 \left(4 \, \mathbf{y}^4 - 70 \, \mathbf{y}^2 \mathbf{z}^2 + 25 \, \mathbf{z}^4 \right) \right) + \\
& 2 \left(-24 \left(\mathbf{x}^2 + \mathbf{y}^2 - 4 \, \mathbf{z}^2 \right) \left(\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 \right)^4 + 30 \, \text{dz}^2 \left(\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 \right)^2 \left(\left(\mathbf{x}^2 + \mathbf{y}^2 \right)^2 - 12 \left(\mathbf{x}^2 + \mathbf{y}^2 \right) \mathbf{z}^2 + 8 \, \mathbf{z}^4 \right) + \right. \\
& \quad \left. 7 \, \text{dz}^4 \left(-5 \left(\mathbf{x}^2 + \mathbf{y}^2 \right)^3 + 120 \left(\mathbf{x}^2 + \mathbf{y}^2 \right)^2 \mathbf{z}^2 - 240 \left(\mathbf{x}^2 + \mathbf{y}^2 \right) \mathbf{z}^4 + 64 \, \mathbf{z}^6 \right) \right) + \\
& 5 \, \text{dy}^2 \left(-21 \, \text{dz}^2 \left(\mathbf{x}^6 - 3 \, \mathbf{x}^2 \, \mathbf{y}^4 - 2 \, \mathbf{y}^6 - 15 \left(\mathbf{x}^2 - 3 \, \mathbf{y}^2 \right) \left(\mathbf{x}^2 + \mathbf{y}^2 \right) \mathbf{z}^2 - 80 \, \mathbf{y}^2 \mathbf{z}^4 + 16 \, \mathbf{z}^6 \right) + \right. \\
& \quad \left. 4 \left(\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 \right)^2 \left(3 \, \mathbf{x}^4 - 4 \, \mathbf{y}^4 + 41 \, \mathbf{y}^2 \mathbf{z}^2 - 18 \, \mathbf{z}^4 - \mathbf{x}^2 \left(\mathbf{y}^2 + 15 \, \mathbf{z}^2 \right) \right) \right) + \\
& 5 \, \text{dx}^2 \left(-4 \left(\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 \right)^2 \left(6 \, \mathbf{x}^4 + 5 \, \mathbf{x}^2 \, \mathbf{y}^2 - \mathbf{y}^4 - 51 \, \mathbf{x}^2 \mathbf{z}^2 + 5 \, \mathbf{y}^2 \mathbf{z}^2 + 6 \, \mathbf{z}^4 \right) + \right. \\
& \quad \left. 7 \, \text{dz}^2 \left(\left(8 \, \mathbf{x}^2 - \mathbf{y}^2 \right) \left(\mathbf{x}^2 + \mathbf{y}^2 \right)^2 + 15 \left(-11 \, \mathbf{x}^4 - 10 \, \mathbf{x}^2 \, \mathbf{y}^2 + \mathbf{y}^4 \right) \mathbf{z}^2 + 240 \, \mathbf{x}^2 \mathbf{z}^4 - 16 \, \mathbf{z}^6 \right) + \right. \\
& \quad \left. 7 \, \text{dy}^2 \left(8 \, \mathbf{x}^6 - 15 \, \mathbf{x}^4 \left(\mathbf{y}^2 + 5 \, \mathbf{z}^2 \right) + \left(\mathbf{y}^2 + \mathbf{z}^2 \right) \left(2 \, \mathbf{y}^4 - 23 \, \mathbf{y}^2 \mathbf{z}^2 + 8 \, \mathbf{z}^4 \right) - 3 \, \mathbf{x}^2 \left(7 \, \mathbf{y}^4 - 100 \, \mathbf{y}^2 \mathbf{z}^2 + 25 \, \mathbf{z}^4 \right) \right) \right) \right), \\
\mathbf{z} & \left(-42 \, \text{dy}^4 \left(\mathbf{x}^6 - 15 \, \mathbf{x}^4 \, \mathbf{y}^2 + 16 \, \mathbf{y}^6 + 10 \, \mathbf{y}^2 \left(3 \, \mathbf{x}^2 - 8 \, \mathbf{y}^2 \right) \mathbf{z}^2 - 3 \left(\mathbf{x}^2 - 15 \, \mathbf{y}^2 \right) \mathbf{z}^4 - 2 \, \mathbf{z}^6 \right) - \right. \\
& 42 \, \text{dx}^4 \left(16 \, \mathbf{x}^6 + \mathbf{y}^6 - 80 \, \mathbf{x}^4 \mathbf{z}^2 - 3 \, \mathbf{y}^2 \mathbf{z}^4 - 2 \, \mathbf{z}^6 - 15 \, \mathbf{x}^2 \left(\mathbf{y}^2 - 3 \, \mathbf{z}^2 \right) \left(\mathbf{y}^2 + \mathbf{z}^2 \right) \right) - \\
& 2 \left(24 \left(3 \left(\mathbf{x}^2 + \mathbf{y}^2 \right) - 2 \, \mathbf{z}^2 \right) \left(\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 \right)^4 - 10 \, \text{dz}^2 \left(\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 \right)^2 \left(15 \left(\mathbf{x}^2 + \mathbf{y}^2 \right)^2 - 40 \left(\mathbf{x}^2 + \mathbf{y}^2 \right) \mathbf{z}^2 + 8 \, \mathbf{z}^4 \right) + \right. \\
& \quad \left. 7 \, \text{dz}^4 \left(35 \left(\mathbf{x}^2 + \mathbf{y}^2 \right)^3 - 210 \left(\mathbf{x}^2 + \mathbf{y}^2 \right)^2 \mathbf{z}^2 + 168 \left(\mathbf{x}^2 + \mathbf{y}^2 \right) \mathbf{z}^4 - 16 \, \mathbf{z}^6 \right) \right) - \\
& 5 \, \text{dy}^2 \left(-4 \left(\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 \right)^2 \left(3 \left(\mathbf{x}^2 - 6 \, \mathbf{y}^2 \right) \left(\mathbf{x}^2 + \mathbf{y}^2 \right) - \left(\mathbf{x}^2 - 41 \, \mathbf{y}^2 \right) \mathbf{z}^2 - 4 \, \mathbf{z}^4 \right) + \right. \\
& \quad \left. 7 \, \text{dz}^2 \left(5 \left(\mathbf{x}^2 - 8 \, \mathbf{y}^2 \right) \left(\mathbf{x}^2 + \mathbf{y}^2 \right)^2 - 15 \left(\mathbf{x}^2 - 15 \, \mathbf{y}^2 \right) \left(\mathbf{x}^2 + \mathbf{y}^2 \right) \mathbf{z}^2 - 12 \left(\mathbf{x}^2 + 13 \, \mathbf{y}^2 \right) \mathbf{z}^4 + 8 \, \mathbf{z}^6 \right) \right) + \\
& 5 \, \text{dx}^2 \left(7 \, \text{dz}^2 \left(5 \left(8 \, \mathbf{x}^2 - \mathbf{y}^2 \right) \left(\mathbf{x}^2 + \mathbf{y}^2 \right)^2 + 15 \left(-15 \, \mathbf{x}^4 - 14 \, \mathbf{x}^2 \, \mathbf{y}^2 + \mathbf{y}^4 \right) \mathbf{z}^2 + 12 \left(13 \, \mathbf{x}^2 + \mathbf{y}^2 \right) \mathbf{z}^4 - 8 \, \mathbf{z}^6 \right) + \right. \\
& \quad \left. 7 \, \text{dy}^2 \left(8 \, \mathbf{x}^6 - 75 \, \mathbf{x}^4 \, \mathbf{y}^2 - 75 \, \mathbf{x}^2 \, \mathbf{y}^4 + 8 \, \mathbf{y}^6 - 15 \left(\mathbf{x}^4 - 20 \, \mathbf{x}^2 \, \mathbf{y}^2 + \mathbf{y}^4 \right) \mathbf{z}^2 - 21 \left(\mathbf{x}^2 + \mathbf{y}^2 \right) \mathbf{z}^4 + 2 \, \mathbf{z}^6 \right) - \right. \\
& \quad \left. 4 \left(\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 \right)^2 \left(18 \, \mathbf{x}^4 - 3 \, \mathbf{y}^4 + \mathbf{y}^2 \mathbf{z}^2 + 4 \, \mathbf{z}^4 + \mathbf{x}^2 \left(15 \, \mathbf{y}^2 - 41 \, \mathbf{z}^2 \right) \right) \right) \right) \right];
\end{aligned}$$

For larger distances the dx,dy,dz (def.: dsomething) terms do not count so much, thus we consider only terms that do not have them, thus we take dsomething=0.

