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The Demographic Transition in Europe: A Neoclassical Dynastic Approach[1]

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This paper investigates the factors that have shaped the demographic transition in a number of European countries (Sweden, England, and France) since the mid 18th century. The analytical framework is a version of the neoclassical growth model with dynastic preferences calibrated to match the Swedish experience. This model is used to study the contribution of various factors to the explanation of the observed demographic patterns, both over time and across countries. The factors considered are mortality changes, technological progress, and the evolution of the cost of children. The analysis suggests that the contribution of observed mortality rates is limited. A substantial part of the demographic-transition facts must be attributed to variation in the cost of children and/or technological change, both over time and across countries. *Journal of Economic Literature* Classification Numbers: J1, O0.

Key Words: demographic transition; mortality; fertility; neoclassical growth model; dynastic preferences; altruism.

1. INTRODUCTION

The natural rate of increase in European populations accelerated markedly between 1750 and 1850, with growth rates peaking during the second half of the 19th century. The subsequent period is characterized

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by manifest declines in reproduction rates, leading to the more modest levels recorded in recent times. This broad picture conceals a diversity of national experiences though. In effect, the specialized literature—e.g., Anderson (1996, pp. 217–224), Chesnais (1992, p. 223), or Woods (1996, pp. 305–307)—has identified distinct regional models for the timing and scale of the changes undergone up to the late 19th century.

The aim of this paper is to investigate the role of different factors in shaping the demographic transformations under way since the mid/late 18th century in European countries. The focus is on three such factors: survival (or mortality) rates, technology growth, and the costs of rearing children. The specific objective is to assess how variation in these factors can account for the diverse features of the demographic transition in Sweden, England, and France.

The analysis is conducted within the framework of a competitive neoclassical economy where—as in Barro and Becker (1989)—dynastic altruism motivates capital accumulation and fertility choices. The approach is quantitative and the conclusions are judged by their ability to match the observed paths of selected economic and demographic variables. A benchmark parametric setting is chosen such that the model's equilibrium is consistent with long-run features of the "typical" Western economy in its pre-transition and post-transition steady states. Then the paths of survival rates, technology growth, and the child-cost parameter are calibrated so that the transition between these two steady states produces the observed paths of the Swedish population natural rate of increase and survival rates and a path of productivity growth that traces the Swedish one as closely as possible. The experiments conducted modify this baseline setup in various ways. First, direct observations on the historical paths of survival rates in England and France are used. Second, for these countries, paths of the child-cost and technology-growth parameters are found that match the model to the observed path of population growth. One can then study how the diversity in the paths of population and productivity growth rates across countries can be traced to differences in the behavior of mortality, technology growth, and child costs.

In the baseline Swedish setup, accompanying the rise in technology growth and the decline in mortality rates since 1740, the cost of children must have followed a U-shaped trajectory with a trough in the period 1880–1914. Concerning national experiences, differences in the speed of mortality declines help explain some of the differences in the observed transitions. A substantial part of these differences, though, must be attributed to the diversity in the evolution of child costs or technology growth.

The main contribution of this paper is to provide a quantitatively oriented interpretation of the historical demographic transition in Europe within a version of the Barro–Becker (1989) model. The quantitative approach has also been followed in a number of papers. Moe (1998) introduces human capital and non-market activities in a model similar to the present one which she applies to data for a developing country. Hansen and Prescott (1998) study the English historical demographic transition and the shift from the pre-industrial regime to the modern mode of production. In their model, fertility and population follow a pre-specified functional form. Fernandez-Villaverde (1999) emphasizes the role of skill-biased technical change and human capital accumulation. That study focuses on a narrower set of observations than the present paper (i.e., England since 1875), thus excluding the early period of soaring population growth. None of these papers evaluates the quantitative implications of the theory over the transition against detailed time series. Eckstein *et al*. (1999) use detailed Swedish data on mortality and *total fertility* to estimate and analyze a non-altruistic life-cycle partial-equilibrium model with exogenous wages and no capital accumulation. I use instead three-country low-frequency observations on mortality and the *natural rate of increase* in population to calibrate the transition of a dynastic general-equilibrium model.

This paper also relates to the literature on the demographic transition that departs from the neoclassical technology and/or dynastic preferences assumed here. This literature includes Becker *et al*. (1990), Dahan and Tsiddon (1998), Ehrlich and Lui (1991), and Galor and Weil (1996, 2000). These works contain important insights into the relation between economic and demographic changes, but their implications have not been evaluated with a quantitative approach.

The rest of the paper is organized as follows. Section 2 documents the facts to be explained. Section 3 presents the model and characterizes the household's optimal choices. Section 4 characterizes the equilibrium and outlines some steady-state effects of parameter changes. Section 5 calibrates the model's benchmark economy. Section 6 reports the results of the numerical experiments and discusses the findings. Section 7 ends the paper with conclusions and directions for future research.

2. THE FACTS

In Europe, the mid 18th century represents a turning point in the secular behavior of demographic series which leads the way toward the subsequent reductions in fertility and mortality rates. Hence, this paper focuses on developments that were initiated about 1750. The sources of the data used in this section are described in Appendix A.

The natural rate of increase (NRI) is the rate of increase in population per 1000 population over one year due to the natural processes of births

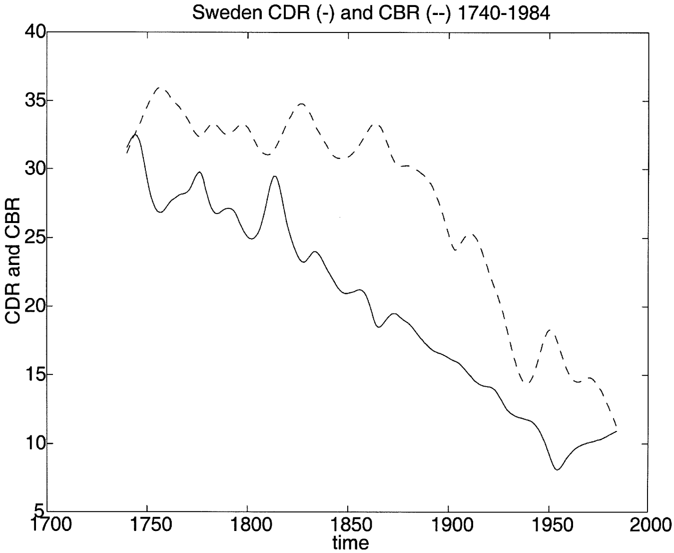

FIG. 1. CDR and CBR for Sweden, 1740–1984.

and deaths. The NRI is thus calculated as the difference between the crude birth rate (CBR) and the crude death rate (CDR).

Figures 1–3 display the HP-filtered (with parameter 100) annual CBR's and CDR's over 1740–1985 for Sweden, England, and France. In the three countries, there is a trend toward lower mortality. In Sweden, the decline is gradual, whereas England goes through stages, with a relatively flat pattern between 1840 and 1880 that becomes steeper afterward. In France, the permanent declines in mortality rates started earlier and from higher levels. Although France tends to catch up with its neighbors by 1840, its mortality rate remains consistently higher throughout.

Concerning fertility, the differences across the three countries are more apparent. In England, the fertility rate rises and stays above pre-transition values for over a century before the onset of the declining trend. In Sweden, the fertility rate remains instead fairly stable during the period that precedes the decided downturn under way since 1875. Whereas in England and Sweden the paths of the CBR and CDR are dissociated over much of the period, the pattern in France is of a near balance between births and deaths. The French declines in death rates are matched by falls in fertility right from the outset.

The patterns of mortality and fertility change that have been reported here are consistent with the accounts in the specialized literature such as Anderson (1996), Chesnais (1992), Perrenoud (1991), and Vallin (1991).

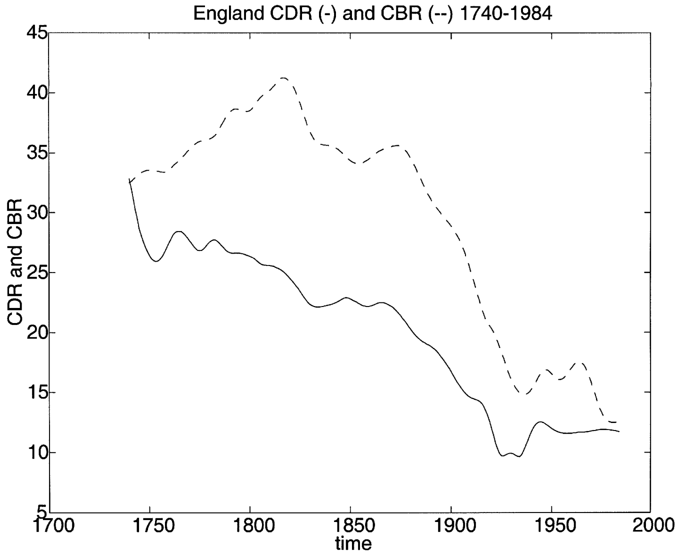

FIG. 2. CDR and CBR for England, 1740–1984.

