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## Creative Destruction and Policy in a Model of Endogenous Growth

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# Creative Destruction and Policy in a Model of Endogenous Growth\*

Xavier Mateos-Planas

## Abstract

This paper extends a model of endogenous growth through the introduction of a component of knowledge that makes new technologies more productive than older vintages. Creative destruction or obsolescence of technologies underlies the growth process. In this setup, the growth effects of various policies are analyzed. These policies include selective subsidies to firms that produce final output, a general lump-sum tax on final-output firms, and openness to trade with a less developed country. The results show the existence of growth effects that have not been studied in the previous literature.

**KEYWORDS:** endogenous growth, obsolescence, embodied technology, vintage, policy

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## 1 Introduction

Technological change and innovation involve the emergence of new products and technologies, along with the gradual displacement of old ones. That the process of replacement of new goods and sectors for older ones is a substantive ingredient of modern economic growth is not a new idea, having been first expressed in Schumpeter (1942)'s celebrated notion of *creative destruction*. In this view, economic progress is the cause as well as the consequence of the rise and decline of sectors and firms. If innovation and obsolescence interact, then policies that interfere with the obsolescence process may have implications for economic growth. For example, industrial policies in the form of selective taxes and subsidies might alter the relative position of old and new sectors.<sup>1</sup> Perhaps less obviously, other policies such as lump-sum business taxation and trade policies may also have a bearing on this *creative destruction* and, thereby, on economic growth.

The aim of this paper is to investigate the implications of these policies for growth in the presence of technological obsolescence. To that end, two objectives will be pursued. The first is to gain an understanding of a model economy undergoing the process of gradual replacement of old by new firms and sectors. Then the second and substantive objective is to analyze, in this context, the consequences of selective subsidies, lump-sum taxes, and openness to trade for the process of growth.

This investigation is based on the analysis of a model that extends previous work on endogenous growth theory. The model borrows its basic structure from Grossman and Helpman (1991) so that privately conducted R&D is the engine of growth. The paper develops a version which incorporates obsolescence of technologies in a deliberately simple form. More precisely, in order to accommodate the existence of new along with old declining sectors or firms, this paper introduces a productivity augmenting component of knowledge that makes new technologies more productive than older vintages. As in previous

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<sup>1</sup> Selective policies have been widespread in the fast-developing experiences of Korea, Singapore and Taiwan (see Young (1992) and Westphal (1990)). In these cases, the government has been involved in channeling credit to selected industries. Sector-specific policies are a central theme in development policy (see, for example, Krueger (1990), Pack and Westphal (1986), and Pack (2000)). OECD (1990) documents that government support and industry subsidies are common practice in developed countries too.

work, new designs for consumption varieties are the purposeful outcome of commercial research. But it is also assumed that commercial research generates, as a by product, knowledge that is useful in the production of output of the latest vintages of goods. Specifically, each new design embodies the level of technology existing at the time it is created.

The paper characterizes equilibrium properties of the economy. There is transitional dynamics and, in the long-run, the economy approaches a balanced growth path. The long-run growth rate of innovation is higher than in the symmetric model of Grossman and Helpman (1991). Two forces with opposite sign underlie this result. The introduction of new goods embodying the leading technology erodes the profits of existing firms over time [i.e., the business stealing effect]. On the other hand, however, new goods enjoy a productivity advantage over existing ones. In this model, this latter effect dominates the net positive impact on the incentives for innovation. One can also characterize the dynamic and static profiles for employment, output and prices over existing vintages. Replacement of old technologies is gradual and employment and profits are higher in new vintages, whereas the price is lower.<sup>2</sup> Regarding welfare, the equilibrium is suboptimal for exactly the same reasons as the economy without vintage effects is. The market induces too little research.

In this setup, I analyze the growth effects of various policies that have a differential impact on technologies at different stages of their life-cycle. I start with an ad-valorem subsidy to final-output firms. When the subsidy favours the production of low-profit goods, older vintages benefit from it and the long-run growth rate is reduced. On the other hand, a subsidy to more profitable goods benefits the more recent vintages and spurs growth. Policies that amplify the destructive effect of growth by widening the advantage of new over old technologies are beneficial to growth. I also explore the role of lump-sum taxation levied on the producers of final goods. In the model without obsolescence, growth decreases unambiguously because profits, and thus the value of innovating firms, are reduced. In the present model, on the contrary, profits for any firm are declining over time. A lump-sum tax will render old firms unprofitable. As the range of competing vintages in operation is narrowed, the returns to innovation may increase. Thus lump-sum taxation can be beneficial to growth. A similar mechanism underlies the positive effects on growth from

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<sup>2</sup> Provided that the focus on final-good rather than capital-good vintages is superfluous, these implications are reminiscent of the the evidence on declining relative price of new equipment reported in Greenwood et al. (1997).

trade with a less developed country.

This model is related to literature on endogenous growth. In that private R&D is the engine of growth and there is horizontal product differentiation, the paper relates to Romer (1990) as well as to Grossman and Helpman (1991). In that the model introduces a notion of vertical differentiation it resembles Stokey (1988), Young (1991), and Aghion and Howitt (1992). Other papers have developed theories that combine both vertical and horizontal differentiation, including Young (1993) and Thompson (2001). Unlike these works, in the present paper vertical differentiation is assumed to arise as an external vintage effect rather than derived from detailed micro-foundations. The modeling strategy in Caballero and Jaffe (1993) and Lai (1998) is very close to the one in this paper. The emphasis and the questions addressed set the present paper apart from those works. There is a significant literature analyzing models of growth with vintage capital investment which include Krusell (1998) and Boucekkinne *et al.* (2001, 2003). Compared with these works, the present paper is more policy-oriented while sacrificing the explicit analysis of capital accumulation. The model has scale effects and is subject to the objections by Jones (1995). The choice of model has been dictated uniquely by simplicity though. The effects on the incentives for R&D activities studied in this paper should nevertheless carry over under Jones (1995)'s assumptions.

Many papers to date have discussed the effects of policies on growth. The main contribution of this paper is to demonstrate in a simple model that the mere existence of technological vintage effects may be significant to understand some of these effects.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the equilibrium of the economy and its welfare properties. Section 4 introduces policies and analyzes their growth effects. Section 5 concludes the paper.

## 2 The model

The two inputs in the economy are a primary factor denoted by  $h$ , and technology or knowledge  $A$ . The technology input has two components. The first is disembodied (scientific or engineering) knowledge useful in the production of further knowledge. The second component consists of knowledge that is

embodied in the production process of particular consumption goods. The introduction of the embodied component of knowledge is the main distinctive feature of the model which I proceed to set up more formally.

The productive side of the economy is composed of two sectors. A research and development (R&D) sector and a consumption-goods sector. The R&D sector uses the primary factor  $h$ , and the existing level of disembodied knowledge  $A(t)$  as inputs to produce designs for new consumption varieties. The measure of varieties at a given point in time  $t$  is  $n(t)$ . Existing varieties are indexed by  $i \in [0, n(t)]$ . The technology of the R&D firms exhibits increasing returns to scale in both inputs and is specified as

$$\dot{n}(t) = \delta h_r(t) A(t). \quad (1)$$

Here  $\delta$  is a productivity parameter in R&D, and  $h_r(t)$  denotes the amount of the primary input  $h$  employed in research at time  $t$ . Additions to the stock of disembodied knowledge result as a by-product or externality from the R&D activity that creates new goods. I choose units so that,

$$\dot{A}(t) = \dot{n}(t). \quad (2)$$

The second is the consumption-goods sector which is composed of the firms that produce differentiated varieties. Each of these firms uses the primary factor as an input to physically produce a particular design brought about by the R&D sector. The flow of output to a firm producing a variety  $i \in [0, n(t)]$  is given by a function which is linear in the amount of primary factor employed there  $h(i, t)$ ,

$$\phi(A(t(i)))h(i, t). \quad (3)$$

Here  $\phi(\cdot)$  is a non-decreasing function of the productive knowledge embodied in the production of this good which I assume corresponds to the stock of knowledge existing at the time the variety  $i$  was invented,  $A(t(i))$ . The motivation for this assumption is the same as for the external effect in R&D. Knowledge has a productivity-augmenting component which affects consumption-goods' technologies unequally.<sup>3</sup>

<sup>3</sup> Young's (1991) view of technological processes is very similar. There technological innovation is defined by the introduction of new and "more advanced" horizontally

The consumption side is standard. The economy is populated by a fixed measure (let it be 1) of identical infinitely-lived agents. Each of them is endowed with  $h$  units of the non-perishable primary factor. Preferences are defined over paths of consumption of different varieties. For utility at a point in time  $t$ , I adopt the standard CES specification:

$$\left[ \int_0^{n(t)} c(i, t)^{\theta/(1+\theta)} di \right]^{(1+\theta)/\theta} \quad \text{with } \theta > 1. \quad (4)$$

The elasticity of substitution between goods is  $1 + \theta$  which characterizes the preference for variety. The infinite stream of instantaneous utilities is discounted and the intertemporal elasticity of substitution is constant. In particular, I make the log assumption and represent intertemporal utility by

$$\int_0^{\infty} e^{-\rho t} \log \left[ \int_0^{n(t)} c(i, t)^{\theta/(1+\theta)} di \right]^{(1+\theta)/\theta} dt, \quad \text{with } \rho < 1 \quad (5)$$

The market structure is also standard. The R&D market sector is competitive. Since the technology for R&D firms exhibits increasing returns to scale, the equilibrium must rely on some form of external effect in R&D. In effect, the stock of disembodied knowledge enters the R&D technology as an externality. Concerning final goods, each innovator must purchase a design from the R&D sector before commencing production. This constitutes a fixed cost which prevents any rule of marginal-cost pricing from being supported in equilibrium. Final-good firms are then assumed to behave as monopolistic competitors. Embodied knowledge creates increasing returns in the production of varieties. This is assumed to be an externality as well. The market for the primary input is competitive. Consumers can freely lend and borrow. Any producer of a particular variety finances its initial cost by issuing equity.