And we get back dipolar form :)

```

FullSimplify[ (tmp /. {dx -> 0, dy -> 0, dz -> 0})  $\frac{1}{64 \pi (x^2 + y^2 + z^2)^{15/2}} dx dy dz \mu_0$  ]
{ -  $\frac{3 dx dy dz \mu_0 x (x^2 + y^2 - 4 z^2)}{4 \pi (x^2 + y^2 + z^2)^{7/2}}$ , -  $\frac{3 dx dy dz \mu_0 y (x^2 + y^2 - 4 z^2)}{4 \pi (x^2 + y^2 + z^2)^{7/2}}$ ,  $\frac{3 dx dy dz \mu_0 z (-3 (x^2 + y^2) + 2 z^2)}{4 \pi (x^2 + y^2 + z^2)^{7/2}}$  }
{nXZdipol[x, y, z, dx, dy, dz], nYZdipol[x, y, z, dx, dy, dz], nZZdipol[x, y, z, dx, dy, dz]}
{ -  $\frac{3 dx dy dz \mu_0 x z}{4 \pi (x^2 + y^2 + z^2)^{5/2}}$ , -  $\frac{3 dx dy dz \mu_0 y z}{4 \pi (x^2 + y^2 + z^2)^{5/2}}$ , -  $\frac{dx dy dz \mu_0 (-x^2 - y^2 + 2 z^2)}{4 \pi (x^2 + y^2 + z^2)^{5/2}}$  }
Simplify[
%- {nXZdipol[x, y, z, dx, dy, dz], nYZdipol[x, y, z, dx, dy, dz], nZZdipol[x, y, z, dx, dy, dz]},
Assumptions -> Element[x, Reals]]
{0, 0, 0}

```

Export to C-form. Dipolar.

Ignore constant multiplicative factor $\frac{1}{64 \pi} \mu_0$

Also exclude factor common for all components: $64 \frac{3 dx dy dz}{4 (x^2 + y^2 + z^2)^{7/2}}$

```

simplified = FullSimplify[ (dNdip (  $\frac{1}{64 \pi} \mu_0$  )-1 (  $64 \frac{3 dx dy dz}{4 (x^2 + y^2 + z^2)^{7/2}}$  )-1 ) ]
{ -x (x^2 + y^2 - 4 z^2), -y (x^2 + y^2 - 4 z^2), -3 (x^2 + y^2) z + 2 z^3 }
CForm[  $64 \frac{3 dx dy dz}{4 (x^2 + y^2 + z^2)^{7/2}}$  ]
(48*dx*dy*dz)/Power(Power(x,2) + Power(y,2) + Power(z,2),3.5)
CForm[simplified[[1]]]
-(x*(Power(x,2) + Power(y,2) - 4*Power(z,2)))
CForm[simplified[[2]]]
-(y*(Power(x,2) + Power(y,2) - 4*Power(z,2)))
CForm[simplified[[3]]]
-3*(Power(x,2) + Power(y,2))*z + 2*Power(z,3)

```

Export to C-form. Cubic form.

First we further simplify it.

Ignore constant multiplicative factor $\frac{1}{64 \pi} \mu_0$

Also exclude factor common for all components: $\frac{1}{(x^2 + y^2 + z^2)^{15/2}} dx dy dz$

$$\text{simplified} = \left\{ x \left(-48 (x^2 + y^2 - 4 z^2) (x^2 + y^2 + z^2)^4 + 7 dx^4 (7 x^6 - 6 x^4 y^2 + 13 y^6 - 3 (47 x^4 - 20 x^2 y^2 + 65 y^4) z^2 + 15 (15 x^2 + 11 y^2) z^4 - 56 z^6) \right), \right. \\ y \left(-48 (x^2 + y^2 - 4 z^2) (x^2 + y^2 + z^2)^4 + 7 dx^4 (13 x^6 - 6 x^2 y^4 + 7 y^6 - 3 (65 x^4 - 20 x^2 y^2 + 47 y^4) z^2 + 15 (11 x^2 + 15 y^2) z^4 - 56 z^6) \right), \\ \left. -z \left(48 (3 (x^2 + y^2) - 2 z^2) (x^2 + y^2 + z^2)^4 - 7 dx^4 (43 x^6 + 30 x^4 y^2 + 30 x^2 y^4 + 43 y^6 - 15 (15 x^4 + 8 x^2 y^2 + 15 y^4) z^2 + 147 (x^2 + y^2) z^4 - 14 z^6) \right) \right\};$$

$$\text{Simplify} \left[\left(\text{dNcc} \left(\frac{1}{64 \pi} \mu_0 \right)^{-1} \left(\frac{1}{(x^2 + y^2 + z^2)^{15/2}} dx^3 \right)^{-1} \right) - \text{simplified} \right]$$

{0, 0, 0}

simplified

Assuming[x > 0 && y > 0 && z > 0 && dx > 0, FullSimplify[simplified]]

FullSimplify[simplified,

Assumptions → Element[x, Reals] && Element[y, Reals] && Element[z, Reals] && dx > 0]

$$\left\{ x \left(-48 (x^2 + y^2 - 4 z^2) (x^2 + y^2 + z^2)^4 + 7 dx^4 (7 x^6 - 6 x^4 y^2 + 13 y^6 - 3 (47 x^4 - 20 x^2 y^2 + 65 y^4) z^2 + 15 (15 x^2 + 11 y^2) z^4 - 56 z^6) \right), \right. \\ y \left(-48 (x^2 + y^2 - 4 z^2) (x^2 + y^2 + z^2)^4 + 7 dx^4 (13 x^6 - 6 x^2 y^4 + 7 y^6 - 3 (65 x^4 - 20 x^2 y^2 + 47 y^4) z^2 + 15 (11 x^2 + 15 y^2) z^4 - 56 z^6) \right), \\ \left. -z \left(48 (3 (x^2 + y^2) - 2 z^2) (x^2 + y^2 + z^2)^4 - 7 dx^4 (43 x^6 + 30 x^4 y^2 + 30 x^2 y^4 + 43 y^6 - 15 (15 x^4 + 8 x^2 y^2 + 15 y^4) z^2 + 147 (x^2 + y^2) z^4 - 14 z^6) \right) \right\}$$

$$\left\{ x \left(-48 (x^2 + y^2 - 4 z^2) (x^2 + y^2 + z^2)^4 + 7 dx^4 (7 x^6 - 6 x^4 y^2 + 13 y^6 - 3 (47 x^4 - 20 x^2 y^2 + 65 y^4) z^2 + 15 (15 x^2 + 11 y^2) z^4 - 56 z^6) \right), \right. \\ y \left(-48 (x^2 + y^2 - 4 z^2) (x^2 + y^2 + z^2)^4 + 7 dx^4 (13 x^6 - 6 x^2 y^4 + 7 y^6 - 3 (65 x^4 - 20 x^2 y^2 + 47 y^4) z^2 + 15 (11 x^2 + 15 y^2) z^4 - 56 z^6) \right), \\ \left. -z \left(48 (3 (x^2 + y^2) - 2 z^2) (x^2 + y^2 + z^2)^4 - 7 dx^4 (43 x^6 + 30 x^4 y^2 + 30 x^2 y^4 + 43 y^6 - 15 (15 x^4 + 8 x^2 y^2 + 15 y^4) z^2 + 147 (x^2 + y^2) z^4 - 14 z^6) \right) \right\}$$

$$\left\{ x \left(-48 (x^2 + y^2 - 4 z^2) (x^2 + y^2 + z^2)^4 + 7 dx^4 (7 x^6 - 6 x^4 y^2 + 13 y^6 - 3 (47 x^4 - 20 x^2 y^2 + 65 y^4) z^2 + 15 (15 x^2 + 11 y^2) z^4 - 56 z^6) \right), \right. \\ y \left(-48 (x^2 + y^2 - 4 z^2) (x^2 + y^2 + z^2)^4 + 7 dx^4 (13 x^6 - 6 x^2 y^4 + 7 y^6 - 3 (65 x^4 - 20 x^2 y^2 + 47 y^4) z^2 + 15 (11 x^2 + 15 y^2) z^4 - 56 z^6) \right), \\ \left. -z \left(48 (3 (x^2 + y^2) - 2 z^2) (x^2 + y^2 + z^2)^4 - 7 dx^4 (43 x^6 + 30 x^4 y^2 + 30 x^2 y^4 + 43 y^6 - 15 (15 x^4 + 8 x^2 y^2 + 15 y^4) z^2 + 147 (x^2 + y^2) z^4 - 14 z^6) \right) \right\}$$

We gained nothing. Happens ...