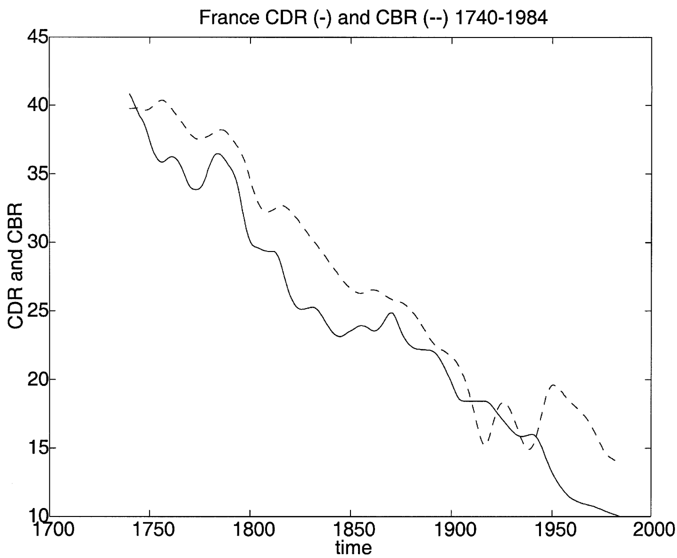

FIG. 3. CDR and CBR for France, 1740–1984.

TABLE I
Natural Rate of Increase (35-year annual averages per 1000, 1740–1984)

Period	England	France	Sweden
1740–1774	6.09	2.71	5.16
1775–1809	11.35	2.93	5.84
1810–1844	14.73	4.99	7.9
1845–1879	12.68	2.49	11.61
1880–1914	12.02	1.14	10.56
1915–1949	6.04	0.39	5.94
1950–1984	3.55	5.79	5.12

The separate experiences in crude rates are reflected in the implied paths of the NRI. Sweden displays moderately rising NRI's until the mid-1800s, then an acceleration that subsides by 1914 and gives way to the characteristic declining trend since. Compared with the Swedish case, England's experience is notorious for its early acceleration in the NRI, which can be traced back to the mid-l700s. The English NRI's move into step with the Swedish figures by the mid-l800s. France undergoes a comparatively stable transition in terms of the NRI, with only a brief and modest rise in the 1820s. For the analysis of this paper, it will prove convenient to focus on 35-year-average figures. Following this convention, the data displayed in Figures 1–3 are summarized in Table I.

Turning now to economic variables, the period considered is one of unprecedented improvements in living standards. Table II displays productivity growth factors for the 35-year periods indicated. It suggests that productivity surged first in England, the fruits of the Industrial Revolution spreading to the rest of the countries with some lag.[2] It is evident, however, that France and Sweden exhibited faster growth of productivity at least since the mid-1800s.

3. THE MODEL

The analytical framework is a version of the neoclassical model of growth with endogenous fertility in Barro and Becker (1989). The two modifications are the introduction of mortality risk and a simplified treatment of

[2] As explained in Appendix A, the pre-1870 figures for Sweden and England are not directly comparable. Nonetheless, the impression given is consistent with Chesnais (1992, p. 455) who dates the first stages of industrialization and growth for Scandinavian countries to the turn of the 19th century. Early writers date instead the beginning of economic growth in England around 1760 [Chesnais (1992, p. 447, footnote 3)].

TABLE II
Average Productivity Growth

Period	England	Sweden	France
1740–1774	1.12	0.9	
1775–1809	1.21	1.20	
1810–1844	1.45	1.47	
1845–1879	1.35	1.45	
1880–1914	1.50	1.82	1.75
1915–1949	1.86	2.25	2.18
1950–1984	1.81	1.99	1.88

the cost of children. This section presents the assumptions of the model and analyzes the decision problem of the households.

3.1. Demographic Structure

The agents in the economy can live for two periods: young age and adulthood. The economy is populated by a continuum of households, each consisting of possibly one adult and her heirs. I denote generations by the period when their members are born. At time $t-1$, a number N_{t-1} of children are born and become young. This value is net of early infant mortality. A member of generation $t-1$ survives to period t with probability π_{t-1}. Thus, only $\pi_{t-1}N_{t-1}$ agents belonging to generation $t-1$ will reach adulthood. In turn, generation $t-1$ bears N_t effective children. The adult population growth factor between two consecutive generations t and $t+1$, g_t, is thus

$$g_t \equiv \frac{\pi_t N_t}{\pi_{t-1} N_{t-1}}. \tag{1}$$

For the sake of defining consistent measures, let a model's period correspond to 35 years. Then the NRI per thousand population between t and $t+1$ can be computed as

$$NRI_t = \left[\frac{N_t\pi_t + N_{t+1}}{N_{t-1}\pi_{t-1} + N_t} - 1\right]\frac{1000}{35} = \left[\frac{g_t\pi_t(1+g_{t+1}/\pi_{t+1})}{g_t+\pi_t} - 1\right]\frac{1000}{35}. \tag{2}$$

The behavior of this variable is the primary focus of this paper. Note first that, given the paths of adult-population growth g_t and survival rates π_t, the definition of the NRI does not require measuring infant mortality rates.[3] Second, without further assumptions, the ambiguous effects of g_t and π_t on NRI_t are reflective of natural composition effects.

[3]The CBR and CDR values do depend on the infant mortality rate, but it nets out when calculating the NRI as CBR–CDR.

A member of generation $t-1$ plans to have n_t children by the end of her first life-time period $t-1$. This planned fertility is net of infant mortality. If she survives to adulthood, with probability π_{t-1}, then the plan will be effectively implemented. However, conditional on that she dies before completing adulthood, which occurs with probability $1-\pi_{t-1}$, the fertility plan can only be carried out if premature death does not occur too soon, which is the case with probability γ_{t-1}. That is, γ_{t-1} is the probability of carrying out the planned fertility conditional on not reaching adulthood. It follows that the motion for the flow of births is

$$N_t = N_{t-1}n_t[\pi_{t-1} + \gamma_{t-1}(1-\pi_{t-1})]. \tag{3}$$

As a consequence, the adult population growth factor between t and $t+1$ in Eq. (1) can be written as

$$g_t = \frac{\pi_t}{\pi_{t-1}} n_t[\pi_{t-1} + \gamma_{t-1}(1-\pi_{t-1})]. \tag{4}$$

When mortality and planned fertility have constant values, the growth rate of the total population, $NRI \times (35/1000)$, coincides with that of the adult population, $g-1$.

3.2. Technology

A single sector produces final output. Capital, K_t, and labor units, L_t, are the inputs employed in period t to produce total output through a neoclassical production function, $F(K_t, A_tL_t)$. It will be assumed that $F(\cdot,\cdot)$ is Cobb–Douglas, with $0<\theta<1$ being the output share of capital and A_t representing labor-augmenting technology,

$$F(K_t, A_tL_t) = K_t^{\theta}(A_tL_t)^{1-\theta}. \tag{5}$$

Technology is assumed to grow at a rate $x_t - 1$ between period t and $t+1$:

$$x_t = \frac{A_{t+1}}{A_t}. \tag{6}$$

Output produced at t can be used for consumption C_t, for accumulation of next-period capital K_{t+1}, or for producing children. Capital depreciates at the rate $\delta \leq 1$. Each birth is assumed to imply a goods cost. This cost amounts to $A_t\eta_t$ for every born child. Since the number of births at t is N_t, total child-rearing expenditures amount to $A_t\eta_tN_t$.

Finally, it remains to specify the technology for rearing children. Many authors argue that child-rearing is intensive in time. In this case, the term η_t should reflect the opportunity cost of the parental time, as in Barro and Becker (1989). Here it is assumed that children entail a cost in terms of goods only. This cost may depend on the level of the economy-wide capital per worker though. With some abuse of notation, one can write

$$\eta_t = \eta_t(\hat{k}_t), \tag{7}$$

with $\hat{k}_t$ denoting capital per effective worker, $\hat{k}_t = K_t/(A_t L_t)$. Notice this formulation admits that the child cost be proportional to the technology-adjusted wage, as in Barro and Becker (1989). By assuming it is a goods cost rather than a time cost, however, I rule out the impact of changes in fertility on hours of labor supplied. This simplifies the explicit computation of the model's transitional dynamics. The time index in $\eta_t(\cdot)$ accounts for the possibility that the parameters of this function change over time.

3.3. Households

Individual agents make choices on fertility and capital transfers. This section describes the household's environment and characterizes the optimal choices.

Budget Constraint

An agent of generation $t-1$ receives at $t-1$ a claim on the amount of wealth k_t from her parents. In the first period of her life, this wealth is allocated between two assets, called conditional annuities, s_t, and simple annuities, a_t. Simple annuities will pay at t a rate of return R_t^a only if she survives. A conditional annuity will pay a rate of return R_t^s except if she dies and, additionally, does not have any children, in which case it will pay nothing. For example, if $\gamma_{t-1} = 1$ conditional annuities yield a return with certainty and are equivalent to simple annuities. If the agent dies before period t but leaves descendants, the return on conditional annuities is devoted to cover the cost of rearing her heirs, $n_t A_t \eta_t$, and to provide them with *accidental* bequests, adding up to $n_t k_{t+1}^A$. If she survives to period t, then she receives the returns from her total wealth and labor income and spends the revenues on her own consumption, child-rearing, and wealth accumulation in the form of voluntary bequests, $n_t k_{t+1}^V$.