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differentiated goods that are producible with lower factor requirements. The model in Caballero and Jaffe (1993) and Lai (1998) differentiates goods by their quality in a similar way. The case where  $\phi(\cdot)$  is a constant corresponds to Romer (1990) or Grossman and Helpman (1991).

### 3 Equilibrium

The equilibrium is defined as paths for prices, allocations of the primary input, and consumption vectors such that: consumers maximize utility; research and final-goods firms maximize profits; all markets clear.

The consumer's behavior is solved in two stages. First, at any point in time  $t$ , consumption expenditure, denoted  $E(t)$ , must be allocated among the existing set of varieties so as to maximize instantaneous current utility. Given prices  $p(i, t)$ 's, the static problem consists of choosing  $c(i, t)$ 's that maximize the value in Eq.(4), provided that  $E(t) = \int_{[0, n(t)]} p(i, t)c(i, t)$  holds. Given  $E(t)$ , the solution to this problem yields a demand function for each variety:

$$c(i, t) = E(t) \frac{p(i, t)^{-\theta-1}}{\int_0^{n(t)} p(j, t)^{-\theta} dj}. \quad (6)$$

The second step in the consumer's problem is the choice of paths of consumption and assets that maximize the value of the discounted stream of instantaneous utilities in Eq. (5). Financial assets yield an instantaneous return  $r(t)$ . A no-arbitrage condition that requires that the consumer be indifferent between financial assets and equity must be satisfied at every point in time. Denote the value at  $t$  of a firm producing a variety born at  $\tau$ ,  $i(\tau)$ , by  $V(i(\tau), t)$ . Assume that the value of a firm coincides with its fundamental value, that is to say,  $V(i(\tau), t) \equiv \int_t^\infty e^{-\int_t^s r(u)du} \pi(i(\tau), s) ds$ , where  $\pi(i(\tau), t)$  is the profit of a firm producing good  $i(\tau)$  at time  $t$ . Then the no-arbitrage condition reads

$$r(t)V(i(t), t) = \pi(i(t), t) + \dot{V}(i(t), t), \quad (7)$$

where the dot notation denotes the derivative with respect to the second time argument only [not  $t$  in  $i(t)$ ]. The second right-hand side term is the gain/loss of a (given) firm  $i(t)$ . The sum of the instantaneous profit and the capital gain/loss must equal the opportunity cost on equity claims of the capital invested in firm  $i(t)$ ,  $r_t V(i(t), t)$ . Provided that this condition holds and given the goods' prices, the consumer selects a path for  $E(t)$  and a path for assets that maximize  $\int_{[0, \infty]} e^{-\rho t} \log(P(t)E(t))dt$  with  $P(t) \equiv [\int_{[0, n(t)]} p(j, t)^{-\theta} dj]^{\frac{1}{\theta}}$ . The solution to the consumer's dynamic problem must satisfy the Euler equation

$$\frac{\dot{E}_t}{E_t} = r_t - \rho. \quad (8)$$

Research firms sell new designs in a competitive market. Denote the price of designs created at  $t$  by  $p_A(t)$ . With free entry and exit, in an equilibrium with positive innovation, profits of engaging in research must be exactly zero. In other words,

$$w(t) = p_A(t)\delta A(t). \quad (9)$$

Each firm producing a differentiated variety maximizes current profits at any time. It faces a demand function given by the consumer's optimal behavior in Eq.(6), so that profits at  $t$  of a firm producing  $i$  can be written as

$$\pi(i, t) = E(t) \frac{p(i, t)^{-\theta-1}}{\int_0^{n(t)} p(j, t)^{-\theta} dj} \left[ p(i, t) - \frac{w(t)}{\phi(A(t(i)))} \right]. \quad (10)$$

Given that the effect of the price choice of a particular producer on the integral is negligible, profit maximization yields the standard mark-up pricing rule

$$p(i, t) = \frac{1 + \theta}{\theta} \frac{w(t)}{\phi(A(t(i)))}. \quad (11)$$

The mark-up is constant across firms but the unit-cost depends on the embodied productivity level given by  $\phi(A_{t(i)})$ . The more productive a firm, the lower the price of the good. This is the source of assymetries in the economy's equilibrium. Using Eq. (11), profits and quantities in Eq. (6) and (10) can be written in a more convenient way as

$$\pi(i, t) = E(t) \frac{[\phi(A(t(i)))]^\theta}{\int [\phi(A(t(j)))]^\theta dj} \frac{1}{1 + \theta}, \quad (12)$$

$$c(i, t) = E_t \frac{[\phi(A(t(i)))]^{1+\theta}}{\int [\phi(A(t(j)))]^\theta dj} \frac{1}{w(t)} \frac{\theta}{1 + \theta}. \quad (13)$$

Since the market for designs is competitive, the price of any new design will be bid up until it equals the value of the profits a monopolist can extract or, in other words, the value of the firm. Formally, in equilibrium, for a design appeared at  $t$ , the following must hold:

$$V(i(t), t) = p_A(t). \quad (14)$$

The market clearing condition for the primary input reads

$$\int_0^{n(t)} h(i, t) di + h_r(t) = h, \quad (15)$$

where, according to Eq.(3), firm  $i$ 's demand of the input is given by,

$$h(i, t) = \frac{c(i, t)}{\phi(A(t(i)))} \quad (16)$$

The equilibrium for the economy is completely described by equations (1), (2), (7)-(9), and (12)-(16). This set of equations can be reduced to two expressions involving  $E$  and  $A$ . The first is a market-clearing expression derived from (1), (2), (9), (15) and (16), with (13). The second is a no-arbitrage condition derived from (7), (8) and (14), with (12). To proceed, one can set the time path for a nominal variable and measure prices against the chosen numeraire. I set  $p_{At} = 1$ . With this normalization, conditions (9), and (7) with (12) and (14) specialize, respectively, to

$$w(t) = \delta A(t), \quad (17)$$

$$r(t) = \pi(i(t), t) - \theta \frac{\phi'(A(t))}{\phi(A(t))} \dot{A}(t). \quad (18)$$

Equation (18) deserves comment.<sup>4</sup> This condition says that the current profit of a just-born firm is higher than the interest rate by a term which depends

<sup>4</sup> The total differential of the fundamental value of an innovator can be calculated as

$$\begin{aligned} \frac{dV(i(t), t)}{dt} &= \dot{V}(i(t), t) + \frac{dV(i(t), t)}{di(t)} \frac{di(t)}{dt} \\ &= \dot{V}(i(t), t) + \left[ \int_t^\infty e^{-\int_t^s r(u) du} \frac{d\pi(i(t), t)}{di(t)} dt \right] \dot{A}(t) \\ &= \dot{V}(i(t), t) + V(i(t), t) \theta \frac{\phi'(A(t))}{\phi(A(t))} \dot{A}(t) \\ &= -\pi(i(t), t) + \left[ r(t) + \theta \frac{\phi'(A(t))}{\phi(A(t))} \dot{A}(t) \right] V(i(t), t) \end{aligned}$$

upon the effect of innovation on the new entrants' productivity. If new firms are more productive, future profits for existing firms will be decreasing over time. Hence, the required current return to a new firm must be higher than the interest rate in order to offset future losses which, in turn, depend on the rate of technological change. Hence this term captures the idea of 'creative destruction'.

From the law-of-motion of knowledge in Eq.(1) and (2), with the market clearing condition (15), using the factor demand functions (16) with (13), and the R&D equilibrium free-entry condition (17), an equilibrium with positive innovation must satisfy the differential equation,

$$\frac{\dot{A}(t)}{A(t)} = \delta h - \frac{\theta}{1 + \theta} \frac{E(t)}{A(t)}. \quad (19)$$

A higher ratio of consumption expenditure to knowledge amounts to higher firms' demand for the primary input. The induced factor allocations then tends to lower the innovation rate. It turns out that this equilibrium equation is identical to the one arising from the model with symmetric varieties in Grossman and Helpman (1991) [i.e. with  $\phi(\cdot)$  a constant].