CForm[simplified[[1]]]

$$x \cdot (-48 \cdot (\text{Power}(x, 2) + \text{Power}(y, 2) - 4 \cdot \text{Power}(z, 2)) \cdot \text{Power}(\text{Power}(x, 2) + \text{Power}(y, 2) + \text{Power}(z, 2), 4) + 7 \cdot \text{Power}(dx, 4) \cdot (7 \cdot \text{Power}(x, 6) - 6 \cdot \text{Power}(x, 4) \cdot \text{Power}(y, 2) + 13 \cdot \text{Power}(y, 6) - 3 \cdot (47 \cdot \text{Power}(x, 4) - 20 \cdot \text{Power}(x, 2) \cdot \text{Power}(y, 2) + 65 \cdot \text{Power}(y, 4)) \cdot \text{Power}(z, 2) + 15 \cdot (15 \cdot \text{Power}(x, 2) + 11 \cdot \text{Power}(y, 2)) \cdot \text{Power}(z, 4) - 56 \cdot \text{Power}(z, 6)))$$

CForm[simplified[[2]]]

```
y*(-48*(Power(x,2) + Power(y,2) - 4*Power(z,2))*Power(Power(x,2) + Power(y,2) + Power(z,2),4) +
7*Power(dx,4)*(13*Power(x,6) - 6*Power(x,2)*Power(y,4) + 7*Power(y,6) -
3*(65*Power(x,4) - 20*Power(x,2)*Power(y,2) + 47*Power(y,4))*Power(z,2) +
15*(11*Power(x,2) + 15*Power(y,2))*Power(z,4) - 56*Power(z,6)))
```

CForm[simplified[[3]]]

```
-(z*(48*(3*(Power(x,2) + Power(y,2)) - 2*Power(z,2))*Power(Power(x,2) + Power(y,2) + Power(z,2),4)
7*Power(dx,4)*(43*Power(x,6) + 30*Power(x,4)*Power(y,2) + 30*Power(x,2)*Power(y,4) + 43*Pow
15*(15*Power(x,4) + 8*Power(x,2)*Power(y,2) + 15*Power(y,4))*Power(z,2) + 147*(Power(x,2)
14*Power(z,6))))
```

Export to C-form. Non-cubic form.

First we further simplify it.

Ignore constant multiplicative factor $\frac{1}{64\pi} \mu_0$

Also exclude factor common for all components: $\frac{1}{(x^2+y^2+z^2)^{15/2}} dx dy dz$

CForm $\left[\frac{1}{(x^2 + y^2 + z^2)^{15/2}} dx dy dz\right]$

(dx*dy*dz)/Power(Power(x,2) + Power(y,2) + Power(z,2),7.5)

$$\begin{aligned}
\ln[1]= \text{simplified} = & \left\{ x \left(-14 dy^4 (x^6 - 15 x^4 y^2 + 16 y^6 - 6 (x^4 - 25 x^2 y^2 + 40 y^4) z^2 - 15 (x^2 - 11 y^2) z^4 - 8 z^6) - \right. \right. \\
& 14 dx^4 (8 x^6 - 15 x^2 (y^2 - 15 z^2) (y^2 + z^2) + 5 (y^2 - 8 z^2) (y^2 + z^2)^2 - 12 x^4 (y^2 + 13 z^2)) + \\
& 2 \left(-24 (x^2 + y^2 - 4 z^2) (x^2 + y^2 + z^2)^4 + 30 dz^2 (x^2 + y^2 + z^2)^2 \left((x^2 + y^2)^2 - 12 (x^2 + y^2) z^2 + 8 z^4 \right) + \right. \\
& \quad \left. 7 dz^4 \left(-5 (x^2 + y^2)^3 + 120 (x^2 + y^2)^2 z^2 - 240 (x^2 + y^2) z^4 + 64 z^6 \right) \right) + \\
& 5 dy^2 \left(7 dz^2 \left(- (x^2 - 8 y^2) (x^2 + y^2)^2 + 15 (x^2 - 11 y^2) (x^2 + y^2) z^2 + 240 y^2 z^4 - 16 z^6 \right) + \right. \\
& \quad \left. 4 (x^2 + y^2 + z^2)^2 (x^4 - 6 y^4 + 51 y^2 z^2 - 6 z^4 - 5 x^2 (y^2 + z^2)) \right) + \\
& 5 dx^2 \left(21 dz^2 (2 x^6 + 3 x^4 y^2 - y^6 + 15 (-3 x^4 - 2 x^2 y^2 + y^4) z^2 + 80 x^2 z^4 - 16 z^6) - \right. \\
& \quad 4 (x^2 + y^2 + z^2)^2 (4 x^4 + x^2 (y^2 - 41 z^2) - 3 (y^2 - 6 z^2) (y^2 + z^2)) + \\
& \quad \left. 7 dy^2 (2 x^6 - 21 x^4 (y^2 + z^2) - 15 x^2 (y^4 - 20 y^2 z^2 + z^4) + (y^2 + z^2) (8 y^4 - 83 y^2 z^2 + 8 z^4)) \right) \left. \right\}, \\
y \left(-14 dx^4 (16 x^6 - 240 x^4 z^2 - 15 x^2 (y^2 - 11 z^2) (y^2 + z^2) + (y^2 - 8 z^2) (y^2 + z^2)^2) - \right. \\
& 14 dy^4 (5 x^6 + 8 y^6 - 156 y^4 z^2 + 225 y^2 z^4 - 40 z^6 - 15 x^4 (y^2 + 2 z^2) - 3 x^2 (4 y^4 - 70 y^2 z^2 + 25 z^4)) + \\
& 2 \left(-24 (x^2 + y^2 - 4 z^2) (x^2 + y^2 + z^2)^4 + 30 dz^2 (x^2 + y^2 + z^2)^2 \left((x^2 + y^2)^2 - 12 (x^2 + y^2) z^2 + 8 z^4 \right) + \right. \\
& \quad \left. 7 dz^4 \left(-5 (x^2 + y^2)^3 + 120 (x^2 + y^2)^2 z^2 - 240 (x^2 + y^2) z^4 + 64 z^6 \right) \right) + \\
& 5 dy^2 \left(-21 dz^2 (x^6 - 3 x^2 y^4 - 2 y^6 - 15 (x^2 - 3 y^2) (x^2 + y^2) z^2 - 80 y^2 z^4 + 16 z^6) + \right. \\
& \quad \left. 4 (x^2 + y^2 + z^2)^2 (3 x^4 - 4 y^4 + 41 y^2 z^2 - 18 z^4 - x^2 (y^2 + 15 z^2)) \right) + \\
& 5 dx^2 \left(7 dz^2 \left((8 x^2 - y^2) (x^2 + y^2)^2 + 15 (-11 x^4 - 10 x^2 y^2 + y^4) z^2 + 240 x^2 z^4 - 16 z^6 \right) - \right. \\
& \quad 4 (x^2 + y^2 + z^2)^2 (6 x^4 - y^4 + 5 y^2 z^2 + 6 z^4 + x^2 (5 y^2 - 51 z^2)) + \\
& \quad \left. 7 dy^2 (8 x^6 - 15 x^4 (y^2 + 5 z^2) + (y^2 + z^2) (2 y^4 - 23 y^2 z^2 + 8 z^4) - 3 x^2 (7 y^4 - 100 y^2 z^2 + 25 z^4)) \right) \left. \right\}, \\
z \left(-42 dy^4 (x^6 - 15 x^4 y^2 + 16 y^6 + 10 y^2 (3 x^2 - 8 y^2) z^2 - 3 (x^2 - 15 y^2) z^4 - 2 z^6) - \right. \\
& 42 dx^4 (16 x^6 + y^6 - 80 x^4 z^2 - 3 y^2 z^4 - 2 z^6 - 15 x^2 (y^2 - 3 z^2) (y^2 + z^2)) - \\
& 2 \left(24 (3 (x^2 + y^2) - 2 z^2) (x^2 + y^2 + z^2)^4 - 10 dz^2 (x^2 + y^2 + z^2)^2 \left(15 (x^2 + y^2)^2 - 40 (x^2 + y^2) z^2 + 8 z^4 \right) + \right. \\
& \quad \left. 7 dz^4 \left(35 (x^2 + y^2)^3 - 210 (x^2 + y^2)^2 z^2 + 168 (x^2 + y^2) z^4 - 16 z^6 \right) \right) - \\
& 5 dy^2 \left(-4 (x^2 + y^2 + z^2)^2 (3 (x^2 - 6 y^2) (x^2 + y^2) - (x^2 - 41 y^2) z^2 - 4 z^4) + \right. \\
& \quad \left. 7 dz^2 \left(5 (x^2 - 8 y^2) (x^2 + y^2)^2 - 15 (x^2 - 15 y^2) (x^2 + y^2) z^2 - 12 (x^2 + 13 y^2) z^4 + 8 z^6 \right) \right) + \\
& 5 dx^2 \left(7 dz^2 \left(5 (8 x^2 - y^2) (x^2 + y^2)^2 + 15 (-15 x^4 - 14 x^2 y^2 + y^4) z^2 + 12 (13 x^2 + y^2) z^4 - 8 z^6 \right) + \right. \\
& \quad \left. 7 dy^2 (8 x^6 - 75 x^4 y^2 - 75 x^2 y^4 + 8 y^6 - 15 (x^4 - 20 x^2 y^2 + y^4) z^2 - 21 (x^2 + y^2) z^4 + 2 z^6) - \right. \\
& \quad \left. 4 (x^2 + y^2 + z^2)^2 (18 x^4 - 3 y^4 + y^2 z^2 + 4 z^4 + x^2 (15 y^2 - 41 z^2)) \right) \left. \right\};
\end{aligned}$$

$$\text{Simplify} \left[\left(\text{dNncc} \left(\frac{1}{64 \pi} \mu_0 \right)^{-1} \left(\frac{1}{(x^2 + y^2 + z^2)^{15/2}} dx dy dz \right)^{-1} \right) - \text{simplified} \right]$$