More formally, the portfolio choice at time $t-1$ by the agent of cohort $t-1$ must satisfy the constraint $s_t + a_t = k_t$. In the event that she dies and has children, which happens with probability $(1-\pi_{t-1})\gamma_{t-1}$, accidental bequests to generation t are determined by $R_t^s s_t = n_t k_{t+1}^A + n_t A_t \eta_t$, where $A_t \eta_t$ is the expenditure on each child and k_{t+1}^A represents the wealth received by each child that survives up to period $t+1$. Hereafter, all prices are expressed in terms of the final good. If she lives up to the second-period t, then she faces the constraint $c_t = w_t + R_t^s s_t + R_t^a a_t -$

$n_t k_{t+1}^V - n_t A_t \eta_t$. Here c_t is her consumption, w_t is labor income, and k_{t+1}^V is the bequest received by each of her children. The budget set for an agent of cohort $t-1$ is given by these three equations. They can be combined to get a more compact expression:

$$c_t = w_t + R_t^a k_t - n_t \frac{R_t^a}{R_t^s} A_t \eta_t - n_t k_{t+1}^V - n_t k_{t+1}^A \left(\frac{R_t^a}{R_t^s} - 1 \right). \tag{8}$$

Preferences and the Decision Problem

The preferences of an individual of cohort $t-1$ reflect a concern for the welfare of all future descendants. Generations are linked through parental altruism which is manifest in the form of intergenerational transfers of wealth. Let V_{t-1} denote the utility attained by a typical member of generation $t-1$ from the perspective of time $t-1$. The separable specification is assumed

$$\begin{aligned} V_{t-1} &\equiv E[u(c_t) + \beta n_t^{1-\epsilon} V_t \mid t-1] \\ &= \pi_{t-1} u(c_t) + \beta E[n_t^{1-\epsilon} V_t \mid t-1], \end{aligned} \tag{9}$$

where E is the expectation operator. This formulation assumes that a parent's utility depends on her own consumption, on the number of children, and on the utility of each child. As of period $t-1$, there is uncertainty about outcomes, so utility is evaluated in expected terms. Expenditures on children do not enter utility but only adult consumption does through $u(\cdot)$.[4] The discount term reflects that individuals care about the welfare of every child. It is assumed that $\epsilon > 0$, and $\epsilon > 1$ if utility is negative.[5] The instantaneous utility function $u(\cdot)$ is assumed to take on the CRRA specification

$$u(c) = \frac{1}{1-\sigma} c^{1-\sigma}, \quad \text{with } \sigma > 0. \tag{10}$$

Households take as given labor income, asset returns, and mortality risk. It is assumed that the per-child cost is also exogenously given to the household.[6] Now members of generation $t-1$ make their choices on the

[4]For the sake of tractability, models with altruistic preferences often assume that the number of children enters separately with the utility of the children [see, for instance, Palivos (1995) and Razin and Ben-Zion (1975)]. On the other hand, the assumption that parents' instantaneous utility does not depend on the number of children is relaxed in Barro and Sala-i-Martin (1995, Chap. 7). Actually, in their formulation, dropping this additional component of utility from children precludes any dynamics for fertility, at least when $\eta_a = 0$.

[5]Barro and Becker (1989) assume $\epsilon < 1$. The current formulation allows us to accommodate relative risk aversion greater than one, a point also made by Alvarez (1999, footnote 2).

[6]In Barro and Sala-i-Martin (1995, Chap. 7), the alternative assumption is made. I think the assumption made here accords better with existing theories.

first-period $t-1$ under uncertainty. For the preferences in (9) and (10), given a value k_t of wealth, the values c_t, k^A_{t+1}, k^V_{t+1}, and n_t are determined as the solution to the Bellman equation

$$\begin{aligned} V_{t-1}(k_t) &= \max\{\pi_{t-1}u(c_t) + \beta E[n_t^{1-\epsilon} V_t(k_{t+1})]\} \\ &= \max\{\pi_{t-1}u(c_t) + \beta n_t^{1-\epsilon}[\pi_{t-1}V_t(k^V_{t+1}) \\ &\qquad + (1-\pi_{t-1})\gamma_{t-1}V_t(k^A_{t+1})]\} \end{aligned} \tag{11}$$

subject to the budget constraint (8). Although the problem is stationary, I keep the subscripts in the value function for clarity.

One can show that, with a competitive insurance market, the equilibrium asset returns R^a and R^s are actuarially fair so $\pi_{t-1}R^a_t = [\pi_{t-1} + (1-\pi_{t-1})\gamma_{t-1}]R^s_t$. There is then full insurance against lifetime uncertainty so that $k_{t+1} = k^A_{t+1} = k^V_{t+1}$ and the FOC's for the choice of k_{t+1} can be written as

$$n_t u'(c_t) = \beta n_t^{1-\epsilon} R_{t+1} u'(c_{t+1}), \tag{12}$$

where R_{t+1} is the return on one-period securities.[7] The budget constraint (8) can then be posed as

$$c_t = w_t + R_t \frac{k_t}{\pi_{t-1}} - \left(\frac{A_t \eta_t}{\pi_t} + \frac{k_{t+1}}{\pi_t}\right)\frac{\pi_t}{\pi_{t-1}} n_t(\pi_{t-1} + (1-\pi_{t-1})\gamma_{t-1}). \tag{13}$$

The present full-insurance setup implies that, for a given real return on equity, mortality risk does not influence optimal intertemporal choices. The net result is that the condition for intergenerational optimality (12) is not influenced by mortality risk.[8]

[7] An extended-family arrangement, as in Ehrlich and Lui (1991), could also have been assumed as an alternative to markets for annuities. Each extended family is composed of a large number of identical households with independent prospects of survival. This mechanism would ensure bequests to every child whether she is orphaned or not by assuming that an implicit contract concerning intra-family transfers is enforced. In a different model, Ehrlich and Lui (1991) show that, with a trigger-strategy penalty on the defectors, such contracts are self-enforcing. The parallel analysis has not been carried out here.

As one referee rightly pointed out, either assumption rules out a potentially important role for capital market imperfections, and is justified on grounds of analytical convenience. In the absence of such insurance institutional arrangements, life-time uncertainty would command variation in the distribution of wealth. See Fuster (1999) and Loury (1981) on this matter.

[8] In Ehrlich and Lui (1991), the return from investing in children's human capital is positively related to survival probabilities and this determines the positive effect of mortality rate reductions on economic outcomes and the tendency of fertility to decline over time. On the contrary, in the current setup the relevant return on investment is independent of survival rates. The calculations of Preston (1980, pp. 324–326) imply that, to a first approach, this assumption may not be grossly misleading.

Fertility Choice

Since there is full insurance, one can write the value function in (11) as

$$V_{t-1}(k_t) = \max\left\{\pi_{t-1}u(c_t) + \beta n_t^{1-\epsilon}(\pi_{t-1} + (1-\pi_{t-1})\gamma_{t-1})V_t(k_{t+1})\right\},$$

the maximization being subject to the constraint in (13). For the choice on n_t to be well defined, the condition $(1-\sigma)/(\sigma-\epsilon) > 0$ has to be satisfied. Therefore, $\epsilon > \sigma$ if and only if $\sigma > 1$. The optimal choice of n_t balances the cost of an additional descendant in terms of parents' utility and the gain in future generations' utility. Using the specification of $u(\cdot)$ in (10), the solution is characterized by

$$c_{t+1} = \frac{1-\sigma}{\sigma-\epsilon}\left[R_{t+1}\frac{A_t\eta_t}{\pi_t} - w_{t+1}\right]. \tag{14}$$

The conditions (12)–(14) characterize the household's behavior, provided that the implied utility value is bounded.

4. EQUILIBRIUM

For analytical convenience, using Eq. (4) the equilibrium is defined in terms of the growth rate of the labor force, g_t, and per-unit-of-effective-worker variables.

DEFINITION. Given $f(\cdot)$, δ, β, σ, ϵ, $\{\eta_t(\cdot), x_t, \pi_t, \gamma_t\}_{t=0}^{\infty}$, and $\hat{k}_0$, a *perfect foresight competitive equilibrium* consists of sequences of quantities for capital per effective worker $\hat{k}_t$, consumption per effective worker $\hat{c}_t$, adult population growth g_t, and prices w_t, r_t, R_t for $t = 0, 1, 2, \ldots$ such that: taking r_t and w_t as given, firms maximize profits; taking R_t and w_t as given, households maximize their dynastic utility; returns on capital and debt are equalized (no arbitrage); and all markets clear.

Market clearing implies that total wealth equals the aggregate capital stock. Since all households are assumed to be identical, $K_t = N_{t-1}k_t$. Each adult provides one unit of labor so that $L_t = N_{t-1}\pi_{t-1}$. Then $\hat{k}_t \equiv K_t/(A_tL_t) = k_t/(A_t\pi_{t-1})$. Output per effective worker is then $f(\hat{k}_t) \equiv F(\hat{k}_t, 1)$. The no-arbitrage condition implies that $R_t = 1-\delta+r_t$. Maximization by competitive firms leads to $r_t = f'(\hat{k}_t)$ and $w_t = A_t[f(\hat{k}_t) - \hat{k}_tf'(\hat{k}_t)]$. Thus, we can represent the equilibrium wage rate and the rental rate of capital as $Aw(\hat{k})$ and $r(\hat{k})$, respectively.