The description of the equilibrium is completed by determining the equilibrium interest rate in the intertemporal condition (8), using the form of the no-arbitrage condition (18) and the expression for the firm's profits (12). To proceed, the form of  $\phi(\cdot)$  must be specified. I assume a linear specification and choose units so that

$$\phi(A) = A. \quad (20)$$

For notational convenience define

$$\Gamma(A(t), A(0)) \equiv \frac{A(t)^{1+\theta}}{A(t)^{1+\theta} - A(0)^{1+\theta}}. \quad (21)$$

where  $A(0)$  denotes the level of knowledge that is embodied in the oldest technology in operation. The profits for the producer of a just-born variety

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where the third equality uses Eq. (12), and the fourth equality Eq. (7). The normalization of  $p_A(t) = 1$  and Eq. (14) lead to Eq. (18).

can be rewritten, upon appropriate change of variables under the integral in Eq.(12), as

$$\pi(i(t), t) = \frac{E(t)}{A(t)}\Gamma(A(t), A(0)), \quad (22)$$

With this result, the equilibrium interest rate in Eq.(18) can be substituted into Eq.(8) to deliver the second equilibrium equation

$$\frac{\dot{E}(t)}{E(t)} = \frac{E(t)}{A(t)}\Gamma(A(t), A(0)) - \theta \frac{\dot{A}(t)}{A(t)} - \rho. \quad (23)$$

This condition differs from the one in the model without obsolescence in the two first right-hand side terms.<sup>5</sup> Equations (19) and (23) characterize the equilibrium as a system of two differential equations in  $A$  and  $E$ .

### 3.1 Long-run growth

This section is concerned with the long-run behavior of the economy. In this model, because firms, irrespective of their age, set the selling price as a mark-up over the unit cost, profits are always non-negative. Thus no firm is ever shut down. This implies that the oldest firm in operation is the one created at the beginning of times and, consequently, the lowest productivity in operation is the constant  $A(0)$ . In an equilibrium with positive innovation,  $\Gamma(A(t), A(0))$  declines over time towards 1. So  $\Gamma(A(t), A(0)) = 1$  holds approximately in the long-run.

Assume that the economy attains a balanced growth path displaying a constant rate of innovation  $\gamma \equiv \dot{A}(t)/A(t)$ . From the factor-market clearing equilibrium condition Eq.(19),  $E$  must grow at the same rate as  $A$  does. In the long-run the profit rate of an innovator is constant too and so is the interest rate. In these circumstances, the equilibrium equations(19) and (23) can be written as

<sup>5</sup> The first term, which is the profit rate of a new firm, in the Grossman and Helpman (1991) model reads  $(1/(1+\theta))E(t)/A(t)\Gamma(A(t), A(0))$  with the  $\Gamma$  term as above but letting  $\theta = 0$ . The second right-hand side term is simply absent if  $\phi(\cdot)$  is constant.

the two following equations:

$$\gamma = \delta h - \frac{\theta}{1 + \theta} \frac{E}{A} \quad (24)$$

$$\gamma = \frac{E}{A} - \theta\gamma - \rho. \quad (25)$$

Whereas Eq.(24) is exactly as in the model with  $\phi(\cdot)$  constant, Eq.(25) is not for the reasons pointed out in the discussion of Eq.(23). On one hand, the profit rate to the current innovator is higher due to its productive advantage over existing vintages. This *creative* effect is captured by the first right hand side term in Eq.(25) above.<sup>6</sup> However, the pace of future innovations will reduce future profits. This *destructive* component is captured in the second right hand side term. On net, the first *creative* effect dominates the second *destructive* effect. This can be seen in Figure 1.

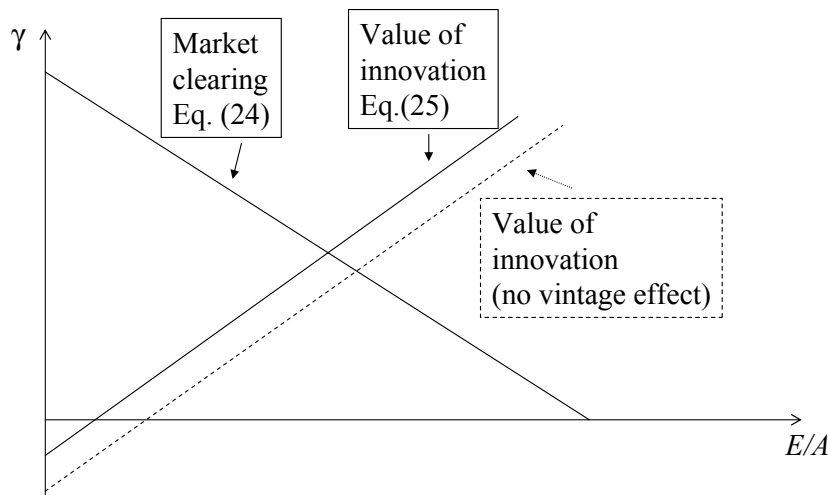


Figure 1. The balanced-growth equilibrium. The dashed line represents the symmetric case.

Combine (24) and (25) and the balanced rate of innovation can be calculated explicitly as

$$\gamma = \frac{1}{1 + \theta} \left[ \delta h - \frac{\theta}{1 + \theta} \rho \right]. \quad (26)$$

<sup>6</sup> It turns out that, if  $\phi$  is constant, this term is smaller by a factor  $1/(1 + \theta)$ .

### 3.2 Product composition

The way the technological knowledge spreads over productive activity gives rise to heterogeneity across firms born at different dates as well as to an evolving pattern for the variables describing the performance of every particular firm. This section characterizes the pattern for prices, quantities, profits and resource allocations both of every firm over time (the dynamic pattern) and across different firms at a given point in time (the static pattern) in the long-run.

Consider first the age-distribution for prices at a given date  $t$ . A firm born at  $\tau$  sets prices as a (constant) mark-up over average cost  $w(t)/A(\tau)$ . As younger (higher  $\tau$ ) goods are produced at lower cost, they are traded at lower prices. The extent to which cost differ across firms depends directly on the innovation path. In a balanced equilibrium with a constant rate of innovation  $\gamma$  the price of goods decreases in the birth-date at that rate. Similarly, the price of any good increases at the rate  $\gamma$  over time.

Consider now the pattern for the output of final goods. From Eq. (13), (17) and (21) one can write  $c(i(\tau), t) = (E/A)(\theta/\delta)(A(\tau)/A(t))^{1+\theta}\Gamma(A(t), A(0))$ . We may then conclude that the quantity of a particular good over time and the quantities at a given point in time of goods of different ages decrease with time and age respectively at the common rate  $-(1 + \theta)\gamma$ . The response of  $c(i(\tau), t)$  to changes in  $\tau$  only depends on how the price varies, all other things held constant. Since  $p(i(\tau), t)$  decreases with  $\tau$ , then  $c(i(\tau), t)$  increases with  $\tau$  at a proportional rate of  $(1 + \theta)\gamma$ . As expected, it depends positively on the elasticity of substitution. The time path for quantities of a particular good hinges on a number of factors. The value of aggregate consumption expenditure grows over time which exerts a positive effect. In opposite direction, as time evolves both the the number of varieties and the own price increase.

Concerning profits, the average cost of any firm is proportional to its price. Then profits accruing to a firm are proportional to the share of consumption expenditure on this good. From (12), (17), and (22) the profit rate of a firm can be written as  $\pi(i(\tau), t) = (E/A)(A(\tau)/A(t))^\theta\Gamma(A(t), A(0))$ . So profits fall with  $t$  and rise with  $\tau$  at the rate  $\theta\gamma_A$ .

The economy is endowed with a fixed amount of primary factor. In a balanced equilibrium, the splitting of resources between research and final-goods is constant. It is interesting to look now at the allocation of primary input among

firms. It results from the tension between two forces. As already seen, younger firms produce a larger amount of goods. On the other hand they are more productive. We have that the first effect dominates so that more advanced firms use a larger amount of primary factor. The quantity of employed resources by individual firms in Eq.(16) then decreases with the age of the firms at a rate  $-\theta\gamma_A$ . For a particular firm the demand of primary factor evolves according to the demand for its product. Then employment in a firm decreases over time at the same rate as consumption does  $-(1 + \theta)\gamma_A$ .

### 3.3 Dynamics and existence

This subsection describes the dynamic adjustment of the economy that starts from an initial level of knowledge  $A(0)$ . The analysis permits to identify existence conditions for the long-run outcomes described above. It is useful to rewrite the equations (19) and (23) as follows:

$$\gamma = \delta h - \frac{\theta}{1 + \theta} \frac{E}{A}$$

$$\frac{d(E/A)}{dt} = [\Gamma(A(t), A(0)) + \theta] \frac{E}{A} - (1 + \theta)\delta h - \rho$$

with  $E/A$  and  $\gamma$  dated at time  $t$ . Notice that the behavior on  $E/A$  is described by a single differential equation with a non-constant coefficient. Starting from an arbitrary initial value of knowledge embodied in the oldest variety produced,  $A(0)$ , the economy features a transition towards the long-run outcomes due uniquely to the fact that the term  $\Gamma$  changes with the value of  $A$ .<sup>7</sup> The graphical representation in Figure 2 of these conditions in the  $A(t)$ - $E/A$  space will suffice to show the economy's behavior.