{0, 0, 0}

simplified

Assuming[x > 0 && y > 0 && z > 0 && dx > 0 && dy > 0 && dz > 0, FullSimplify[simplified]]

FullSimplify[simplified,

Assumptions → Element[x, Reals] && Element[y, Reals] && Element[z, Reals] && dx > 0 && dy > 0 && dz > 0]

$$\begin{aligned}
 & \{x \left(-14 \, dy^4 \left(x^6 - 15 \, x^4 \, y^2 + 16 \, y^6 - 6 \left(x^4 - 25 \, x^2 \, y^2 + 40 \, y^4 \right) z^2 - 15 \left(x^2 - 11 \, y^2 \right) z^4 - 8 \, z^6 \right) - \right. \\
 & \quad 14 \, dx^4 \left(8 \, x^6 - 15 \, x^2 \left(y^2 - 15 \, z^2 \right) \left(y^2 + z^2 \right) + 5 \left(y^2 - 8 \, z^2 \right) \left(y^2 + z^2 \right)^2 - 12 \, x^4 \left(y^2 + 13 \, z^2 \right) \right) + \\
 & \quad 2 \left(-24 \left(x^2 + y^2 - 4 \, z^2 \right) \left(x^2 + y^2 + z^2 \right)^4 + 30 \, dz^2 \left(x^2 + y^2 + z^2 \right)^2 \left(\left(x^2 + y^2 \right)^2 - 12 \left(x^2 + y^2 \right) z^2 + 8 \, z^4 \right) + \right. \\
 & \quad \quad 7 \, dz^4 \left(-5 \left(x^2 + y^2 \right)^3 + 120 \left(x^2 + y^2 \right)^2 z^2 - 240 \left(x^2 + y^2 \right) z^4 + 64 \, z^6 \right) \left. + \right. \\
 & \quad 5 \, dy^2 \left(7 \, dz^2 \left(\left(x^2 + y^2 \right)^2 \left(-x^2 + 8 \, y^2 \right) + 15 \left(x^2 - 11 \, y^2 \right) \left(x^2 + y^2 \right) z^2 + 240 \, y^2 z^4 - 16 \, z^6 \right) + \right. \\
 & \quad \quad 4 \left(x^2 + y^2 + z^2 \right)^2 \left(x^4 - 6 \, y^4 + 51 \, y^2 z^2 - 6 \, z^4 - 5 \, x^2 \left(y^2 + z^2 \right) \right) \left. \right) + \\
 & \quad 5 \, dx^2 \left(21 \, dz^2 \left(2 \, x^6 + 3 \, x^4 \, y^2 - y^6 + 15 \left(-3 \, x^4 - 2 \, x^2 \, y^2 + y^4 \right) z^2 + 80 \, x^2 z^4 - 16 \, z^6 \right) - \right. \\
 & \quad \quad 4 \left(x^2 + y^2 + z^2 \right)^2 \left(4 \, x^4 + x^2 \left(y^2 - 41 \, z^2 \right) - 3 \left(y^2 - 6 \, z^2 \right) \left(y^2 + z^2 \right) \right) + \\
 & \quad \quad \left. \left. 7 \, dy^2 \left(2 \, x^6 - 21 \, x^4 \left(y^2 + z^2 \right) - 15 \, x^2 \left(y^4 - 20 \, y^2 z^2 + z^4 \right) + \left(y^2 + z^2 \right) \left(8 \, y^4 - 83 \, y^2 z^2 + 8 \, z^4 \right) \right) \right) \right), \\
 & y \left(-14 \, dx^4 \left(16 \, x^6 - 240 \, x^4 z^2 - 15 \, x^2 \left(y^2 - 11 \, z^2 \right) \left(y^2 + z^2 \right) + \left(y^2 - 8 \, z^2 \right) \left(y^2 + z^2 \right)^2 \right) - \right. \\
 & \quad 14 \, dy^4 \left(5 \, x^6 + 8 \, y^6 - 156 \, y^4 z^2 + 225 \, y^2 z^4 - 40 \, z^6 - 15 \, x^4 \left(y^2 + 2 \, z^2 \right) - 3 \, x^2 \left(4 \, y^4 - 70 \, y^2 z^2 + 25 \, z^4 \right) \right) + \\
 & \quad 2 \left(-24 \left(x^2 + y^2 - 4 \, z^2 \right) \left(x^2 + y^2 + z^2 \right)^4 + 30 \, dz^2 \left(x^2 + y^2 + z^2 \right)^2 \left(\left(x^2 + y^2 \right)^2 - 12 \left(x^2 + y^2 \right) z^2 + 8 \, z^4 \right) + \right. \\
 & \quad \quad 7 \, dz^4 \left(-5 \left(x^2 + y^2 \right)^3 + 120 \left(x^2 + y^2 \right)^2 z^2 - 240 \left(x^2 + y^2 \right) z^4 + 64 \, z^6 \right) \left. + \right. \\
 & \quad 5 \, dy^2 \left(-21 \, dz^2 \left(x^6 - 3 \, x^2 \, y^4 - 2 \, y^6 - 15 \left(x^2 - 3 \, y^2 \right) \left(x^2 + y^2 \right) z^2 - 80 \, y^2 z^4 + 16 \, z^6 \right) + \right. \\
 & \quad \quad 4 \left(x^2 + y^2 + z^2 \right)^2 \left(3 \, x^4 - 4 \, y^4 + 41 \, y^2 z^2 - 18 \, z^4 - x^2 \left(y^2 + 15 \, z^2 \right) \right) \left. \right) + \\
 & \quad 5 \, dx^2 \left(7 \, dz^2 \left(\left(8 \, x^2 - y^2 \right) \left(x^2 + y^2 \right)^2 + 15 \left(-11 \, x^4 - 10 \, x^2 \, y^2 + y^4 \right) z^2 + 240 \, x^2 z^4 - 16 \, z^6 \right) - \right. \\
 & \quad \quad 4 \left(x^2 + y^2 + z^2 \right)^2 \left(6 \, x^4 - y^4 + 5 \, y^2 z^2 + 6 \, z^4 + x^2 \left(5 \, y^2 - 51 \, z^2 \right) \right) \left. + \right. \\
 & \quad \quad \left. \left. 7 \, dy^2 \left(8 \, x^6 - 15 \, x^4 \left(y^2 + 5 \, z^2 \right) + \left(y^2 + z^2 \right) \left(2 \, y^4 - 23 \, y^2 z^2 + 8 \, z^4 \right) - 3 \, x^2 \left(7 \, y^4 - 100 \, y^2 z^2 + 25 \, z^4 \right) \right) \right) \right), \\
 & z \left(-42 \, dy^4 \left(x^6 - 15 \, x^4 \, y^2 + 16 \, y^6 + 10 \, y^2 \left(3 \, x^2 - 8 \, y^2 \right) z^2 - 3 \left(x^2 - 15 \, y^2 \right) z^4 - 2 \, z^6 \right) - \right. \\
 & \quad 42 \, dx^4 \left(16 \, x^6 + y^6 - 80 \, x^4 z^2 - 3 \, y^2 z^4 - 2 \, z^6 - 15 \, x^2 \left(y^2 - 3 \, z^2 \right) \left(y^2 + z^2 \right) \right) - \\
 & \quad 2 \left(24 \left(3 \left(x^2 + y^2 \right) - 2 \, z^2 \right) \left(x^2 + y^2 + z^2 \right)^4 - 10 \, dz^2 \left(x^2 + y^2 + z^2 \right)^2 \left(15 \left(x^2 + y^2 \right)^2 - 40 \left(x^2 + y^2 \right) z^2 + 8 \, z^4 \right) + \right. \\
 & \quad \quad 7 \, dz^4 \left(35 \left(x^2 + y^2 \right)^3 - 210 \left(x^2 + y^2 \right)^2 z^2 + 168 \left(x^2 + y^2 \right) z^4 - 16 \, z^6 \right) \left. - \right. \\
 & \quad 5 \, dy^2 \left(-4 \left(x^2 + y^2 + z^2 \right)^2 \left(3 \left(x^2 - 6 \, y^2 \right) \left(x^2 + y^2 \right) - \left(x^2 - 41 \, y^2 \right) z^2 - 4 \, z^4 \right) + \right. \\
 & \quad \quad 7 \, dz^2 \left(5 \left(x^2 - 8 \, y^2 \right) \left(x^2 + y^2 \right)^2 - 15 \left(x^2 - 15 \, y^2 \right) \left(x^2 + y^2 \right) z^2 - 12 \left(x^2 + 13 \, y^2 \right) z^4 + 8 \, z^6 \right) \left. + \right. \\
 & \quad 5 \, dx^2 \left(7 \, dz^2 \left(5 \left(8 \, x^2 - y^2 \right) \left(x^2 + y^2 \right)^2 + 15 \left(-15 \, x^4 - 14 \, x^2 \, y^2 + y^4 \right) z^2 + 12 \left(13 \, x^2 + y^2 \right) z^4 - 8 \, z^6 \right) + \right. \\
 & \quad \quad 7 \, dy^2 \left(8 \, x^6 - 75 \, x^4 \, y^2 - 75 \, x^2 \, y^4 + 8 \, y^6 - 15 \left(x^4 - 20 \, x^2 \, y^2 + y^4 \right) z^2 - 21 \left(x^2 + y^2 \right) z^4 + 2 \, z^6 \right) - \\
 & \quad \quad \left. \left. 4 \left(x^2 + y^2 + z^2 \right)^2 \left(18 \, x^4 - 3 \, y^4 + y^2 z^2 + 4 \, z^4 + x^2 \left(15 \, y^2 - 41 \, z^2 \right) \right) \right) \right) \}
 \end{aligned}$$