Then clearing in output market, $F(K_t, A_tL_t) + (1-\delta)K_t = C_t + K_{t+1} + N_tA_t\eta_t$, or, equivalently, the household's budget constraint (13), can be written as

$$f(\hat{k}_t) + (1-\delta)\hat{k}_t = \hat{c}_t + g_t\left(\hat{k}_{t+1}x_t + \frac{\eta_t(\hat{k}_t)}{\pi_t}\right), \qquad t = 0, 1, 2, \ldots. \tag{15}$$

Optimal fertility in (14) dictates

$$\hat{c}_t = \frac{1-\sigma}{\sigma-\epsilon}\left[(1-\delta+r(\hat{k}_t))\frac{\eta_{t-1}(\hat{k}_{t-1})}{x_{t-1}\pi_{t-1}} - w(\hat{k}_t)\right], \qquad t = 1, 2, \ldots, \tag{16}$$

and the first-order condition for the intertemporal allocation of consumption (12) can be posed as

$$g_t = \frac{\pi_t}{\pi_{t-1}}(\pi_{t-1} + \gamma_{t-1}(1-\pi_{t-1}))[\beta(1-\delta+r(\hat{k}_{t+1}))]^{1/\epsilon}\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}x_t\right)^{-\sigma/\epsilon}, \quad t = 0, 1, 2, \ldots, \tag{17}$$

where use has been made of (4) to substitute n_t away. The term $\eta_t(\hat{k})$ will now be specified as

$$\eta_t(\hat{k}) = \eta_{at} + \eta_{bt}w(\hat{k}), \quad \text{with} \quad \eta_{at}, \eta_{bt} \geq 0. \tag{18}$$

Thus, an equilibrium is described by paths of $\hat{k}_t$, $\hat{c}_t$, and g_t that satisfy the four equations (15)–(18) above for given initial capital intensity $\hat{k}_0$ and a $\hat{c}_0$ consistent with the appropriate transversality condition. Under the functional forms assumed, $f(\hat{k}) = \hat{k}^\theta$, $r(\hat{k}) = \theta\hat{k}^{\theta-1}$, and $w(\hat{k}) = (1-\theta)\hat{k}^\theta$. The description of the equilibrium is completed by a condition ensuring that the household's utility is not unbounded for the paths implied by the above equations. This condition will be stated in terms of steady-state outcomes.

Define a steady state as an equilibrium where all per-unit-of-effective-worker variables, prices, and fertility and mortality rates remain constant over time. A steady state is characterized by the set of equations

$$\Lambda(\hat{k}) \equiv f(\hat{k}) + (1-\delta)\hat{k} - \hat{c}(\hat{k}) - \left(\hat{k}x + \frac{\eta(\hat{k})}{\pi}\right)g(\hat{k}) = 0, \tag{19}$$

with

$$\hat{c}(\hat{k}) = \frac{1-\sigma}{\sigma-\epsilon}\left[(1-\delta+r(\hat{k}))\frac{\eta(\hat{k})}{x\pi} - w(\hat{k})\right], \tag{20}$$

$$g(\hat{k}) = (\pi + \gamma(1-\pi))[\beta(1-\delta+r(\hat{k}))]^{1/\epsilon}x^{-\sigma/\epsilon}, \tag{21}$$

and

$$\eta(\hat{k}) = \eta_a + \eta_b w(\hat{k}). \tag{22}$$

Bounded utility requires that

$$g(\hat{k})x < 1 - \delta + r(\hat{k}). \tag{23}$$

As in Barro and Becker (1989), there may be a multiplicity of steady states. Although the number of steady states cannot be bounded analytically, none of the numerical setups considered in this research has been found to have more than two steady states. The equilibrium system is of second order as long as η_b is positive.[9] Local stability analysis has been performed by studying the two roots of the linearized system. Although analytical results have not been obtained, the linearized system has been investigated numerically. For all the economies considered in this research, the condition $\Lambda'(\hat{k}) > 0$ (see definition in Eq. (19)) characterizes saddle-path stable steady states.[10]

The steady-state effects of a change in the rate of technical progress and the child cost are discussed in Barro and Becker (1989). A higher rate of technical progress, x, implies, by Eq. (17), a steeper profile for consumption which, for given r, calls for lower discount or lower fertility. But x may also lead to lower capital accumulation and a higher interest which exerts the opposite effect on equilibrium fertility. The effects of changes in the child-cost parameters on population growth are straightforward. The treatment of mortality here is more elaborate than that in Barro and Becker (1989), which opens new theoretical possibilities. On the one hand, higher survival probabilities have a direct impact on the population growth rate [see Eq. (4)]. In this model, stable steady states are such that the marginal value of the increased quantity of children declines relative to the return from quality. Therefore, parents tend to increase the quality of the children which materializes in larger transfers of capital per capita and, possibly, lower fertility. On the other hand, however, increasing the survival rate implies a reduction in the perceived per-parent cost of raising and endowing a given amount of children. This price effect induces a substitution of quantity for quality of children which tends to increase fertility rates. The net result depends on the relative importance of the two effects. When mortality drops happen to be concentrated at advanced adult ages (γ large), the impact on population growth is small and the cost-reduction or substitution effect dominates.[11] In general, specific statements about the implications of changes in parameters can only be made in a quantitative version of the model.

5. CALIBRATION

This section determines the parameters of the benchmark economy that will be used for the analysis. The working hypothesis is that the economies under study were all at a steady state before the changes initiated about 1740 and that they converge toward a new steady state that

[9] In Barro and Becker (1989), where children involve a time cost, the system is of third order.

[10] These results are consistent with those conjectured in Barro and Becker (1989).

[11] The implicit assumption in Barro and Becker (1989) is that $\gamma = 1$.

is consistent with post-transition figures. Furthermore, it is assumed that the three economies—Sweden, England, and France—are alike at both the pre-transition and the post-transition steady states. This will permit one to focus the analysis on the factors shaping the transition between the two steady states. The parameters to be determined thus are two sets of steady-state parameters and sequences for the parameters that are allowed to change over time. These time-varying parameters are the child-cost parameters, the survival probabilities, and the rate of technological change. Whereas the steady-state parameters are chosen to match the average behavior of a typical Western economy, the sequences of parameters are calibrated to transition observations for Sweden. Sweden is chosen as the benchmark because the evidence reviewed in Section 2 indicates this is the "normal" case that lies between the two more extreme experiences of England and France.

5.1. Steady-State Calibration

At a steady state, the parameters of the model are π, γ, θ, δ, x, σ, β, ϵ, η_a, and η_b. For the final, or post-transition, steady state, these numbers are pinned down so that the model is consistent with the following implications. I will consider a final steady state where survival rates are virtually 100%. The capital share is assumed to be 40%, as in Cooley and Prescott (1995). Given the period length of 35 years, a depreciation rate of 100% seems reasonable. The coefficient of risk aversion of 3 is in the middle range of values used in the literature. The assumption of 2% long run growth per year is made. I target a zero population growth so that $g = 1$. The investment–output ratio of 0.15 is close to the average for developed countries over the last two decades. The share of the cost of a child in adult consumption must be 0.588. Here I have used the equivalence Oxford scale which weights the first adult as 1, the second as 0.7, and each child as 0.5 (see Van Praag and Warnaar, 1997). These observations still leave a degree of freedom in the choice of η_a and η_b. To resolve the indeterminacy, I will arbitrarily assume $\eta_a = 0$.

To calibrate the values of π, γ, η_b, and x at the initial steady state, one needs observations about the pre-transition situation in England, France, and Sweden. Following Lucas (1998), a population increase of 3.0 per 1000 per year in the pre-industrial period seems reasonable. For the model's 35-year periods, the target is thus set to $g = 1.11$. On the other hand, I impose zero technical change, as output had remained largely stagnant before the Industrial Revolution. The observations on survival rates from the life tables summarized in Table VI of Appendix A for Sweden in the mid-1700s point to a choice of $\pi = 0.6$ and $\gamma = 0.50$. Here π represents the probability of survival between ages 1 and 40, and γ is interpreted as

TABLE III
Calibration Steady States

	σ	θ	δ	β	ϵ
Constant settings:	3.0	0.4	1.0	1.1488	3.78
Changing settings:	x	π	γ	η_a	η_b
Initial ($g = 1.11$)	1.0	0.6	0.5	0	0.335
Final ($g = 1.0$)	2.0	1.0	1.0	0	0.5

the conditional probability of dying after age 15, provided that death occurs between ages 1 and 40. This calibration involves an annual gross interest rate of 2.4%, smaller than the 5% of the final steady state. Other studies, such as Hansen and Prescott (1998), entertain similar implications for the interest rate. Table III summarizes the choices of steady-state parameters.

The changes between the two steady states reflect the interplay of some key comparative-static effects of the benchmark economy. The effect of the increase in survival probabilities between the initial and final steady states would, on its own, lead to higher population growth and lower capital intensity. Thus, in spite of the theoretical case made at the end of Section 4, the calibrated model rules out the possibility that the reduction in mortality can explain the main characteristics of the demographic transition. This result appears to be robust to changes in parametric settings. Other factors must have contributed to the transformations undergone. The observed rise in technical progress alone does indeed lead to lower population growth along with lower capital intensity. But the joint changes in mortality and technical change lead to a final population growth rate that exceeds the benchmark zero-growth case. Thus, a net increase in the child-cost parameter is needed. This tends to partly balance the negative effect of faster technical change and lower mortality on capital intensity.

5.2. Calibrating the Transition

This section describes the procedure leading to the choice of the paths $\{\pi_t, \gamma_t, \eta_{bt}, x_t\}$ over the transition period. One has to establish a correspondence between the model's timing and historical periods. I choose to map the model's periods into the historical 35-year intervals for which the figures on NRI's were averaged and calculated in Table I. In this way, $t = 1$ corresponds to the period 1740–1774, $t = 2$ to 1775–1809, and so on. Periods -1 and 0 correspond to the pre-transition stage.