Represent the  $d(E/A)/dt = 0$  schedule as a concave, increasing, and bounded-from-above function of  $A(t)$  with the upper bound being  $[(1 + \theta)\delta h + \rho]/(1 + \theta)$ . The arrows indicate that from a point above it,  $E/A$  tends to increase

<sup>7</sup> The case of  $\Gamma$  being constant corresponds to the assumption that the measure of varieties is the level of technological knowledge. This is the case in Grossman and Helpman (1991) where the equilibrium must feature an instantaneous jump to the balanced-growth outcomes. Thus for there to be a transition it is necessary to distinguish the measure of goods or firms from the level of knowledge.

and the opposite holds from a point below. On the other hand, only points above the horizontal line at  $E/A$  with value  $[(1 + \theta)/\theta]\delta h$  are consistent with a path with positive innovation [i.e. increasing  $A$  over time]. In the case this horizontal line falls below the upper bound for the  $d(E/A)/dt = 0$  curve, for any choice of initial  $E/A$  either  $A$  falls, or  $E$  attains negative values, or innovation becomes zero in finite time. Thus a necessary condition for positive innovation to be sustainable is  $((1 + \theta)/\theta)\delta h > \delta h + (1/(1 + \theta))\rho$ . Under this condition the graphical analysis shows that there exists an initial value for  $E/A$  such that the economy follows a path of perpetual growth. This path must be above the  $d(E/A)/dt = 0$  curve, approaching it from below as  $A$  increases. Thus consumption expenditure grows faster than knowledge and the ratio between the two approaches the long-run value, which implies that the rate of profit on new goods decreases as the measure of varieties produced increases. Accordingly the innovation rate declines monotonically towards its long-run value.

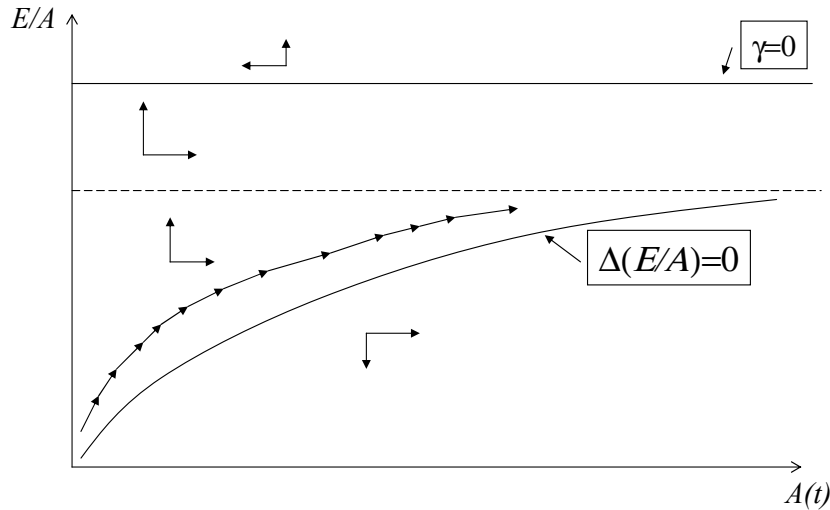


Figure 2. Dynamics of  $z = E/A$ .

### 3.4 Welfare

The economy contains potential departures of the market outcome from the socially optimal allocations. It is not hard to show, however, that the market equilibrium differs from the social optimum for the same reasons as in the economy without vintage effects. In particular, the static allocation of input across

vintages is optimal. On the other hand, the planner's allocation of inputs to research exceeds the equilibrium one, thus making research insufficient in equilibrium. But the wedge is determined by the standard intertemporal spillover effect. This can be seen by solving the planner's optimal control problem to find that the optimal growth rate exceeds the equilibrium growth rate by factor  $1 + \theta$ , just as in the symmetric version analyzed by Grossman and Helpman (1991). The details of the argument are in appendix A. Market prices and interest rate reflect the creative destruction behind obsolescence in a socially efficient way. Thus the same type of R&D subsidy would be a first best policy.<sup>8</sup>

#### 4 Policies

The virtue of the model developed thus far is that clarity is preserved while incorporating obsolescence and heterogeneity of firms or sectors. This is interesting because it permits the analysis of the growth effects of policies that may have a differential impact on different vintages. First, as pointed out in the Introduction, selective subsidies are observed in practice. Subsidies are an instrument that governments use to alleviate the loss of profits and employment in certain firms and sectors. In other cases, this type of interventions are deployed to prop up high-profit sectors. Second, business taxation often has a lump-sum component in the form of license fees and charges calculated on fixed coefficients. This form of fixed cost may induce certain firms to shut down.<sup>9</sup> Third, openness to international trade may induce a reallocation of production of certain goods away from the domestic economy. In the model, selective subsidies, lump-sum taxation, or trade will be shown to bring about growth effects by interfering the process of creative destruction.<sup>10</sup>

<sup>8</sup> Lai (1998) discusses at length welfare effects in a similar context. The present result suggests that the presence of new welfare effects is related to aspects of his model other than obsolescence.

<sup>9</sup> Policy distortions affecting firms' fixed costs would be equivalent to the lump-sum taxation considered here. Lai (1998) considers such fixed costs in a related model but stops short of analyzing its impact on the allocation of resources, which is the focus of the present paper.

<sup>10</sup> Throughout it will be assumed the government balances its budget using lump-sum non-distortionary taxes on or transfers to households as needed.

#### 4.1 Production subsidies

Assume a firm  $i$  is subsidized at the rate  $\eta(i, t)$ , which may change over time. Given that the demand for this good  $c(i, t)$  is given as in Eq.(6), profit maximization implies mark-up pricing on a variable cost,  $w(t)/[\phi(A(t(i)))(1 + \eta(i, t))]$ , which now includes the fact that a positive ad-valorem subsidy leads the firm to sell at a lower price. Assuming, as in Eq.(20), that  $\phi(A) = A$ , the analogous to Eq. (12) and (13) for consumption and profits are, respectively,

$$c(i, t) = \frac{E(t)}{A(t)} \frac{\theta}{1 + \theta} \frac{1}{\delta} \frac{(1 + \eta(i, t))^{1+\theta} A(t(i))^{1+\theta}}{\int^{n(t)} (1 + \eta(j, t))^\theta A(t(j))^\theta dj} \quad (27)$$

and

$$\pi(i, t) = E(t) \frac{1}{1 + \theta} \frac{(1 + \eta(i, t))^{1+\theta} A(t(i))^\theta}{\int^{n(t)} (1 + \eta(j, t))^\theta A(t(j))^\theta dj}. \quad (28)$$

It is apparent that the market position and thus the value of a firm will depend on the overall distribution of subsidy rates over vintages. Certainly, most often subsidies are not set for all the goods produced in the economy, but they are designed to protect specific sectors or groups of producers. The details of the analysis are in appendix B.

##### 4.1.1 Protection of declining sectors

For example, governments have used subsidies to help declining sectors. The target consists of sectors that have experienced a process of job destruction and reduction in profit rates. In terms of the model, the beneficiaries of such a policy would be the oldest vintages. Provided that this policy will certainly keep old firms from declining too fast, we want to assess the implications for the overall performance of the economy. To this end, assume that  $\eta(i, t) = \eta$  for  $i$  such that  $t(i) < t - T$ , and  $\eta(i, t) = 0$  otherwise. That is, all firms aged

$T$  and over benefit from the constant subsidy.<sup>11</sup> It is convenient to define

$$\Gamma^{s1}(A(t)) \equiv \frac{A(t)^{1+\theta}}{(1+\eta)^\theta [A(t-T)^{1+\theta} - A(0)^{1+\theta}] + [A(t)^{1+\theta} - A(t-T)^{1+\theta}]} \quad (29)$$

which, like  $\Gamma(\cdot, \cdot)$  defined in Eq.(21), reflects the competitiveness of the leading technology relative to the whole set of existing firms. In this case, the presence of  $\eta$  shows that older firms are now relatively more competitive thus reducing the edge of the state-of-the-art firms. I will focus on long-run balanced outcomes with constant innovation rate  $\gamma$  so the role of  $A_0$  can be safely ignored. The expressions for quantities and profits in Eq.(12) and (13) can now be written as

$$c(i, t) = \frac{E(t)}{A(t)} \theta \frac{1}{\delta} \left( \frac{A(t(i))}{A(t)} \right)^{1+\theta} (1 + \eta(i, t))^{1+\theta} \Gamma^{s1}(A(t)) \quad (30)$$

$$\pi(i, t) = \frac{E(t)}{A(t)} \left( \frac{A(t(i))}{A(t)} \right)^\theta (1 + \eta(i, t))^{1+\theta} \Gamma^{s1}(A(t)) \quad (31)$$

These equations can be used to develop the two main equilibrium equations. Similarly to the derivation of Eq. (19) or (24), the block of conditions (1), (2), (9), (15), (16), and (30) now leads up to a condition that reflects the market clearing allocation of the primary input to research

$$\gamma = \delta h - \frac{\theta}{1+\theta} \frac{E}{A} \frac{1 + e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta} - 1)}{1 + e^{-\gamma(1+\theta)T}(\eta^{1+\theta} - 1)} \quad (32)$$

The second term on the right-hand side describes the allocation of inputs to the production of consumption goods rather than research. The expression shows that the presence of the subsidy increases the demand of inputs for final-good production and is detrimental to research. This negative effect is smaller the older the recipient firms since those firms take up relatively fewer resources. Graphically, the negatively-sloped line in Figure 1 shifts downwards.