$$\begin{aligned}
& \{ x \left(-14 \, dy^4 \left(x^6 - 15 \, x^4 \, y^2 + 16 \, y^6 - 6 \left(x^4 - 25 \, x^2 \, y^2 + 40 \, y^4 \right) z^2 - 15 \left(x^2 - 11 \, y^2 \right) z^4 - 8 \, z^6 \right) - \right. \\
& \quad 14 \, dx^4 \left(8 \, x^6 - 15 \, x^2 \left(y^2 - 15 \, z^2 \right) \left(y^2 + z^2 \right) + 5 \left(y^2 - 8 \, z^2 \right) \left(y^2 + z^2 \right)^2 - 12 \, x^4 \left(y^2 + 13 \, z^2 \right) \right) + \\
& \quad 2 \left(-24 \left(x^2 + y^2 - 4 \, z^2 \right) \left(x^2 + y^2 + z^2 \right)^4 + 30 \, dz^2 \left(x^2 + y^2 + z^2 \right)^2 \left(\left(x^2 + y^2 \right)^2 - 12 \left(x^2 + y^2 \right) z^2 + 8 \, z^4 \right) + \right. \\
& \quad \quad 7 \, dz^4 \left(-5 \left(x^2 + y^2 \right)^3 + 120 \left(x^2 + y^2 \right)^2 z^2 - 240 \left(x^2 + y^2 \right) z^4 + 64 \, z^6 \right) \left. + \right. \\
& \quad 5 \, dy^2 \left(7 \, dz^2 \left(- \left(x^2 - 8 \, y^2 \right) \left(x^2 + y^2 \right)^2 + 15 \left(x^2 - 11 \, y^2 \right) \left(x^2 + y^2 \right) z^2 + 240 \, y^2 z^4 - 16 \, z^6 \right) + \right. \\
& \quad \quad 4 \left(x^2 + y^2 + z^2 \right)^2 \left(x^4 - 6 \, y^4 + 51 \, y^2 z^2 - 6 \, z^4 - 5 \, x^2 \left(y^2 + z^2 \right) \right) \left. \right) + \\
& \quad 5 \, dx^2 \left(21 \, dz^2 \left(2 \, x^6 + 3 \, x^4 \, y^2 - y^6 + 15 \left(-3 \, x^4 - 2 \, x^2 \, y^2 + y^4 \right) z^2 + 80 \, x^2 z^4 - 16 \, z^6 \right) - \right. \\
& \quad \quad 4 \left(x^2 + y^2 + z^2 \right)^2 \left(4 \, x^4 + x^2 \left(y^2 - 41 \, z^2 \right) - 3 \left(y^2 - 6 \, z^2 \right) \left(y^2 + z^2 \right) \right) + \\
& \quad \quad \left. \left. 7 \, dy^2 \left(2 \, x^6 - 21 \, x^4 \left(y^2 + z^2 \right) - 15 \, x^2 \left(y^4 - 20 \, y^2 z^2 + z^4 \right) + \left(y^2 + z^2 \right) \left(8 \, y^4 - 83 \, y^2 z^2 + 8 \, z^4 \right) \right) \right) \right\}, \\
& y \left(-14 \, dx^4 \left(16 \, x^6 - 240 \, x^4 z^2 - 15 \, x^2 \left(y^2 - 11 \, z^2 \right) \left(y^2 + z^2 \right) + \left(y^2 - 8 \, z^2 \right) \left(y^2 + z^2 \right)^2 \right) - \right. \\
& \quad 14 \, dy^4 \left(5 \, x^6 + 8 \, y^6 - 156 \, y^4 z^2 + 225 \, y^2 z^4 - 40 \, z^6 - 15 \, x^4 \left(y^2 + 2 \, z^2 \right) - 3 \, x^2 \left(4 \, y^4 - 70 \, y^2 z^2 + 25 \, z^4 \right) \right) + \\
& \quad 2 \left(-24 \left(x^2 + y^2 - 4 \, z^2 \right) \left(x^2 + y^2 + z^2 \right)^4 + 30 \, dz^2 \left(x^2 + y^2 + z^2 \right)^2 \left(\left(x^2 + y^2 \right)^2 - 12 \left(x^2 + y^2 \right) z^2 + 8 \, z^4 \right) + \right. \\
& \quad \quad 7 \, dz^4 \left(-5 \left(x^2 + y^2 \right)^3 + 120 \left(x^2 + y^2 \right)^2 z^2 - 240 \left(x^2 + y^2 \right) z^4 + 64 \, z^6 \right) \left. + \right. \\
& \quad 5 \, dy^2 \left(-21 \, dz^2 \left(x^6 - 3 \, x^2 \, y^4 - 2 \, y^6 - 15 \left(x^2 - 3 \, y^2 \right) \left(x^2 + y^2 \right) z^2 - 80 \, y^2 z^4 + 16 \, z^6 \right) + \right. \\
& \quad \quad 4 \left(x^2 + y^2 + z^2 \right)^2 \left(3 \, x^4 - 4 \, y^4 + 41 \, y^2 z^2 - 18 \, z^4 - x^2 \left(y^2 + 15 \, z^2 \right) \right) \left. \right) + \\
& \quad 5 \, dx^2 \left(7 \, dz^2 \left(\left(8 \, x^2 - y^2 \right) \left(x^2 + y^2 \right)^2 + 15 \left(-11 \, x^4 - 10 \, x^2 \, y^2 + y^4 \right) z^2 + 240 \, x^2 z^4 - 16 \, z^6 \right) - \right. \\
& \quad \quad 4 \left(x^2 + y^2 + z^2 \right)^2 \left(6 \, x^4 - y^4 + 5 \, y^2 z^2 + 6 \, z^4 + x^2 \left(5 \, y^2 - 51 \, z^2 \right) \right) \left. + \right. \\
& \quad \quad \left. \left. 7 \, dy^2 \left(8 \, x^6 - 15 \, x^4 \left(y^2 + 5 \, z^2 \right) + \left(y^2 + z^2 \right) \left(2 \, y^4 - 23 \, y^2 z^2 + 8 \, z^4 \right) - 3 \, x^2 \left(7 \, y^4 - 100 \, y^2 z^2 + 25 \, z^4 \right) \right) \right) \right) \right\}, \\
& z \left(-42 \, dy^4 \left(x^6 - 15 \, x^4 \, y^2 + 16 \, y^6 + 10 \, y^2 \left(3 \, x^2 - 8 \, y^2 \right) z^2 - 3 \left(x^2 - 15 \, y^2 \right) z^4 - 2 \, z^6 \right) - \right. \\
& \quad 42 \, dx^4 \left(16 \, x^6 + y^6 - 80 \, x^4 z^2 - 3 \, y^2 z^4 - 2 \, z^6 - 15 \, x^2 \left(y^2 - 3 \, z^2 \right) \left(y^2 + z^2 \right) \right) - \\
& \quad 2 \left(24 \left(3 \left(x^2 + y^2 \right) - 2 \, z^2 \right) \left(x^2 + y^2 + z^2 \right)^4 - 10 \, dz^2 \left(x^2 + y^2 + z^2 \right)^2 \left(15 \left(x^2 + y^2 \right)^2 - 40 \left(x^2 + y^2 \right) z^2 + 8 \, z^4 \right) + \right. \\
& \quad \quad 7 \, dz^4 \left(35 \left(x^2 + y^2 \right)^3 - 210 \left(x^2 + y^2 \right)^2 z^2 + 168 \left(x^2 + y^2 \right) z^4 - 16 \, z^6 \right) \left. - \right. \\
& \quad 5 \, dy^2 \left(-4 \left(x^2 + y^2 + z^2 \right)^2 \left(3 \left(x^2 - 6 \, y^2 \right) \left(x^2 + y^2 \right) - \left(x^2 - 41 \, y^2 \right) z^2 - 4 \, z^4 \right) + \right. \\
& \quad \quad 7 \, dz^2 \left(5 \left(x^2 - 8 \, y^2 \right) \left(x^2 + y^2 \right)^2 - 15 \left(x^2 - 15 \, y^2 \right) \left(x^2 + y^2 \right) z^2 - 12 \left(x^2 + 13 \, y^2 \right) z^4 + 8 \, z^6 \right) \left. + \right. \\
& \quad 5 \, dx^2 \left(7 \, dz^2 \left(5 \left(8 \, x^2 - y^2 \right) \left(x^2 + y^2 \right)^2 + 15 \left(-15 \, x^4 - 14 \, x^2 \, y^2 + y^4 \right) z^2 + 12 \left(13 \, x^2 + y^2 \right) z^4 - 8 \, z^6 \right) + \right. \\
& \quad \quad 7 \, dy^2 \left(8 \, x^6 - 75 \, x^4 \, y^2 - 75 \, x^2 \, y^4 + 8 \, y^6 - 15 \left(x^4 - 20 \, x^2 \, y^2 + y^4 \right) z^2 - 21 \left(x^2 + y^2 \right) z^4 + 2 \, z^6 \right) - \\
& \quad \quad \left. \left. 4 \left(x^2 + y^2 + z^2 \right)^2 \left(18 \, x^4 - 3 \, y^4 + y^2 z^2 + 4 \, z^4 + x^2 \left(15 \, y^2 - 41 \, z^2 \right) \right) \right) \right) \left. \right\}
\end{aligned}$$