The benchmark paths for π_t and γ_t (survival probabilities) are set according to the information gathered in Table VI of Appendix A for Sweden. Now, whereas the paths of π_t and γ_t have been determined from direct observations, the two remaining paths of x_t (technological change) and η_{bt} (child cost) are calibrated so that the implied transition is consistent with

TABLE IV
Targets for Productivity Growth and the NRI

Time	NRI	Productivity growth	Time	NRI	Productivity growth
−1 (1670–1704)	3.14	1.00	5 (1880–1914)	10.56	1.82
0 (1705–1739)	3.14	1.00	6 (1915–1949)	5.94	2.25
1 (1740–1774)	5.16	0.90	7 (1950–1984)	5.12	1.99
2 (1775–1809)	5.84	1.20	8 (1985–2019)	3.00	2.00
3 (1810–1844)	7.90	1.47	9 (2020–2054)	1.00	2.00
4 (1845–1879)	11.61	1.45	10 (2055–2089)	0.00	2.00

targets for the NRI and productivity growth. The target for the path of the NRI uses the Swedish figures in Table I. I assume the NRI becomes zero two periods after period 7 (1950–1984). Concerning average productivity growth, I will use the 35-year-average figures in Table II. More specifically, Table IV shows the chosen targets for productivity growth and the NRI.

Given the paths of the survival rates π_t and γ_t and the target for NRI_t, Eq. (2) permits one to calculate the path of the adult-population growth factor g_t that is consistent with that target. To implement the calibration procedure, I target g_t rather than NRI_t. A second practical observation is that productivity growth between t and $t+1$ is calculated as $x_t(\hat{k}_{t+1}/\hat{k}_t)^{\theta}$. Third, calibrating the child-cost parameter (η_{bt}) and technology growth (x_t) requires computing the transition of the model when the system is forced by changes in π_t, γ_t, η_{bt}, and x_t over time. To perform these calculations, I assume perfect foresight so that the changes initiated after period 0 are known and anticipated by agents as of period 1. This seems a natural assumption given the low frequency of the model's implications. The system is saddle-path stable and the equilibrium path is found using an iterative shooting algorithm. Details on computation are provided in Appendix B.

Now one can find the paths of the child cost and the growth rate of technology that are consistent with the targets in Table IV given the Swedish vital statistics. More specifically, the paths of η_{bt} (child cost) and x_t (technological change) have been calibrated in order to produce an accurate match of the NRI's. Concerning productivity growth, the procedure has to be less demanding. A good fit to the NRI is only consistent with paths of productivity growth that systematically undershoot the targeted figures. The compromise adopted here is to pick out a setting that, while consistent with the target NRI path, traces as closely as possible the pattern of the target productivity-growth path. The choice of parameters is displayed in Table V.

TABLE V
Benchmark Paths of Parameters

Time	π	γ	x	η_b
−1	0.600	0.500	1.000	0.335
0	0.600	0.500	1.000	0.335
1	0.607	0.510	1.011	0.335
2	0.614	0.568	1.100	0.170
3	0.678	0.608	1.350	0.220
4	0.698	0.546	1.450	0.200
5	0.774	0.624	2.200	0.130
6	0.881	0.700	2.300	0.190
7	0.967	0.756	1.600	0.315
8	1.000	1.000	1.750	0.350
9	1.000	1.000	1.850	0.450
10	1.000	1.000	1.880	0.470
11	1.000	1.000	2.001	0.500

5.3. Features of the Benchmark Model

The parameters in Tables III and V characterize the choice of benchmark. To provide a visual impression, these benchmark paths are depicted in Fig. 4. Figures 4(a) and (b) simply display the historical Swedish figures for survival rates. Figure 4(c) shows that strong growth of technology starts in period 3 (1810–1844) and increases to reach a peak that exceeds the long-run rate in period 6 (1915–1949). In periods 7 and 8 (1950–2000), the growth of technology slows down to hit a low of 1.6% per year. Subsequently, technology accelerates toward its long-run growth rate of 2% per year. As displayed in Fig. 4(d), the evolution of the child-cost parameter has a clear U-shaped profile. Until period 5 (1880–1914), there is a marked reduction in the child cost. Afterward, sharp increases take place which drive the child cost above its pre-transition levels from period 8 (1985–2020) onward.

Figure 5 represents the paths of the key endogenous variables of the model implied by the calibration considered, along with their targeted counterparts. The path of the NRI in Fig. 5(a) closely matches its target. The main feature of the path of the NRI is a marked hump shape with a peak in period 4 (1845–1879). The growth rate of productivity exhibits a similar pattern but with the peak occurring in period 6 (1915–1949). As shown in Fig. 5(b), the match of the productivity-growth targets is far from perfect, thus reflecting the tension in the calibration mentioned above. The search for a setting that does appreciably better than this has proved elusive.

The characteristics of this calibration can be interpreted as follows. Sharp declines in the child-cost component over the first periods of the transition are necessary to explain the early burst of population growth. This is

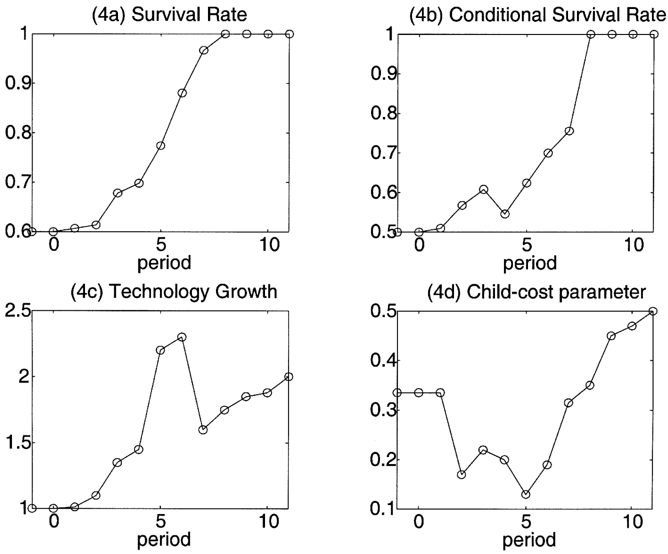

FIG. 4. Exogenous benchmark parameters: (a) π, (b) γ, (c) x, and (d) η_b.

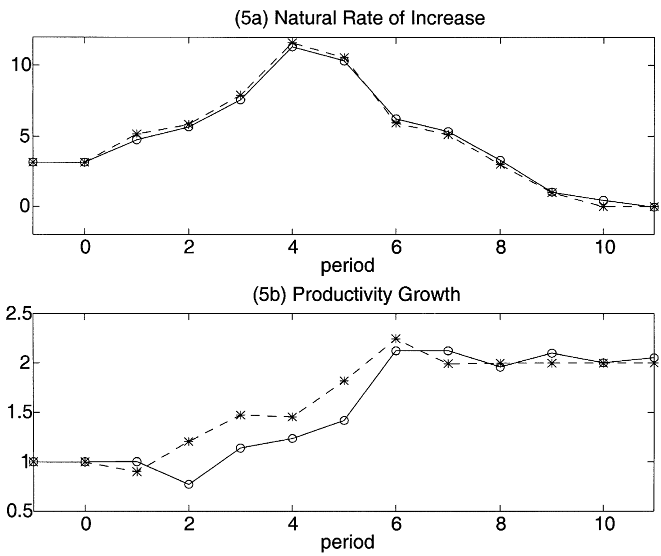

FIG. 5. Endogenous benchmark paths ($\circ$): (a) NRI and (b) productivity growth. The values indicated with $*$ correspond to the benchmark calibration targets.

because the contribution to population growth of the rising rate of technical change in those times is negative. Given that the effect on the NRI of observed mortality declines is not strong enough, the size of reductions in child costs must have been large at least until about 1880. Subsequently, rising costs have decisively contributed to the falls in population growth and, to some extent, the acceleration of productivity growth through capital accumulation. In any event, mortality declines have tended to boost the NRI and, hence, their contribution to the size of early population rises must have been positive.

6. NUMERICAL EXPERIMENTS

This section studies the role of three exogenous factors—mortality, technological change, and child cost—in explaining the diverse features of the demographic transition across European countries which are summarized in Table I. Sweden is taken as the reference model and assumed to be represented by the baseline setup calibrated in Section 5. England and France will then be studied separately. More specifically, three experiments will be conducted for each of the two countries. Experiment 1 modifies the benchmark setup using the survival probabilities (in Appendix A) of the country under study and computes the implied path of the natural rate of increase (NRI). Comparing this path with that corresponding to the Swedish benchmark and with that corresponding to the country's actual NRI experience will be indicative of the importance of mortality rates. The differences between the country's NRI path and the Swedish one that cannot be accounted for by observed differences in survival probabilities will then have to be attributed to differences in the other two factors. However, unlike for survival rates, there are no direct observations for the rate of technological change or for the child-cost parameter. The other two experiments consist of fitting exercises of either unobserved parameter's path. Experiment 2 calculates, under the country's survival conditions, the path of the child-cost parameter that fits the equilibrium of the otherwise benchmark economy to the country's observed trajectory for the NRI shown in Table I. Experiment 3 fits instead the path of the rate of technology growth to the country's NRI experience.

Thus, Experiments 2 and 3 deliver outcomes associated with explanations of the demographic transition based on child costs and technology growth, respectively. By construction, both hypotheses are consistent with the data on the NRI. The implications of the two experiments for productivity growth can then be compared with the evidence on relative productivity-growth performances contained in Table II to help judge the relative merit of the two hypotheses.