<sup>11</sup> The same exercise could thus be conducted in the Grossman and Helpman (1991) model without vintage effects. But this would require that subsidies can be targeted directly at firms of specific ages.

The second condition is related to the equilibrium returns to innovation activities. Again, just like for the derivation of Eq. (23) or (25), the set of conditions (8), (7), (14) and, now, (31) lead to the following:

$$\gamma = \frac{E}{A} \frac{1 + e^{-\rho T} e^{-\gamma(1+\theta)T} ((1+\eta)^{1+\theta} - 1)}{1 + e^{-\gamma(1+\theta)T} ((1+\eta)^\theta - 1)} - \theta\gamma - \rho \quad (33)$$

The term accompanying  $E/A$  picks out the role of the subsidy for the returns to innovation. The net sign of this effect is ambiguous. On one hand, the subsidy increases the present value of the firm and thus the return to creating a new product. However, the subsidy also increases the competitiveness of the rest of firms which tends to erode the flow of profits. The direct benefits occur in the future and are discounted, thus when discount is high or the period to qualify for the subsidy,  $T$ , is long then the subsidy scheme is more likely to be detrimental for the return to investment. In terms of Figure 1, the positively sloped curve may shift either way.

In general equilibrium, nonetheless, the net growth effect of these subsidies is unambiguously negative. Combining the two above conditions Eq.(32) and (33) yields an analytical expression for the determination of the innovation rate  $\gamma$  in equilibrium:

$$\frac{1 + e^{-\rho T} e^{-\gamma(1+\theta)T} ((1+\eta)^{1+\theta} - 1)}{1 + e^{-\gamma(1+\theta)T} ((1+\eta)^{1+\theta} - 1)} = \frac{(1+\theta)\gamma + \rho}{\delta h - \gamma} \frac{\theta}{1+\theta}.$$

This expression shows that for positive and finite values of  $T$  the long-run rate of innovation is smaller with a subsidy  $\eta > 0$  than when there is no subsidy or  $\eta = 0$ . It also shows that the rate of innovation declines with the subsidy rate  $\eta$  in a stable steady state.<sup>12</sup>

The forces at work are as follows. On one hand, the value of firms becomes higher because the subsidy increases profits from age  $T$  on. On the other hand, since subsidized firms increase their demand for inputs, there are fewer

<sup>12</sup> More precisely, this holds if the term on the left-hand side is a flatter function of  $\gamma$  than the term on the right-hand side. This is certainly the case in the unique and stable equilibrium corresponding to an economy without subsidies. With a positive subsidy rate one can construct examples with two steady states, including one with the reverse relative slopes of the terms in the equation. My best guess is that such a steady state must be unstable.

resources left for research. It turns out that the effect on the value of the firms is small relative to the impact on the demand for the labor input. The reason is that the former is discounted since it occurs far in the future. It is important to stress is that it is not the presence of subsidies per se that reduces the allocation of resources to growth-generating activities, but its distribution across firms of different vintages. If all goods were subsidized (i.e.  $T = 0$ ) then the subsidy rate  $\eta$  would not have any bearing on equilibrium innovation.

For a given subsidy rate  $\eta$ , the growth effect of restricting the subsidy to older firms (i.e., a rise in  $T$ ) is generally ambiguous. From the above expression it is clear that as  $T \rightarrow \infty$  the innovation rate  $\gamma$  approaches the value corresponding to the zero-subsidy situation with  $T = 0$ . More specifically,  $\gamma$  first declines with  $T$  and eventually must increase. The intuition for the negative growth effect has already been discussed in the previous paragraph. The intuition for the positive growth effect rests on the fact that when the subsidy shifts to a narrower set of older firms there is less of a diversion of inputs away from research.

#### 4.1.2 Support to high-profit sectors

The opposite case that subsidies are targeted at the most profitable firms is considered in this section. Such a scheme creates a wider advantage of innovators over the average existing firm. This produces a larger incentive for undertaking research. To make this case, assume that at  $t$  there is a positive subsidy rate  $\eta$  only for firms born after  $t - T$ . Assume that  $\eta(i, t) = \eta$  for  $i$  such that  $t(i) > t - T$ , and  $\eta(i, t) = 0$  otherwise.

It is useful to define

$$\Gamma^{s2}(A_t) \equiv \frac{A(t)^{1+\theta}}{A(t-T)^{1+\theta} - A(0)^{1+\theta} + (1+\eta)^\theta [A(t)^{1+\theta} - A(t-T)^{1+\theta}]} \quad (34)$$

Again, this is reflective of the relative competitive position of firms. The qualitative effect of the subsidy is as in  $\Gamma^{s1}$  as defined in Eq.(29) above, but here this subsidy improves the relative competitiveness of younger rather than older vintages. The equilibrium expressions for quantities and profits in Eq. (30) and (31) also apply provided that  $\Gamma^{s1}$  is replaced by  $\Gamma^{s2}$ . Proceeding as in the pre-

vious cases, the market-clearing related condition leads to the expression

$$\gamma = \delta h - \frac{\theta}{1 + \theta} \frac{E}{A} \frac{(1 + \eta)^{1+\theta} (1 - e^{-\gamma(1+\theta)T}) + e^{-\gamma(1+\theta)t}}{(1 + \eta)^\theta (1 - e^{-\gamma(1+\theta)T}) + e^{-\gamma(1+\theta)t}} \quad (35)$$

which describes the incentives governing the allocation of inputs to research and final-goods production. As before, for given  $E/A$ , the subsidy leads to less resources to research. Graphically, the positive sloped curve in Figure 1 shifts downwards. On the other hand, the study of the return to innovations produces the no-arbitrage equilibrium equation:

$$\gamma = \frac{E}{A} \frac{(1 + \eta)^{1+\theta} (1 - e^{-\rho T} e^{-\gamma(1+\theta)T}) + e^{-\rho T} e^{-\gamma(1+\theta)T}}{(1 + \eta)^\theta (1 - e^{-\gamma(1+\theta)T}) + e^{-\gamma(1+\theta)T}} - \theta \gamma - \rho \quad (36)$$

The effect of  $\eta$  here is unambiguously positive. The innovator enjoys the subsidy edge right from the start of its activity, and the drop in profit due to the future withdrawal of the subsidy is discounted. Graphically, the negatively-sloped curve shifts upwards in Figure 1.

In general equilibrium, the direct impact on the profitability of research dominates so that the subsidy increases the rate of innovation. This can be seen explicitly by combining Eq.(35) and (36) to obtain

$$\frac{(1 + \eta)^{1+\theta} - e^{-\rho T} e^{-\gamma(1+\theta)T} ((1 + \eta)^{1+\theta} - 1)}{(1 + \eta)^{1+\theta} - e^{-\gamma(1+\theta)T} ((1 + \eta)^{1+\theta} - 1)} = \frac{(1 + \theta)\gamma + \rho}{\delta h - \gamma} \frac{\theta}{1 + \theta}.$$

This expression shows that for a positive and finite  $T$  the long-run rate of innovation increases with the subsidy in a stable steady state (see footnote 12 above). Again, a subsidy that applies to everyone ( $T \rightarrow \infty$ ) has no growth effect, showing the importance of the selective character of this policy.

For a given subsidy rate  $\eta$ , the growth effect of an extension of the subsidy to older firms (i.e., a rise in  $T$ ) is ambiguous. From the expression above it is clear that as  $T \rightarrow \infty$  the innovation rate approaches the value corresponding to the zero-subsidy situation with  $T = 0$ . More specifically,  $\gamma$  increases initially and must eventually decline. The reason for a positive growth effect has already been discussed in a preceding paragraph. The intuition for the negative effect of  $T$  on  $\gamma$  is that when the subsidy applies to a wider range of firms there is

a higher diversion of inputs away from research, which counteracts the effect from the rise in the return to innovating.<sup>13</sup>

#### 4.2 Lump-sum taxes

Here I assume there is a fixed tax  $f$  collected on firms producing consumption goods. Since a firm's gross profit declines over time, this fixed tax per period will imply that current net profits for some firms will become negative. At this point, any such a firm will shut down and stop operating. The firms that will drop out of the produced set will be those aged  $T$  and above for some  $T$  that has to be determined. More formally, Eq.(11) still characterizes the pricing rule of a firm  $i$ . Provided that the range of varieties is now finite, an analysis similar to that in section 4.1 delivers the following equations for gross profit rates and quantities:

$$c(i, t) = \frac{E(t)}{A(t)} \frac{\theta}{\delta} \left( \frac{A(t(i))}{A_t} \right)^{1+\theta} \Gamma^f(A(t), A(t-T)) \quad (37)$$

$$\pi(i, t) = \frac{E(t)}{A(t)} \left( \frac{A(t(i))}{A(t)} \right)^\theta \Gamma^f(A(t), A(t-T)) \quad (38)$$

<sup>13</sup> A more realistic setting would recognize the presence of firm-specific learning-by-doing (LBD). However the basic insight from the subsidy experiments just described should stand up to such an extension. Although the typical innovator would see its profits increase initially, as long as LBD is bounded, profits will eventually start declining, much like the firms of this paper. The crucial point here is that, on average, the most productive firms will still belong in the subset of youngest firms (although at a finer level the relation age-profits may not be monotonic due to LBD). Consider now a subsidy targeted at a set of high-profit firms. The profitability threshold can always be chosen to include all the firms under a certain age. I believe the mechanism analyzed in the paper should carry over to this situation to deliver the same type of results. It is true nonetheless that a subsidy to a narrower subset of high-profit firms might exclude a range of the newest firms which have not yet realized large enough gains from learning. I guess the results for this case should be analogous to those following a subsidy targeted at an intermediate range of firms in the paper's model. This experiment would thus have aspects of subsidies to both young and old firms, and the result should largely depend on the specific range of firms selected.

with

$$\Gamma^f(A(t), A(t-T)) \equiv \frac{A(t)^{1+\theta}}{A(t)^{1+\theta} - A(t-T)^{1+\theta}} = \frac{1}{1 - e^{-\gamma(1+\theta)T}}, \quad (39)$$

where the second equality follows from the constant-growth assumption.