$$\begin{aligned}
& \{ x \left(-14 \, dy^4 \left(x^6 - 15 \, x^4 \, y^2 + 16 \, y^6 - 6 \left(x^4 - 25 \, x^2 \, y^2 + 40 \, y^4 \right) z^2 - 15 \left(x^2 - 11 \, y^2 \right) z^4 - 8 \, z^6 \right) - \right. \\
& \quad 14 \, dx^4 \left(8 \, x^6 - 15 \, x^2 \left(y^2 - 15 \, z^2 \right) \left(y^2 + z^2 \right) + 5 \left(y^2 - 8 \, z^2 \right) \left(y^2 + z^2 \right)^2 - 12 \, x^4 \left(y^2 + 13 \, z^2 \right) \right) + \\
& \quad 2 \left(-24 \left(x^2 + y^2 - 4 \, z^2 \right) \left(x^2 + y^2 + z^2 \right)^4 + 30 \, dz^2 \left(x^2 + y^2 + z^2 \right)^2 \left(\left(x^2 + y^2 \right)^2 - 12 \left(x^2 + y^2 \right) z^2 + 8 \, z^4 \right) + \right. \\
& \quad \quad 7 \, dz^4 \left(-5 \left(x^2 + y^2 \right)^3 + 120 \left(x^2 + y^2 \right)^2 z^2 - 240 \left(x^2 + y^2 \right) z^4 + 64 \, z^6 \right) \left. + \right. \\
& \quad 5 \, dy^2 \left(7 \, dz^2 \left(- \left(x^2 - 8 \, y^2 \right) \left(x^2 + y^2 \right)^2 + 15 \left(x^2 - 11 \, y^2 \right) \left(x^2 + y^2 \right) z^2 + 240 \, y^2 z^4 - 16 \, z^6 \right) + \right. \\
& \quad \quad 4 \left(x^2 + y^2 + z^2 \right)^2 \left(x^4 - 6 \, y^4 + 51 \, y^2 z^2 - 6 \, z^4 - 5 \, x^2 \left(y^2 + z^2 \right) \right) \left. \right) + \\
& \quad 5 \, dx^2 \left(21 \, dz^2 \left(2 \, x^6 + 3 \, x^4 \, y^2 - y^6 + 15 \left(-3 \, x^4 - 2 \, x^2 \, y^2 + y^4 \right) z^2 + 80 \, x^2 z^4 - 16 \, z^6 \right) - \right. \\
& \quad \quad 4 \left(x^2 + y^2 + z^2 \right)^2 \left(4 \, x^4 + x^2 \left(y^2 - 41 \, z^2 \right) - 3 \left(y^2 - 6 \, z^2 \right) \left(y^2 + z^2 \right) \right) + \\
& \quad \quad \left. \left. 7 \, dy^2 \left(2 \, x^6 - 21 \, x^4 \left(y^2 + z^2 \right) - 15 \, x^2 \left(y^4 - 20 \, y^2 z^2 + z^4 \right) + \left(y^2 + z^2 \right) \left(8 \, y^4 - 83 \, y^2 z^2 + 8 \, z^4 \right) \right) \right) \right\}, \\
& y \left(-14 \, dx^4 \left(16 \, x^6 - 240 \, x^4 z^2 - 15 \, x^2 \left(y^2 - 11 \, z^2 \right) \left(y^2 + z^2 \right) + \left(y^2 - 8 \, z^2 \right) \left(y^2 + z^2 \right)^2 \right) - \right. \\
& \quad 14 \, dy^4 \left(5 \, x^6 + 8 \, y^6 - 156 \, y^4 z^2 + 225 \, y^2 z^4 - 40 \, z^6 - 15 \, x^4 \left(y^2 + 2 \, z^2 \right) - 3 \, x^2 \left(4 \, y^4 - 70 \, y^2 z^2 + 25 \, z^4 \right) \right) + \\
& \quad 2 \left(-24 \left(x^2 + y^2 - 4 \, z^2 \right) \left(x^2 + y^2 + z^2 \right)^4 + 30 \, dz^2 \left(x^2 + y^2 + z^2 \right)^2 \left(\left(x^2 + y^2 \right)^2 - 12 \left(x^2 + y^2 \right) z^2 + 8 \, z^4 \right) + \right. \\
& \quad \quad 7 \, dz^4 \left(-5 \left(x^2 + y^2 \right)^3 + 120 \left(x^2 + y^2 \right)^2 z^2 - 240 \left(x^2 + y^2 \right) z^4 + 64 \, z^6 \right) \left. + \right. \\
& \quad 5 \, dy^2 \left(-21 \, dz^2 \left(x^6 - 3 \, x^2 \, y^4 - 2 \, y^6 - 15 \left(x^2 - 3 \, y^2 \right) \left(x^2 + y^2 \right) z^2 - 80 \, y^2 z^4 + 16 \, z^6 \right) + \right. \\
& \quad \quad 4 \left(x^2 + y^2 + z^2 \right)^2 \left(3 \, x^4 - 4 \, y^4 + 41 \, y^2 z^2 - 18 \, z^4 - x^2 \left(y^2 + 15 \, z^2 \right) \right) \left. \right) + \\
& \quad 5 \, dx^2 \left(7 \, dz^2 \left(\left(8 \, x^2 - y^2 \right) \left(x^2 + y^2 \right)^2 + 15 \left(-11 \, x^4 - 10 \, x^2 \, y^2 + y^4 \right) z^2 + 240 \, x^2 z^4 - 16 \, z^6 \right) - \right. \\
& \quad \quad 4 \left(x^2 + y^2 + z^2 \right)^2 \left(6 \, x^4 - y^4 + 5 \, y^2 z^2 + 6 \, z^4 + x^2 \left(5 \, y^2 - 51 \, z^2 \right) \right) \left. + \right. \\
& \quad \quad \left. \left. 7 \, dy^2 \left(8 \, x^6 - 15 \, x^4 \left(y^2 + 5 \, z^2 \right) + \left(y^2 + z^2 \right) \left(2 \, y^4 - 23 \, y^2 z^2 + 8 \, z^4 \right) - 3 \, x^2 \left(7 \, y^4 - 100 \, y^2 z^2 + 25 \, z^4 \right) \right) \right) \right) \right\}, \\
& z \left(-42 \, dy^4 \left(x^6 - 15 \, x^4 \, y^2 + 16 \, y^6 + 10 \, y^2 \left(3 \, x^2 - 8 \, y^2 \right) z^2 - 3 \left(x^2 - 15 \, y^2 \right) z^4 - 2 \, z^6 \right) - \right. \\
& \quad 42 \, dx^4 \left(16 \, x^6 + y^6 - 80 \, x^4 z^2 - 3 \, y^2 z^4 - 2 \, z^6 - 15 \, x^2 \left(y^2 - 3 \, z^2 \right) \left(y^2 + z^2 \right) \right) - \\
& \quad 2 \left(24 \left(3 \left(x^2 + y^2 \right) - 2 \, z^2 \right) \left(x^2 + y^2 + z^2 \right)^4 - 10 \, dz^2 \left(x^2 + y^2 + z^2 \right)^2 \left(15 \left(x^2 + y^2 \right)^2 - 40 \left(x^2 + y^2 \right) z^2 + 8 \, z^4 \right) + \right. \\
& \quad \quad 7 \, dz^4 \left(35 \left(x^2 + y^2 \right)^3 - 210 \left(x^2 + y^2 \right)^2 z^2 + 168 \left(x^2 + y^2 \right) z^4 - 16 \, z^6 \right) \left. - \right. \\
& \quad 5 \, dy^2 \left(-4 \left(x^2 + y^2 + z^2 \right)^2 \left(3 \left(x^2 - 6 \, y^2 \right) \left(x^2 + y^2 \right) - \left(x^2 - 41 \, y^2 \right) z^2 - 4 \, z^4 \right) + \right. \\
& \quad \quad 7 \, dz^2 \left(5 \left(x^2 - 8 \, y^2 \right) \left(x^2 + y^2 \right)^2 - 15 \left(x^2 - 15 \, y^2 \right) \left(x^2 + y^2 \right) z^2 - 12 \left(x^2 + 13 \, y^2 \right) z^4 + 8 \, z^6 \right) \left. + \right. \\
& \quad 5 \, dx^2 \left(7 \, dz^2 \left(5 \left(8 \, x^2 - y^2 \right) \left(x^2 + y^2 \right)^2 + 15 \left(-15 \, x^4 - 14 \, x^2 \, y^2 + y^4 \right) z^2 + 12 \left(13 \, x^2 + y^2 \right) z^4 - 8 \, z^6 \right) + \right. \\
& \quad \quad 7 \, dy^2 \left(8 \, x^6 - 75 \, x^4 \, y^2 - 75 \, x^2 \, y^4 + 8 \, y^6 - 15 \left(x^4 - 20 \, x^2 \, y^2 + y^4 \right) z^2 - 21 \left(x^2 + y^2 \right) z^4 + 2 \, z^6 \right) - \\
& \quad \quad \left. \left. 4 \left(x^2 + y^2 + z^2 \right)^2 \left(18 \, x^4 - 3 \, y^4 + y^2 z^2 + 4 \, z^4 + x^2 \left(15 \, y^2 - 41 \, z^2 \right) \right) \right) \right) \left. \right\}
\end{aligned}$$

We gained nothing. Happens ...