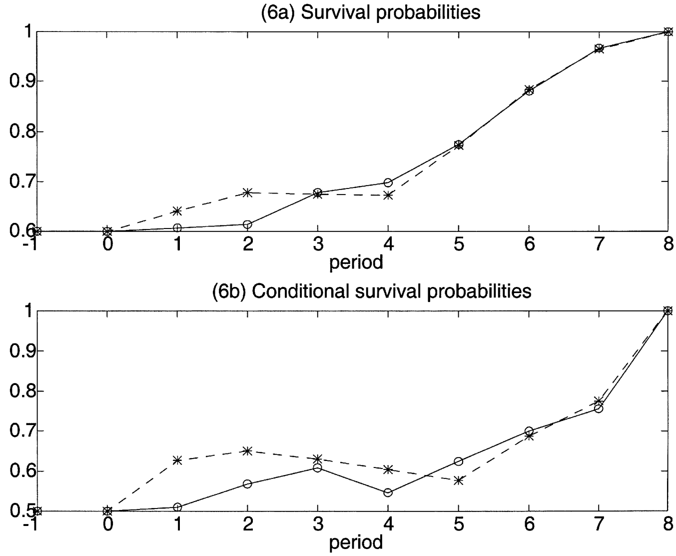

FIG. 6. Survival rates of England used in Experiment 1 ($*$) and of Sweden used in the benchmark setting ($\circ$): (a) π and (b) γ.

6.1. England

Relative to Sweden, England displays an earlier and larger boost in population growth up until 1915–1949.

Experiment 1 (Role of Mortality). Figure 6 compares the survival probabilities in Sweden and England. It is apparent that, relative to Sweden, England features faster early rises for about two periods. Using the English vital statistics in the otherwise benchmark setup produces the Experiment-1 NRI path represented in Fig. 7. For comparison purposes, the figure also displays the target NRI's for England (based on the data in Table I) and the NRI's for Sweden (already shown in Fig. 5(a)). Relative to the Swedish benchmark, the Experiment-1 NRI path stays higher for the first two periods of the transition. Subsequently, the Swedish and the Experiment-1 trajectories become very close together. This result is not surprising since the differences in mortality rates in Fig. 6 are most visible in periods 1 (1740–1774) and 2 (1775–1809). The target English NRI path and the Experiment-1 NRI path show a certain similarity in periods 1 (1740–1774) and 2 (1775–1809). In the periods that follow, however, Experiment 1 fails to predict the observed persistence of the English NRI's above those in Sweden. Thus, a complete explanation of the English demographic transition after period 2 (1775–1809) requires resorting to differences in the child-cost component and/or rates of technical change.

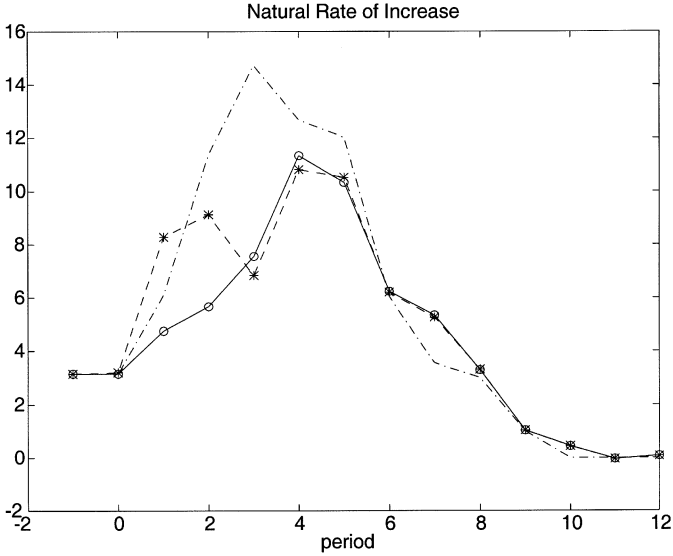

FIG. 7. NRI produced by Experiment 1 for England (∗) versus NRI produced by the Swedish benchmark setting (○) and the English target (– · – ·).

Experiment 2 (Child-Cost Hypothesis). The observation from Fig. 7 that Experiment 1 widely undershoots the English NRI's in periods 3 (1810–1844) and 4 (1845–1879) suggests that the child-cost parameter should be reduced in either or both of these periods. In effect, this is a prominent feature of the path that fits the English NRI experience using this country's survival rates in the otherwise benchmark setting. Experiment 2 searches this one trajectory for the parameter η_{bt} which is displayed against the benchmark path in Fig. 8.

Experiment 3 (Technological-Change Hypothesis). Figure 7 indicates that, in order to bring about the high NRI's observed in England between periods 2 (1775–1809) and 4 (1845–1879), the rate of technical change must have been lower in England than in Sweden at some date in this interval of time. Experiment 3 finds the path of the rate of technical change that matches the NRI's of England using the benchmark setup modified with this country's survival rates. Figure 9 displays it along with the benchmark path. The path of the rate of technical change fitted in Experiment 3 does indeed feature lower values than the benchmark Swedish path in periods 3 (1810–1844) and 4 (1845–1879). The Experiment-3 path exhibits larger swings throughout.

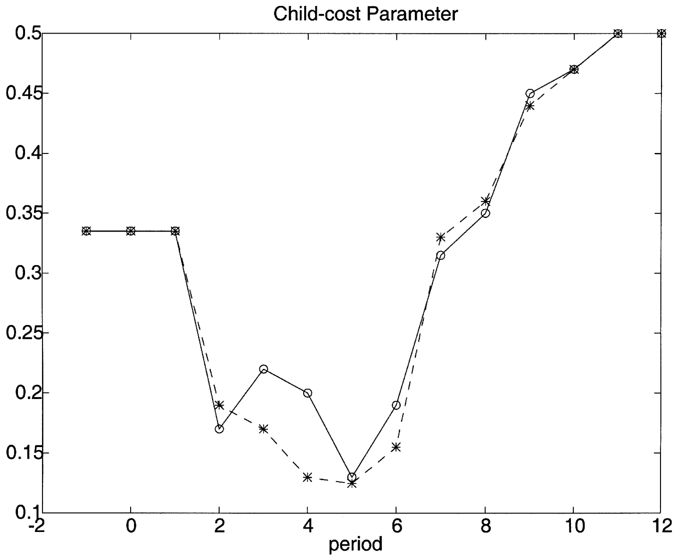

FIG. 8. Child cost η_b for England in Experiment 2 ($*$) versus Swedish benchmark setting ($\circ$).

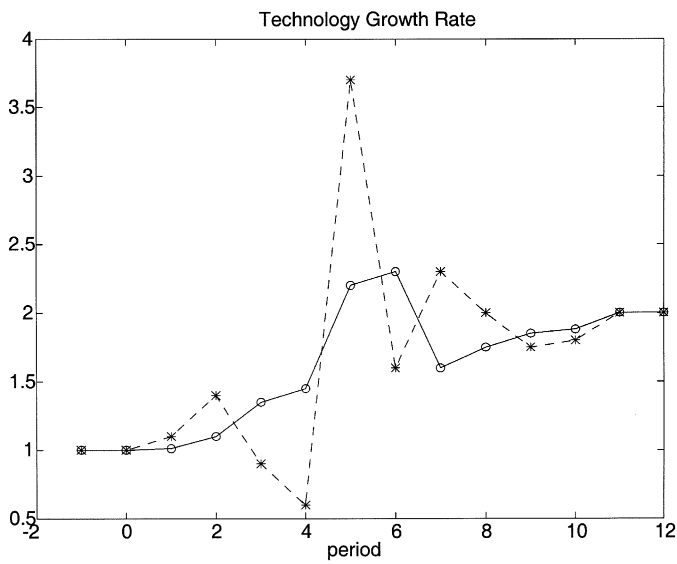

FIG. 9. Rate of technical change x for England in Experiment 3 ($*$) versus Swedish benchmark setting ($\circ$).

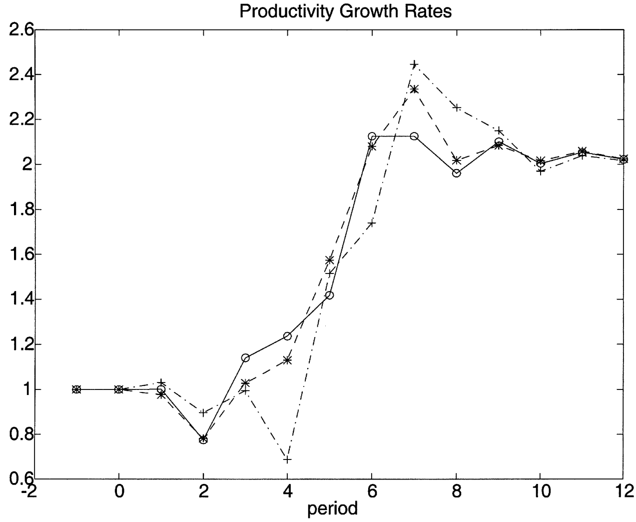

FIG. 10. Productivity growth rate for England produced by Experiment 2 (child-cost hypothesis) (∗) and by Experiment 3 (technological-change hypothesis) (+) and for Sweden produced by the benchmark setting (○).

Implications for Productivity Growth

Figure 10 shows the paths of productivity growth implied by Experiments 2 and 3 along with the trajectory associated with the Swedish benchmark case (already displayed in Fig. 5(b)). These implications from the model are to be compared with the relative patterns that emerge from the data in Table II which can be summarized as follows: relative to Sweden, England has a rate of productivity growth that is similar until period 3 (1810–1844) and lower between period 4 (1845–1879) and period 7 (1950–1984).

Figure 10 indicates that, until period 3 (1810–1844), the two hypotheses are broadly consistent with the observed pattern. Between periods 4 (1845–1879) and 6 (1915–1949), Experiment 3 implies that productivity growth falls short of the benchmark figures in two out of the three periods, whereas the path from Experiment 2 traces closely the benchmark figures with only mild departures. Thus, the technological-change hypothesis does somehow better in these periods. In period 7 (1950–1984), both hypotheses fail to produce the observed pattern, as they imply productivity growth rates that, relative to Sweden, are too high.