Now one can calculate the age  $T$  at which a firm will be discontinued. To that end, use Eq.(38) to set

$$\pi(i(t), t+T) = \frac{E(t+T)}{A(t+T)} \left( \frac{A(t)}{A(t+T)} \right)^\theta \Gamma^f(A(t+T), A(t)) - f = 0,$$

which, on a balanced-growth path with  $A(t+T) = \exp(\gamma T)A(t)$  and constant  $E/A$ , delivers the relation between  $f$  and  $T$ , given  $\gamma$  and  $E/A$ ,

$$\frac{E}{A} e^{-\gamma\theta T} \frac{1}{1 - e^{-\gamma(1+\theta)T}} = f. \quad (40)$$

One can show that the equation related to market clearing can be found by rearranging (1), (2), (15), (16) and (37) to yield the same Eq.(24) as in the model without policies.

The condition related to the return from innovations will be affected though. Using Eq.(7) with the normalization  $V[i(t)]_t = 1$  and Eq.(38), one can show that

$$r = \frac{E(t)}{A(t)} \Gamma^f(A(t), A(t-T)) - \theta\gamma - \theta\gamma \frac{1}{r} f (1 - e^{-rT}). \quad (41)$$

Comparing this to Eq.(18) with (20) and (22) reveals a number of effects of the tax  $f$  on the equilibrium return to innovations. The term  $\Gamma^f(., .)$  indicates that a new firm will be more profitable since the range of competitors is narrowed. The last term accounts for the negative effects of the tax on the profits firms can produce. With Eq.(8) and (39), the above expression leads to another equilibrium equation:

$$\gamma = \frac{E(t)}{A(t)} \frac{1}{1 - e^{-\gamma(1+\theta)T}} - \theta\gamma - \theta\gamma \frac{1}{r} f (1 - e^{-(\gamma+\rho)T}) - \rho \quad (42)$$

Given the tax  $f$ , the balanced-growth equilibrium consists of values for  $T$ ,  $\gamma$  and  $E/A$  that satisfy Eq. (40), (24) and (42). As shown in appendix C, with a positive lump-sum tax, the innovation rate will be higher than without the tax if and only if

$$\theta \frac{\gamma}{\gamma + \rho} (e^{\gamma T} - e^{-\rho T}) < 1. \quad (43)$$

For  $T$  small enough,  $\gamma$  is higher than without  $f$ . Using the shut-down condition Eq.(40), for every such a  $T$ , and the corresponding  $\gamma$  and  $E/A$ , a value of the tax  $f$  can be found that generates this equilibrium. It can then be concluded that there exists a value  $f$  such that growth increases with a lump-sum tax. For any such tax level, innovation increases as competitors are driven out of the market and release resources for research. This effect dominates the negative effect of lower profits and a shorter productive life on the returns to an innovator.

This result stands in stark contrast with the one implied by the model without obsolescence and symmetric goods of Grossman and Helpman (1991). In that case there is no purge of competitors so that a lump-sum tax is unambiguously detrimental for growth through its negative effect on profit rates.

### 4.3 Trade and openness

This section considers the case that the economy engages in trade with a foreign country that has a lower level of technological development. Formally, denote by  $\tilde{A}(t)$  the level of knowledge reached by this foreign partner at time  $t$ . The assumption is that  $A(t)/\tilde{A}(t) > 1$ . Since the focus is on balanced-growth situations, I will assume that this gap is constant over time so both countries are growing at the same rate. Therefore, it takes a constant period of length  $T$  for the foreign country to catch up with the domestic country's technology, so that  $\tilde{A}(t + T) = A(t)$  all  $t$ . This implies that  $A(t)/\tilde{A}(t) = \exp(\gamma T)$ . I will further assume that the wage in the foreign economy,  $\tilde{w}$ , is lower than that in the domestic economy  $w$ , and that the gap is such that all goods that are technologically feasible in the foreign country [i.e.  $i \leq \tilde{A}$ ] will be produced there. In other words, over time old varieties formerly produced domestically will eventually be produced in the foreign country. This scenario thus captures the features of the international product cycle studied by Vernon (1966) and Stokey (1991). Firms in both countries operate under conditions

of monopolistic competition, so the pricing rule for goods will now be similar to Eq.(11) with (20):

$$p(i, t) = \frac{1 + \theta}{\theta} \frac{1}{A(t(i))} \times \begin{cases} w(t) & i > \tilde{A}(t) \\ \tilde{w}(t) & i \leq \tilde{A}(t) \end{cases}$$

Define the term that represents the relative competitiveness of an innovator in the domestic country as follows:

$$\begin{aligned} \Gamma^o(A(t), \tilde{A}(t)) &\equiv \frac{A(t)^{1+\theta}}{\left(\frac{w(t)}{\tilde{w}(t)}\right)^\theta (\tilde{A}(t)^{1+\theta} - A(0)^{1+\theta}) + (A(t)^{1+\theta} - \tilde{A}(t)^{1+\theta})} \\ &= \frac{e^{\gamma(1+\theta)T}}{\left(\frac{w(t)}{\tilde{w}(t)}\right)^\theta + e^{\gamma(1+\theta)T} - 1} < 1, \end{aligned}$$

where the equality follows from the assumptions made about the technological gap, and the inequality is from the assumption  $\tilde{w} < w$ . Observe that for the closed economy analyzed in section 3, the balanced-growth value of  $\Gamma$  in Eq. (21) is unity. This indicates that a domestic innovator in the open economy faces fiercer competition from low-wage producers located in the foreign country. On its own, that would tend to have a detrimental effect on the incentives to conduct research. The net effect depends on a larger number of interactions. To work out these effects, start again with the condition related to clearing in the domestic market for the primary input. Appendix D shows that this leads to

$$\begin{aligned} \gamma &= \delta h - \frac{\theta}{1 + \theta} \frac{E}{A} \Gamma^o(A(t), \tilde{A}(t)) \left[ 1 - \left( \frac{\tilde{A}(t)}{A(t)} \right)^{1+\theta} \right] \\ &= \delta h - \frac{\theta}{1 + \theta} \frac{E}{A} \Gamma^o(A(t), \tilde{A}(t)) (1 - e^{-\gamma(1+\theta)T}) \end{aligned} \quad (44)$$

Comparison with Eq.(24) corresponding to the closed economy indicates that in the open economy foreign competition in the production of final goods frees up resources for research. In terms of Figure 1, the curve with negative slope shifts upwards with trade.

The second equilibrium condition that relates to the market value of innovation

for the open economy reads

$$\gamma = \frac{E}{A} \Gamma^o(A(t), \tilde{A}(t))(1 - e^{-(r+\gamma\theta)T}) - \theta\gamma - \rho. \quad (45)$$

This differs from Eq.(25) that corresponds to the closed economy. As mentioned earlier, trade with a low-wage country reduces current profits as well as the lifetime span of an innovating firm in the high-wage domestic economy. Graphically, the curve with positive slope in Figure 1 will shift downwards with openness. Combining the two last equations, the net effect of trade can be determined. It is convenient to define

$$\varphi(T, \gamma) \equiv \frac{1 - e^{-\gamma(1+\theta)T}}{1 - e^{-(\rho+(1+\theta)\gamma)T}}.$$

Then the equilibrium growth rate must satisfy

$$\gamma = \frac{1}{1 + \theta\varphi(T, \gamma)} \left[ \delta h - \frac{\theta}{1 + \theta} \rho\varphi(T, \gamma) \right],$$

which is to be compared with the closed-economy expression in Eq.(26). Since  $\varphi(T, \gamma) < 1$ , growth for the open economy is higher. This outcome is driven by the diversion of resources from final-goods production into research that occurs when foreign competition intensifies the process of economic obsolescence of existing domestic technologies.