In[2]:= CForm[simplified[[1]]]

```
Out[2]/CForm= x*(-14*Power(dy,4)*(Power(x,6) - 15*Power(x,4)*Power(y,2) + 16*Power(y,6) -
6*(Power(x,4) - 25*Power(x,2)*Power(y,2) + 40*Power(y,4))*Power(z,2) -
15*(Power(x,2) - 11*Power(y,2))*Power(z,4) - 8*Power(z,6)) -
14*Power(dx,4)*(8*Power(x,6) - 15*Power(x,2)*(Power(y,2) - 15*Power(z,2))*
(Power(y,2) + Power(z,2)) +
5*(Power(y,2) - 8*Power(z,2))*Power(Power(y,2) + Power(z,2),2) -
12*Power(x,4)*(Power(y,2) + 13*Power(z,2))) +
2*(-24*(Power(x,2) + Power(y,2) - 4*Power(z,2))*
Power(Power(x,2) + Power(y,2) + Power(z,2),4) +
30*Power(dz,2)*Power(Power(x,2) + Power(y,2) + Power(z,2),2)*
(Power(Power(x,2) + Power(y,2),2) - 12*(Power(x,2) + Power(y,2))*Power(z,2) +
8*Power(z,4)) + 7*Power(dz,4)*
(-5*Power(Power(x,2) + Power(y,2),3) +
120*Power(Power(x,2) + Power(y,2),2)*Power(z,2) -
240*(Power(x,2) + Power(y,2))*Power(z,4) + 64*Power(z,6))) +
5*Power(dy,2)*(7*Power(dz,2)*(Power(Power(x,2) + Power(y,2),2)*
(-Power(x,2) + 8*Power(y,2)) +
15*(Power(x,2) - 11*Power(y,2))*(Power(x,2) + Power(y,2))*Power(z,2) +
240*Power(y,2)*Power(z,4) - 16*Power(z,6)) +
4*Power(Power(x,2) + Power(y,2) + Power(z,2),2)*
(Power(x,4) - 6*Power(y,4) + 51*Power(y,2)*Power(z,2) - 6*Power(z,4) -
5*Power(x,2)*(Power(y,2) + Power(z,2)))) +
5*Power(dx,2)*(21*Power(dz,2)*(2*Power(x,6) + 3*Power(x,4)*Power(y,2) - Power(y,6) +
15*(-3*Power(x,4) - 2*Power(x,2)*Power(y,2) + Power(y,4))*Power(z,2) +
80*Power(x,2)*Power(z,4) - 16*Power(z,6)) -
4*Power(Power(x,2) + Power(y,2) + Power(z,2),2)*
(4*Power(x,4) + Power(x,2)*(Power(y,2) - 41*Power(z,2)) -
3*(Power(y,2) - 6*Power(z,2))*(Power(y,2) + Power(z,2))) +
7*Power(dy,2)*(2*Power(x,6) - 21*Power(x,4)*(Power(y,2) + Power(z,2)) -
15*Power(x,2)*(Power(y,4) - 20*Power(y,2)*Power(z,2) + Power(z,4)) +
(Power(y,2) + Power(z,2))*
(8*Power(y,4) - 83*Power(y,2)*Power(z,2) + 8*Power(z,4))))
```

In[3]:= CForm[simplified[[2]]]

```
Out[3]/CForm= y*(-14*Power(dx,4)*(16*Power(x,6) - 240*Power(x,4)*Power(z,2) -
  15*Power(x,2)*(Power(y,2) - 11*Power(z,2))*(Power(y,2) + Power(z,2)) +
  (Power(y,2) - 8*Power(z,2))*Power(Power(y,2) + Power(z,2),2)) -
  14*Power(dy,4)*(5*Power(x,6) + 8*Power(y,6) - 156*Power(y,4)*Power(z,2) +
  225*Power(y,2)*Power(z,4) - 40*Power(z,6) -
  15*Power(x,4)*(Power(y,2) + 2*Power(z,2)) -
  3*Power(x,2)*(4*Power(y,4) - 70*Power(y,2)*Power(z,2) + 25*Power(z,4))) +
  2*(-24*(Power(x,2) + Power(y,2) - 4*Power(z,2))*
  Power(Power(x,2) + Power(y,2) + Power(z,2),4) +
  30*Power(dz,2)*Power(Power(x,2) + Power(y,2) + Power(z,2),2)*
  (Power(Power(x,2) + Power(y,2),2) - 12*(Power(x,2) + Power(y,2))*Power(z,2) +
  8*Power(z,4)) + 7*Power(dz,4)*
  (-5*Power(Power(x,2) + Power(y,2),3) +
  120*Power(Power(x,2) + Power(y,2),2)*Power(z,2) -
  240*(Power(x,2) + Power(y,2))*Power(z,4) + 64*Power(z,6))) +
  5*Power(dy,2)*(-21*Power(dz,2)*(Power(x,6) - 3*Power(x,2)*Power(y,4) -
  2*Power(y,6) - 15*(Power(x,2) - 3*Power(y,2))*(Power(x,2) + Power(y,2))*
  Power(z,2) - 80*Power(y,2)*Power(z,4) + 16*Power(z,6)) +
  4*Power(Power(x,2) + Power(y,2) + Power(z,2),2)*
  (3*Power(x,4) - 4*Power(y,4) + 41*Power(y,2)*Power(z,2) - 18*Power(z,4) -
  Power(x,2)*(Power(y,2) + 15*Power(z,2)))) +
  5*Power(dx,2)*(7*Power(dz,2)*((8*Power(x,2) - Power(y,2))*
  Power(Power(x,2) + Power(y,2),2) +
  15*(-11*Power(x,4) - 10*Power(x,2)*Power(y,2) + Power(y,4))*Power(z,2) +
  240*Power(x,2)*Power(z,4) - 16*Power(z,6)) -
  4*Power(Power(x,2) + Power(y,2) + Power(z,2),2)*
  (6*Power(x,4) - Power(y,4) + 5*Power(y,2)*Power(z,2) + 6*Power(z,4) +
  Power(x,2)*(5*Power(y,2) - 51*Power(z,2))) +
  7*Power(dy,2)*(8*Power(x,6) - 15*Power(x,4)*(Power(y,2) + 5*Power(z,2)) +
  (Power(y,2) + Power(z,2))*
  (2*Power(y,4) - 23*Power(y,2)*Power(z,2) + 8*Power(z,4)) -
  3*Power(x,2)*(7*Power(y,4) - 100*Power(y,2)*Power(z,2) + 25*Power(z,4))))
```