6.2. France

France did not experience the standard pattern of demographic transition observed elsewhere. In effect, as documented is Table I, the NRI there did not undergo a period of rapid increases.

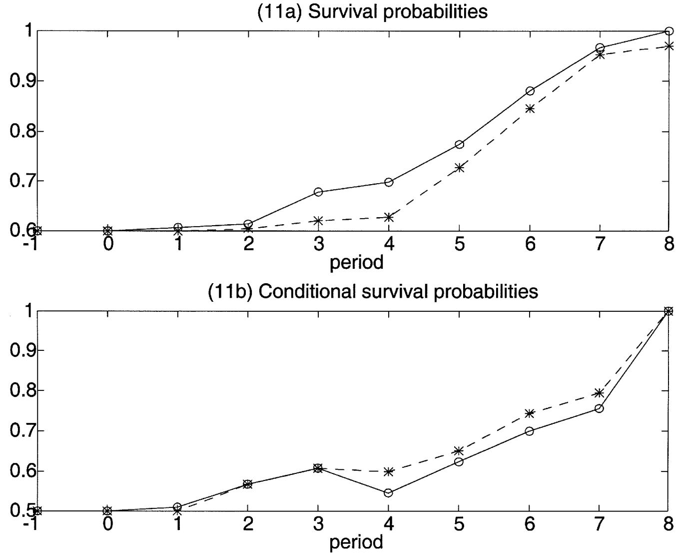

FIG. 11. Survival rates of France used in Experiment 1 ($*$) and of Sweden used in the benchmark setting ($\circ$): (a) π and (b) γ.

Experiment 1 (Role of Mortality). As described in Appendix A, the data on survival probabilities for France are incomplete and figures for early periods are, to some degree, speculative. Figure 11 depicts the choice of survival rates for France and, for comparison purposes, Sweden.

Experiment 1 feeds these French vital statistics into the benchmark economy and produces the corresponding path of the NRI depicted in Fig. 12. For comparison purposes, the target NRI's for France (based on the data in Table I) and the NRI's for Sweden are also displayed in this figure. Between periods 1 (1740–1774) and 4 (1845–1879), the Experiment-1 NRI's are lower than the Swedish NRI's and closer to the French target. Intuitively, low survival rates in those periods contributed to moderate population growth in France. However, Experiment 1 still shows a counterfactual surge of the NRI well above the values observed for France in periods 4 (1845–1879) and 5 (1880–1914). Thus, in order to account for the substantial departures of the French NRI's from the Swedish ones, other factors must have been at work.

Experiment 2 (Child-Cost Hypothesis). As shown in Fig. 12, Experiment 1 overshoots to various degrees the path of the French NRI. This suggests that the path of the child-cost parameter, η_b, that matches the NRI in France under this country's mortality conditions must lie, overall, above the Swedish benchmark path. Experiment 2 finds such a path which is displayed in Fig. 13 along with the benchmark path. As anticipated, the

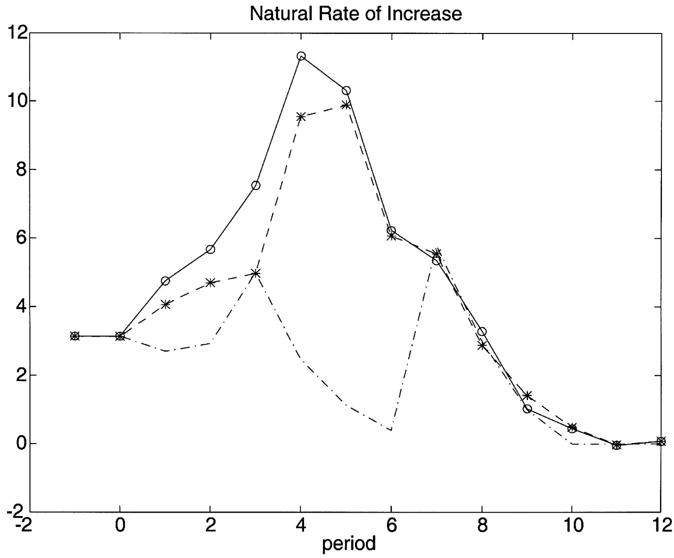

FIG. 12. NRI produced by Experiment 1 for France (∗) versus NRI produced by the Swedish benchmark setting (○) and the French target (– · – ·).

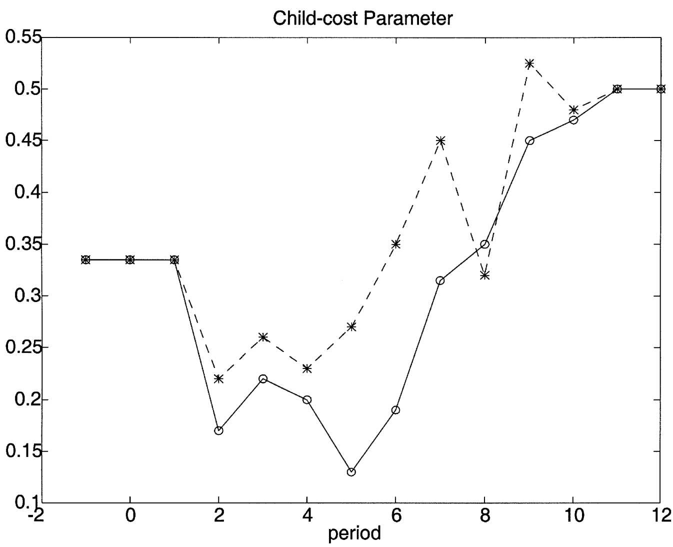

FIG. 13. Child-cost η_b for France in Experiment 2 (∗) versus Swedish benchmark setting (○).

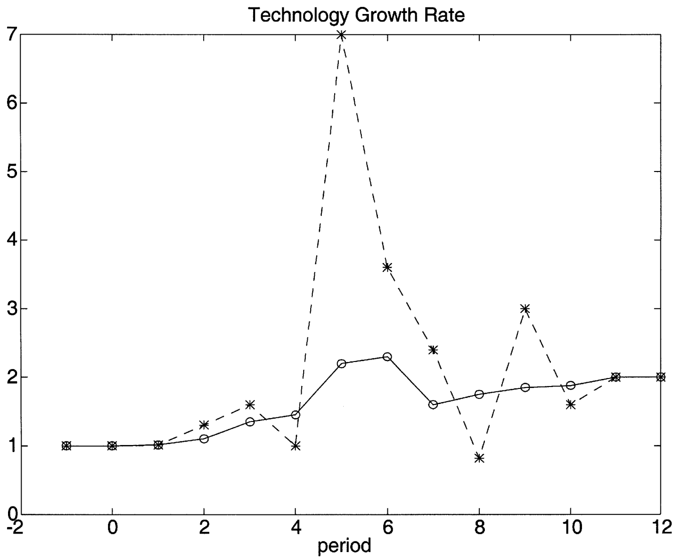

FIG. 14. Rate of technical change x for France in Experiment 3 ($*$) versus Swedish benchmark setting ($\circ$).

child-cost parameter path that fits the French NRI observations stays above the one corresponding to the Swedish benchmark in the central periods of the transition.

Experiment 3 (Technological-Change Hypothesis). It is clear from Fig. 12 that technological change in France must have been particularly rapid to overcome the forces of fertility growth in the central periods of the transition. Experiment 3 finds the trajectory for the rate of technology growth that is required to match the French NRI's under the local mortality conditions. It is displayed in Fig. 14 along with the benchmark series of technology growth. Between periods 5 (1880–1914) and 7 (1950–1984), the path of the rate of technology growth fitted in Experiment 3 exceeds the Swedish benchmark figures by a wide margin.

Implications for Productivity Growth

Figure 15 shows the paths of productivity growth implied by Experiments 2 and 3 along with the trajectory associated with the Swedish benchmark. These outcomes are to be compared with the relative pattern that emerges from the data in Table II. Given that French data are available only since period 5 (1880–1914), the feature that stands out is that of a close proximity between the Swedish and French rates of productivity growth.

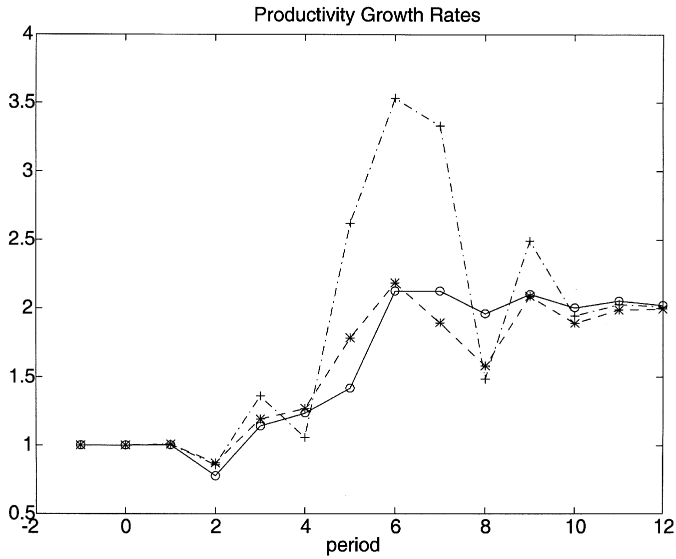

FIG. 15. Productivity growth rate for France produced by Experiment 2 (child-cost hypothesis) (∗) and by Experiment 3 (technological-change hypothesis) (+) and for Sweden produced by the benchmark setting (○).

An inspection of Fig. 15 indicates that the child-cost hypothesis is broadly consistent with the relative pattern observed for France and Sweden. In effect, Experiment 2 implies productivity growth rates that deviate only mildly from the Swedish benchmark. On its part, the technological-change hypothesis performs worse on this dimension. Experiment 3 generates a productivity-growth path for France that widely outperforms the one for Sweden between periods 5 (1880–1914) and 7 (1950–1984).