## 5 Conclusions

This paper studies a model of growth where technological change has an embodied component that makes new vintages of firms more productive. The phenomenon of obsolescence is a feature of the growth process whereby new sectors or firms replace existing ones. The transitional and long-run properties of the economy are investigated. The analysis identifies policies that have effects on growth through their influence on the course of the *creative destruction* associated with the ongoing process of emergence and obsolescence of firms.

In the model, subsidies targeted at more profitable firms will have a positive growth effect. On the contrary, subsidies aimed at older low-profit vintages will be detrimental for growth. Changes in the range of recipient firms may

have a non-monotonic effect on the growth rate though. These growth effects arise entirely from the selective nature of the policies. A uniform non-selective subsidy would have no growth effect. Similarly, a lump-sum tax on firms in the economy that forces the old less-profitable firms to shut down may have a positive effect on growth. It is interesting that, in the absence of obsolescence, a lump-sum tax of the type analyzed here would reduce the growth rate unambiguously. The degree of openness to international trade of a more developed economy with a less developed partner will also be growth-enhancing for similar reasons. Foreign competition moves the production with old technologies to the foreign country and releases resources for innovation at home.

The main contribution of this paper is to demonstrate in a minimal model that, once obsolescence is accounted for, new and potentially important growth-effects of policies show up. The study of these effects necessarily demands a model that, like the one proposed in this paper, accommodates heterogeneous firms or sectors. The model used in this paper is very stylized and is thus bound to have limitations. Further research should extend the policy analysis to models where the vintage effects have deeper microeconomic foundations and which can thus be used for quantitative analysis.

## 6 Colophon

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## A Welfare analysis

The planner's static problem consists of maximizing  $\int_0^n c(j)^{\theta/(1+\theta)} dj$  subject to the constraint  $h_c = \int_0^n (c(j)/A(t(j))) dj$ , where  $h_c$  denotes the amount of primary input available in the consumption-good sector. The solution to this problem delivers the

optimal allocation of this input across existing sectors at a time  $t$ :

$$c(i, t) = h_c \frac{A(t(i))^{1+\theta}}{\frac{1}{1+\theta}(A(t)^{1+\theta} - A(0)^{1+\theta})}.$$

In the market equilibrium the same condition holds provided that there  $h_c = (\theta/(1+\theta))(h + (1/(1+\theta))(\rho/\delta))$ . This allows one to calculate the indirect utility as

$$\left[ \int_0^{n_t} c(i, t)^{\theta/(1+\theta)} di \right]^{(1+\theta)/\theta} = h_c A(t)^{(1+\theta)/\theta} \left( \frac{1}{1+\theta} \right)^{1/\theta}.$$

For the sake of comparison, in the symmetric case the exponent on  $A$  is a smaller  $1/\theta$ .

The planner's dynamic problem is the choice of the paths for  $A$  and  $h_c$  that maximize the representative household's utility. The Hamiltonian associated with this problem is  $H = e^{-\rho t} \log((1 - h_r)A^{(1+\theta)/\theta}) + \mu(\delta h_r A)$ . The solution on a BGP leads to the optimal  $h_r^* = h - (\theta/(1+\theta))(\rho/\delta)$ . The resulting optimal growth rate is  $\gamma^* = \delta(h - (\theta/(1+\theta))(\rho/\delta))$ . Comparing with the market growth rate  $\gamma$  in Eq.(26) shows that  $\gamma^*/\gamma = 1+\theta$ . The same holds for the symmetric economy analyzed by Grossman and Helpman (1991).

## B Subsidies

*B.1 Eq. (27) and (28):* If  $i$  is subsidized at the rate  $\eta(i)$  then profits are

$$\pi(i, t) = c(i, t) \left[ p(i, t)(1 + \eta(i, t)) - \frac{w(t)}{\phi(A(t(i)))} \right]$$

Eq.(6) still holds for the demand of each intermediate. Then profits maximization implies mark-up pricing

$$p(i, t) = \frac{1+\theta}{\theta} \frac{1}{1+\eta(i, t)} \frac{w(t)}{\phi(A(t(i)))}$$

Assuming, as in Eq.(20), that  $\phi(A) = A$ , the quantity produced and sold by the firm is

$$c(i, t) = E(t) \frac{\theta}{1+\theta} \frac{1}{w(t)} \frac{(1+\eta(i, t))^{1+\theta} A(t(i))^{1+\theta}}{\int^{n(t)} (1+\eta(j, t))^\theta A(t(j))^\theta dj}.$$

Using Eq.(17), Eq.(27) obtains. With these equations, the profit rate in Eq.(12) can be written as in Eq.(28).

*B.2 Eq.(30) and (31):* For section 4.1.1 on the the protection of declining sectors, it is convenient to define the limit of  $\Gamma^{s1}(A(t))$  as  $t$  increases as  $\Gamma_{\infty}^{s1} \equiv [(1 + \eta)^{\theta} (e^{-\gamma(1+\theta)T} - e^{-\gamma(1+\theta)t}) + 1 - e^{-\gamma(1+\theta)T}]^{-1}$ . From Eq.(27) one can derive Eq.(30):

$$\begin{aligned} c(i, t) &= E(t)\theta \frac{1}{w(t)} \frac{(1 + \eta(i, t))^{1+\theta} A(t(i))^{1+\theta}}{(1 + \eta(i, t))^{\theta} [A(t - T)^{1+\theta} - A(0)^{1+\theta}] + [A(t)^{1+\theta} - A(t - T)^{1+\theta}]} \\ &= E(t)\theta \frac{1}{w(t)} \left( \frac{A(t(i))}{A(t)} \right)^{1+\theta} (1 + \eta(i, t))^{1+\theta} \Gamma_{\infty}^{s1} \end{aligned}$$

where the last equality follows from assuming constant growth outcomes. Using Eq.(28), one can then recover the expression for profits Eq.(31) as follows:

$$\begin{aligned} \pi(i, t) &= \frac{1}{\theta} \frac{w(t)}{A(t(i))} c(i, t) \\ &= E(t) \frac{(1 + \eta(i, t))^{1+\theta} A(t(i))^{\theta}}{(1 + \eta(i, t))^{\theta} [A(t - T)^{1+\theta} - A(0)^{1+\theta}] + [A(t)^{1+\theta} - A(t - T)^{1+\theta}]} \\ &= \frac{E(t)}{A(t)} \left( \frac{A(t(i))}{A(t)} \right)^{\theta} (1 + \eta(i, t))^{1+\theta} \Gamma_{\infty}^{s1} \end{aligned}$$

where the last equality follows from the constant-growth assumption.

*B.3 Market clearing Eq.(32):* With Eq.(30), the block of equilibrium equations (1), (2), (15), (16), and (9) can now be developed. First, with Eq.(30) and (16) the demand of labor by final-good firms can be written as

$$\begin{aligned} \int^{n(t)} h(i, t) di &= E(t)\theta \frac{1}{w(t)} \Gamma_{\infty}^{s1} \left( \frac{1}{A(t)} \right)^{1+\theta} \\ &\quad \times \left\{ \int_{t-T}^{t-T} A(\tau)^{\theta} (1 + \eta)^{1+\theta} \dot{A}(\tau) d\tau + \int_{t-T}^t A(\tau)^{\theta} \dot{A}(\tau) d\tau \right\} \\ &= E(t)\theta \frac{1}{w(t)} \Gamma_{\infty}^{s1} \left( \frac{1}{A(t)} \right)^{1+\theta} \frac{1}{1 + \theta} \\ &\quad \times \left\{ (1 + \eta)^{1+\theta} [A(t - T)^{1+\theta} - A(0)^{1+\theta}] + [A(t)^{1+\theta} - A(t - T)^{1+\theta}] \right\} \end{aligned}$$

Using that  $w = \delta A$  in Eq.(9) with the normalization  $p_A(t) = 1$ , and the R&D technology in Eq.(1) and (2), market clearing Eq.(15) can be posed as

$$h = \gamma \frac{1}{\delta} + \frac{\theta}{1 + \theta} \frac{1}{\delta} \frac{E(t)}{A(t)} \Gamma_{\infty}^{s1} \times \left[ (1 + \eta)^{1+\theta} \left( e^{-\gamma(1+\theta)T} - e^{-\gamma(1+\theta)t} \right) + 1 - e^{-\gamma(1+\theta)T} \right]$$

This can be rearranged as an expression that leads to Eq.(32).