In[4]:= CForm[simplified[[3]]]

```
Out[4]/CForm= z*(-42*Power(dy,4)*(Power(x,6) - 15*Power(x,4)*Power(y,2) + 16*Power(y,6) +
  10*Power(y,2)*(3*Power(x,2) - 8*Power(y,2))*Power(z,2) -
  3*(Power(x,2) - 15*Power(y,2))*Power(z,4) - 2*Power(z,6)) -
42*Power(dx,4)*(16*Power(x,6) + Power(y,6) - 80*Power(x,4)*Power(z,2) -
  3*Power(y,2)*Power(z,4) - 2*Power(z,6) -
  15*Power(x,2)*(Power(y,2) - 3*Power(z,2))*(Power(y,2) + Power(z,2))) -
2*(24*(3*(Power(x,2) + Power(y,2)) - 2*Power(z,2))*
  Power(Power(x,2) + Power(y,2) + Power(z,2),4) -
  10*Power(dz,2)*Power(Power(x,2) + Power(y,2) + Power(z,2),2)*
  (15*Power(Power(x,2) + Power(y,2),2) - 40*(Power(x,2) + Power(y,2))*Power(z,2) +
  8*Power(z,4)) + 7*Power(dz,4)*
  (35*Power(Power(x,2) + Power(y,2),3) -
  210*Power(Power(x,2) + Power(y,2),2)*Power(z,2) +
  168*(Power(x,2) + Power(y,2))*Power(z,4) - 16*Power(z,6))) -
5*Power(dy,2)*(-4*Power(Power(x,2) + Power(y,2) + Power(z,2),2)*
  (3*(Power(x,2) - 6*Power(y,2))*(Power(x,2) + Power(y,2)) -
  (Power(x,2) - 41*Power(y,2))*Power(z,2) - 4*Power(z,4)) +
  7*Power(dz,2)*(5*(Power(x,2) - 8*Power(y,2))*Power(Power(x,2) + Power(y,2),2) -
  15*(Power(x,2) - 15*Power(y,2))*(Power(x,2) + Power(y,2))*Power(z,2) -
  12*(Power(x,2) + 13*Power(y,2))*Power(z,4) + 8*Power(z,6))) +
5*Power(dx,2)*(7*Power(dz,2)*(5*(8*Power(x,2) - Power(y,2))*
  Power(Power(x,2) + Power(y,2),2) +
  15*(-15*Power(x,4) - 14*Power(x,2)*Power(y,2) + Power(y,4))*Power(z,2) +
  12*(13*Power(x,2) + Power(y,2))*Power(z,4) - 8*Power(z,6)) +
  7*Power(dy,2)*(8*Power(x,6) - 75*Power(x,4)*Power(y,2) -
  75*Power(x,2)*Power(y,4) + 8*Power(y,6) -
  15*(Power(x,4) - 20*Power(x,2)*Power(y,2) + Power(y,4))*Power(z,2) -
  21*(Power(x,2) + Power(y,2))*Power(z,4) + 2*Power(z,6)) -
  4*Power(Power(x,2) + Power(y,2) + Power(z,2),2)*
  (18*Power(x,4) - 3*Power(y,4) + Power(y,2)*Power(z,2) + 4*Power(z,4) +
  Power(x,2)*(15*Power(y,2) - 41*Power(z,2))))))
```

Tests: MFM signal functions

Newells analytical formulas (n_{xx} , n_{xy} , etc.) valid for any rectangular prism. See A. J. Newell, W. Williams, and D. J. Dunlop, "A Generalization of the Demagnetizing Tensor for Nonuniform Magnetization," *Journal of Geophysical Research Solid Earth* 98, 9551-9555 (1993).

For *Mathematica* definitions of these functions ask K. L.

Apparently, n_{xx} does not include the minus sign (added below for the comparison) and vacuum permeability (assumed one below for comparison).

Additionally, C-function does not have the whole $\frac{1}{64\pi}\mu_0$ prefactor, thus we have removed it from this list as well

$\{0., 0., 0.000599996\}$
 $\{0.0000572669778, 0.000114533967, 0.000563697320\}$
 $\{0.0000572676, 0.000114535, 0.0005637\}$
 $\{0.000057267, 0.000114534, 0.000563697\}$
 $\{0.000057267, 0.000114534, 0.000563697\}$
 $\{5.99718831 \times 10^{-10}, 1.19943766 \times 10^{-9}, 5.99625123 \times 10^{-8}\}$
 $\{5.99719 \times 10^{-10}, 1.19944 \times 10^{-9}, 5.99625 \times 10^{-8}\}$
 $\{5.99719 \times 10^{-10}, 1.19944 \times 10^{-9}, 5.99625 \times 10^{-8}\}$
 $\{5.99719 \times 10^{-10}, 1.19944 \times 10^{-9}, 5.99625 \times 10^{-8}\}$
 $\{0.144977741, 0.290787940, 0.0428488726\}$
 $\{0.144929, 0.289858, 0.0420761\}$
 $\{0.145008, 0.290777, 0.0428204\}$
 $\{0.145008, 0.290777, 0.0428204\}$
 $\{0., -1.89478 \times 10^{-14}, 0.00369921\}$
 $\{0., 0., 0.0036\}$
 $\{0., 0., 0.0036992\}$
 $\{0.000359093432, 0.000711292579, 0.00346986543\}$
 $\{0.000343606, 0.000687212, 0.0033822\}$
 $\{0.000359085, 0.000711297, 0.00346985\}$
 $\{3.59994340 \times 10^{-9}, 7.19921164 \times 10^{-9}, 3.59872472 \times 10^{-7}\}$
 $\{3.59831 \times 10^{-9}, 7.19663 \times 10^{-9}, 3.59775 \times 10^{-7}\}$
 $\{3.59994 \times 10^{-9}, 7.19921 \times 10^{-9}, 3.59872 \times 10^{-7}\}$
 $\{0.000772096153, 0.000857013485, -0.000991196789\}$
 $\{0.000764562, 0.000834068, -0.0010047\}$
 $\{0.0007721, 0.000857018, -0.000991195\}$
 $\{7.26262371 \times 10^{-8}, 7.98368584 \times 10^{-8}, -9.23669403 \times 10^{-8}\}$
 $\{7.26197 \times 10^{-8}, 7.98163 \times 10^{-8}, -9.23793 \times 10^{-8}\}$
 $\{7.26262 \times 10^{-8}, 7.98369 \times 10^{-8}, -9.23669 \times 10^{-8}\}$
 $\{-0.0228352822, -0.00816931169, -0.375508957\}$
 $\{-0.0789409, -0.0592057, -0.293836\}$

```
{-0.0266845, -0.0146723, -0.379502}
```

Following results were used for test in the C-code

```
N[(D[-64 π {nxz[x, y, z, dx, dy, dz], nyz[x, y, z, dx, dy, dz], nzz[x, y, z, dx, dy, dz]}, z]) /.
  {x → 0, y → 0, z → 20, dx → 1, dy → 1, dz → 1}]
N[(D[-64 π {nxz[x, y, z, dx, dy, dz], nyz[x, y, z, dx, dy, dz], nzz[x, y, z, dx, dy, dz]}, z]) /.
  {x → 1, y → 2, z → 20, dx → 1, dy → 1, dz → 1}, 9]
N[(D[-64 π {nxz[x, y, z, dx, dy, dz], nyz[x, y, z, dx, dy, dz], nzz[x, y, z, dx, dy, dz]}, z]) /.
  {x → 1, y → 2, z → 200, dx → 1, dy → 1, dz → 1}, 9]
N[(D[-64 π {nxz[x, y, z, dx, dy, dz], nyz[x, y, z, dx, dy, dz], nzz[x, y, z, dx, dy, dz]}, z]) /.
  {x → 1, y → 2, z → 3, dx → 1, dy → 1, dz → 1}, 9]
N[(D[-64 π {nxz[x, y, z, dx, dy, dz], nyz[x, y, z, dx, dy, dz], nzz[x, y, z, dx, dy, dz]}, z]) /.
  {x → 0, y → 0, z → 20, dx → 1, dy → 2, dz → 3}]
N[(D[-64 π {nxz[x, y, z, dx, dy, dz], nyz[x, y, z, dx, dy, dz], nzz[x, y, z, dx, dy, dz]}, z]) /.
  {x → 1, y → 2, z → 20, dx → 1, dy → 2, dz → 3}, 9]
N[(D[-64 π {nxz[x, y, z, dx, dy, dz], nyz[x, y, z, dx, dy, dz], nzz[x, y, z, dx, dy, dz]}, z]) /.
  {x → 1, y → 2, z → 200, dx → 1, dy → 2, dz → 3}, 9]
N[(D[-64 π {nxz[x, y, z, dx, dy, dz], nyz[x, y, z, dx, dy, dz], nzz[x, y, z, dx, dy, dz]}, z]) /.
  {x → 11, y → 12, z → 13, dx → 3, dy → 2, dz → 1}, 9]
N[(D[-64 π {nxz[x, y, z, dx, dy, dz], nyz[x, y, z, dx, dy, dz], nzz[x, y, z, dx, dy, dz]}, z]) /.
  {x → 111, y → 122, z → 133, dx → 3, dy → 2, dz → 1}, 9]
N[(D[-64 π {nxz[x, y, z, dx, dy, dz], nyz[x, y, z, dx, dy, dz], nzz[x, y, z, dx, dy, dz]}, z]) /.
  {x → 4, y → 3, z → 2, dx → 3, dy → 2, dz → 1}, 9]
{0., 0., 0.000599996}
{0.0000572669778, 0.000114533967, 0.000563697320}
{5.99718831 × 10-10, 1.19943766 × 10-9, 5.99625123 × 10-8}
{0.144977741, 0.290787940, 0.0428488726}
{0., -1.89478 × 10-14, 0.00369921}
{0.000359093432, 0.000711292579, 0.00346986543}
{3.59994340 × 10-9, 7.19921164 × 10-9, 3.59872472 × 10-7}
{0.000772096153, 0.000857013485, -0.000991196789}
{7.26262371 × 10-8, 7.98368584 × 10-8, -9.23669403 × 10-8}
{-0.0228352822, -0.00816931169, -0.375508957}
```

Footnote

Written by Kristof M. Lebecki, 2012, Universitat Konstanz, Germany.