*B.4 No-arbitrage Eq.(33):* With Eq.(31), the block of equations (7), (8) with (14) can now be developed. Let  $V(\tau, t)$  denote the present (fundamental) value of a firm in cohort  $\tau$  at time  $t$ . I find that

$$\frac{dV(t, t)}{dt} = \dot{V}(t, t) + \gamma V(t, t) + e^{-\int_t^{t+T} r_u du} \pi(t, t+T)^- - e^{-\int_t^{t+T} r_u du} \pi(t, t+T)^+,$$

the dot notation indicating derivative in the 2nd argument, and with the profits of the marginal firms aged around vintage  $T$  related by

$$\pi(t, t+T)^+ = (1 + \eta)^{1+\theta} \pi(t, t+T)^-,$$

and, by Eq.(31),

$$\pi(t, t+T)^- = \frac{E(t+T)}{A(t+T)} \left( \frac{A(t)}{A(t+T)} \right)^{\theta} \Gamma^{s1}(A(t+T)).$$

I use the normalization  $p_A(t) = 1$  and Eq.(14) so  $V(t, t) = 1$ . In a BGP with constant  $r$  and  $\gamma$

$$\begin{aligned} \frac{dV(t, t)}{dt} &= \dot{V}(t, t) + \theta\gamma + e^{-rT} (\pi(t, t+T)^- - \pi(t, t+T)^+) = \\ &= \dot{V}(t, t) + \theta\gamma \\ &\quad + e^{-rT} \frac{E(t+T)}{A(t+T)} \left( \frac{A(t)}{A(t+T)} \right)^{\theta} \Gamma^{s1}(A(t+T)) (1 - (1 + \eta)^{1+\theta}) = 0 \end{aligned}$$

With the no-arbitrage condition Eq.(7),  $r = \pi(t, t) + \dot{V}(t, t)$ , and that, by Eq.(31),  $\pi(t, t) = \frac{E(t)}{A(t)} \Gamma^{s1}(A(t))$ , one can write

$$r = \frac{E(t)}{A(t)} \Gamma^{s1}(A(t)) - \theta\gamma + e^{-rT} \frac{E(t+T)}{A(t+T)} \left( \frac{A(t)}{A(t+T)} \right)^{\theta} \Gamma^{s1}(A(t+T)) ((1 + \eta)^{1+\theta} - 1)$$

Using that, by Eq.(8),  $r = \gamma + \rho$  on a BGP, Eq.(33) follows.

*B.5 Market clearing Eq.(35):* For section 4.1.2 on support to high-profit sectors it is useful to define the limit of  $\Gamma^{s2}(A_t)$  as  $t$  increases as  $\Gamma_\infty^{s2} \equiv [e^{-\gamma(1+\theta)T} + (1+\eta)^\theta(1-e^{-\gamma(1+\theta)T})]^{-1}$ . Provided that, after replacing  $\Gamma^{s1}$  by  $\Gamma^{s2}$ ,  $c(i, t)$  is still given by Eq.(30), the set of equations (1), (2), (15), (16) and (9) delivers the market-clearing condition  $h = \gamma \frac{1}{\delta} + \frac{\theta}{1+\theta} \frac{1}{\delta} \frac{E(t)}{A(t)} \Gamma^{s2}(A(t)) \times [e^{-\gamma(1+\theta)T} + (1+\eta)^{1+\theta}(1-e^{-\gamma(1+\theta)T})]$ , which can be written more conveniently as an expression that leads to Eq.(35).

*B.6 No arbitrage Eq.(36):* With the normalization  $p_A(t) = 1$  and using the appropriate version of Eq.(31) for profits, I follow steps similar to those involved in the derivation of Eq.(33) in Section B.4. First, I obtain

$$\frac{dV(t, t)}{dt} = \dot{V}(t, t) + \theta\gamma + e^{-rT} \frac{E}{A} \left( \frac{A(t)}{A(t+T)} \right)^\theta \Gamma_\infty^{s2} ((1+\eta)^{1+\theta} - 1) = 0.$$

Provided that, by Eq.(31),  $\pi(t, t) = (E/A) \Gamma_\infty^{s2} (1+\eta)^{1+\theta}$ , the no-arbitrage condition  $r = \pi(t, t) + \dot{V}(t, t)$  from Eq.(7) implies that

$$r = (E/A) \Gamma_\infty^{s2} \left[ (1+\eta)^{1+\theta} - e^{-rT} e^{-\gamma\theta T} ((1+\eta)^{1+\theta} - 1) \right] - \theta\gamma.$$

Using that, by Eq.(8), on a BGP  $r = \gamma + \rho$ , one arrives at a expression that delivers Eq.(36).

## C Lump-sum taxation

*C.1 Eq.(41) and (42):* Differentiate the fundamental value of the firm to obtain

$$\begin{aligned} \frac{dV(i(t), t)}{dt} &= \dot{V}(i(t), t) + \theta\gamma \int_t^{t+T} e^{-r(s-t)} [\pi(i(t), s) + f] ds \\ &= \dot{V}(i(t), t) + \theta\gamma V(i(t), t) + \theta\gamma f \frac{1}{r} (1 - e^{-rT}) \end{aligned}$$

where use of Eq.(38) has been made in the second equality. Using Eq.(7) and the normalization  $V(i(t), t) = 1$  constant, Eq.(41) follows.

Eq.(42) follows from Eq.(41) with (7) and (39) when  $t \rightarrow \infty$ :

$$\begin{aligned}\gamma &= \frac{E(t)}{A(t)} \Gamma^f(A(t), A(t-T)) - \theta\gamma - \theta\gamma \frac{1}{r} f \left(1 - e^{-(\gamma+\rho)T}\right) - \rho \\ &= \frac{E(t)}{A(t)} \frac{1}{1 - e^{-\gamma(1+\theta)T}} - \theta\gamma - \theta\gamma \frac{1}{r} f \left(1 - e^{-(\gamma+\rho)T}\right) - \rho\end{aligned}$$

*C.2 Eq.(43):* Combine the market-clearing and the valuation conditions, Eq.(24) and (42), to substitute  $E/A$  out and obtain

$$\frac{(1+\theta)\gamma + \rho}{(\delta h - \gamma) \frac{1+\theta}{\theta}} = \frac{1}{1 - e^{-\gamma(1+\theta)T}} - \frac{\theta\gamma \frac{1}{r} f(1 - e^{-(\gamma+\rho)T})}{(\delta h - \gamma) \frac{1+\theta}{\theta}}.$$

On the other hand, the shut-down condition (40) with the market-clearing condition (24) can be combined as

$$\frac{f}{(\delta h - \gamma) \frac{1+\theta}{\theta}} = \frac{e^{-\gamma\theta T}}{1 - e^{-\gamma(1+\theta)T}}.$$

The two last expressions allow one to write an equation in  $\gamma$  and  $T$  as follows:

$$\frac{(1+\theta)\gamma + \rho}{(\delta h - \gamma) \frac{1+\theta}{\theta}} = \frac{1}{1 - e^{-\gamma(1+\theta)T}} \left[ 1 - \theta \frac{\gamma}{\gamma + \rho} e^{-\gamma\theta T} (1 - e^{-(\gamma+\rho)T}) \right].$$

In the case without the tax  $f$ , the equilibrium  $\gamma$  is characterized by this equation when the right-hand side is equal to 1 (i.e.,  $T \rightarrow \infty$ ). Therefore, under the lump-sum tax, the innovation rate will be higher if and only if

$$\theta \frac{\gamma}{\gamma + \rho} e^{-\gamma\theta T} (1 - e^{-(\gamma+\rho)T}) < e^{-\gamma(1+\theta)T}$$

which, after rearranging, reads as Eq.(43). The left side of Eq.(43) is an increasing function of  $T$ , and for  $T$  small enough it must hold.

## D Trade

*D.1 Market clearing Eq. (44):* Use the expression for  $p(i, t)$  in section 4.3 into Eq. (6) to derive the demand for each variety as:

$$c(i, t) = \theta \frac{E(t)}{w(t)} \left( \frac{A(t(i))}{A(t)} \right)^{1+\theta} \Gamma^o(A(t), \tilde{A}(t)).$$

The block of equations Eq.(1), (2), (9), (15), and (16) can now be developed as follows. Find the individual demand for labor from Eq. (16):

$$h(i, t) = \frac{c(i, t)}{A(t(i))} = \theta \frac{E(t)}{w(t)} \left( \frac{1}{A(t)} \right)^{1+\theta} \Gamma^o(A(t), \tilde{A}(t)) A(t(i))^\theta.$$

The market-clearing condition in Eq. (15) can be written with Eq. (9) as

$$\int_{\tilde{n}(t)}^{n(t)} h(i, t) di = \frac{\theta}{1+\theta} \frac{1}{\delta} \frac{E(t)}{A(t)} \Gamma^o(A(t), \tilde{A}(t)) \left[ 1 - \left( \frac{\tilde{A}(t)}{A(t)} \right)^{1+\theta} \right],$$

which leads to Eq. (44).

*D.2 No arbitrage Eq. (45):* Use Eq. (10) with the expression for  $p(i, t)$  in section 4.3 to obtain a firm's profit as

$$\pi(i, t) = \frac{E(t)}{A(t)} \left( \frac{A(t(i))}{A(t)} \right)^\theta \Gamma^o(A(t), \tilde{A}(t)).$$

The block of equations Eq. (7), (8), and (14) can now be developed. Following calculations analogous to those in footnote 4 (or in appendix B.4), Eq. (7) can be written as

$$r = \pi(i(t), t) - \theta\gamma - e^{-\int_t^{t+T} r(s) ds} \pi(i(t), t+T) = \frac{E(t)}{A(t)} \Gamma^o(A(t), \tilde{A}(t)) \left[ 1 - e^{-(r+\gamma\theta)T} \right] - \theta\gamma,$$

where the second equality follows under balanced growth. Substitution of this into Eq. (8) on a balanced-growth path (i.e.,  $\gamma = r - \rho$ ), leads to Eq. (45).

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