7. CONCLUSIONS

In this paper, the neoclassical model of growth with dynastic preferences is analyzed quantitatively to assess the role of different factors in the demographic transition of a set of representative European countries since the mid 18th century. The factors considered are the evolution of survival probabilities, technical change, and the costs of rearing children.

A benchmark setting is selected such that, given the historical path of mortality rates in Sweden, the model matches the low-frequency time series for the natural rate of increase (NRI) in population and, as far as possible, productivity growth in that country. In this setup, the child-cost parameter must have first decreased between, roughly, 1740–1774 and 1880–1914 and then risen toward the current high values. For its part, the rate of technology growth must have reached its peak in the periods 1880–1914 and 1915–1949.

Observed cross-country differences in survival probabilities go some way to explain the relatively fast population growth in England and the relatively moderate pattern in France during the early periods of the transition. Quantitatively, the contribution of variation in survival rates is modest though. Variation in child costs and in technology growth are considered as alternative hypotheses to explain the remaining differences in NRI's relative to the Swedish benchmark. It is found that, to explain the relatively faster population growth in England until about 1845–1879, the fall in the child-cost component must have been larger or technical progress must have been slower there in the periods 1810–1844 and 1845–1879. On the other hand, the relatively mild shifts of the NRI observed in France imply that the child cost must have remained high and steady there throughout or that the local rate of technical change in the periods 1880–1914 and 1915–1949 must have widely exceeded that of Sweden and England.

Regarding the implications for cross-country differences in productivity growth, neither the technology-growth hypothesis nor the child-cost hypothesis have a manifest superiority when confronted with the evidence. However, it must be noted that the hypothesis based on technology-growth differences requires large swings in technology growth rates across time and regions, perhaps far larger than may be defensible on intuitive grounds. All the same, the productivity-growth implications must be interpreted with care since, as demonstrated in Fig. 5(b), accommodating observed productivity growth rates at the same time as population growth rates remains a challenge to this model.

In any event, two of the explanatory factors entertained here—technology-growth and child-cost parameters—have not been related to observed variables. Measurement work is thus required to empirically evaluate the model and the explanations proposed within it. However, since the model is deliberately simple, this measurement effort is likely to still leave substantial features of the data to be explained by unmeasured factors. Exploring more elaborate settings—such as the ones considered in the works cited in the Introduction—may help relate fundamental parameters to observations and lead the way to a sharper identification of the forces driving the demographic transition. Future research should pursue these leads with the systematic quantitative approach adopted in this paper.

APPENDIX A

Data Description

Figure 1. CBR and CDR for Sweden 1740–1984. Official registrations in Chesnais (1992, Tables A1.1–A1.7).

Figure 2. CBR and CDR for England 1740–1984. Period 1740–1849, Wrigley and Schofield (1981, Table A3.3). Period 1850–1984, official registrations in Chesnais (1992, Tables A1.3–A1.7).

Figure 3. CBR and CDR for France 1740–1984. Official registrations in Chesnais (1992, Tables A1.1–A1.7).

Table I. 35-year annual averages 1740–1984 for NRI. Sources as for Figs. 1–3.

Table II. Three sources are used. First, productivity figures from Maddison (1991) in his Tables C11 and C12 and 3.1 and 3.3. For England, the figures are adjusted for 1700, 1780, and 1820 as in Hansen and Prescott (1998). Second, the series for the log of Swedish wages used to construct Fig. 3 in Eckstein *et al.* (1999), $\log(w(1))$. Third, OECD data on real GDP growth 1988/1999 and changes in employment at http://www.oecd.org/publications/figures/2000/english.

For England, productivity values have been interpolated to Maddison's series in order to match the dates in Table II of the main text. Then average productivities have been calculated for each period. From there, average growth rates of increase are calculated for each of the first six periods in the table. For the period 1950–1984, one would ideally want a measure of average productivity for 1985–2020. This is unavailable, and I use instead a weighted average of the UK 1988–1999 productivity growth rate calculated from the OECD source and the 1973–1987 growth of productivity calculated from Maddison (1991).

For Sweden, Maddison (1991) does not provide early data, and I use Eckstein *et al.* (1999) to construct the entries in Table II up until the period 1845–1879. The procedure is the same as the one used for England. For 1880–1914, I have two figures. One, 1.595, is obtained from Eckstein *et al.* (1999) as just described, and the other, 1.819, comes from Maddison (1991) as described for the case of England. For comparison across countries, the number in Table II is Maddison's. The entry for 1915–1949 is similarly constructed from the data in Maddison (1991). The figure for 1950–1984 uses the OECD source as in the case of England.

For France, I have not found early data. The figures displayed are constructed just as for England.

TABLE VI
Survival Probability between Ages 1 and 40 (π) and Probability That Non-survivors Die Past Age 15 (γ)

Period	π	γ
	Sweden	
1778–1782	0.607	0.509
1783–1812	0.614	0.568
1813–1847	0.678	0.608
1848–1882	0.698	0.546
1883–1917	0.774	0.624
1918–1952	0.881	0.700
1953–1965	0.967	0.756
	England	
1740–1779	*0.641*	*0.627*
1780–1809	*0.678*	*0.650*
1861–1881	0.673	0.604
1891–1911	0.772	0.577
1921–1947	0.884	0.687
1950–1963	0.965	0.776
	France	
1851–1878	0.628	0.599
1879–1913	0.727	0.651
1920–1947	0.846	0.744
1949–1965	0.953	0.795

Survival Probabilities

The analysis in Sections 5 and 6 requires direct information on survival probabilities of agents within different age groups. In particular, life tables provide the data that permits one to construct the probability of survival between ages 1 and 40 for the corresponding cohort (π in the model's notation). The probability that a member of the cohort dies after age 15, conditional on dying between ages 1 and 40 (γ in the model's notation), can also be calculated. The series that can be constructed from available sources are summarized in Table VI. Figures in *italics* for England indicate entries that were constructed with indirect information as documented next.

I will now describe how Table VI has been constructed. One source are values for the variable l_x (number of surviving to exact age x out of 100,000 born) for $x = 1, 15, 40$ in Keyfit and Flieger (1968) for Sweden, England, and France. Denote by $\pi(x\text{–}y)$ the survival probability between ages x and y. Then $\pi(0\text{–}1)$, $\pi(0\text{–}15)$, and $\pi(0\text{–}40)$ can be constructed from the life tables. In this notation, $\pi(1\text{–}40)$ corresponds to the model's π and can be calculated as

$$\pi \equiv \pi(1\text{–}40) = \frac{\pi(0\text{–}40)}{\pi(0\text{–}1)}.$$

Similarly,

$$\gamma \equiv \frac{1-\pi(15\text{–}40)}{1-\pi(1\text{–}40)} = \left(1-\frac{\pi(0\text{–}40)}{\pi(0\text{–}15)}\right)\frac{1}{1-\pi(1\text{–}40)}.$$

The figures are averaged over the periods represented and across sexes.

For England, this source only provides data from 1861 onward. To reconstruct early data, the periods 1740–1779 and 1780–1809 are based on the Wrigley *et al.* (1997, Table 6.1, p. 215, and Table 6.19, p. 290) figures for ${}_nq_x$, which is the probability of dying for an individual of exact age x, before reaching age $x+n$. These provide ${}_1q_0$, ${}_4q_1$, ${}_5q_5$, ${}_5q_{10}$, ${}_5q_{25}$, ${}_5q_{30}$, and ${}_5q_{35}$, for 1740–1749, 1750–1759, 1760–1769, 1770–1779, 1780–1789, 1790–1799, and 1800–1809, respectively. But figures for mortality rates between ages 15 and 25 (${}_{10}q_{15}$) are missing. Thus, ${}_5q_{15}$ and ${}_5q_{20}$ will be calculated as a proportion of ${}_1q_0$, the proportion being the average factor of proportion observed in periods 1861, 1871, and 1881 in the Keyfit–Flieger (1968) life tables: ${}_5q_{15}/{}_1q_0 = 0.191497919$ and ${}_5q_{20}/{}_1q_0 = 0.24447499$.

The figures from England used to draw Fig. 6 include interpolated values for the missing observations in period 3 (1810–1844). These interpolated values are 0.675 and 0.630 for π and γ, respectively.

To construct Fig. 11, I need to make some assumptions on French survival rates as well for periods 1–3. There are clear indications—such as existing data on CDR's and infant survival rates—that, in the early periods, France was lagging behind its neighbors. Let us then suppose that France had a path of survival probabilities π that remained below the benchmark between periods 1 and 3. Tentatively, I set $\pi_1 = 0.6$, $\pi_2 = 0.605$, and $\pi_3 = 0.621$ instead of the Swedish benchmark 0.607, 0.614, and 0.678, respectively. For lack of better information, I set γ as in the benchmark economy. For periods 4–7, Table VI provides information directly.

APPENDIX B

Computation of the Transition

This appendix briefly outlines the procedure to compute the transitions when the pre-transitional steady state is disturbed by changes at time T. In the paper's timing, $T = 1$. I will assume the economy is initially in a steady state with constant π_i, x_i, η_{bi}, γ_i, $\hat{k}_i$, and $\hat{c}_i$. I consider that at time T the path of these parameters changes and these changes are known as of T. Note that the new values of x_T, γ_T, and π_T are known at time T before they are realized at time $T+1$. There is perfect foresight about the future course of events. The only aspect that has to be worked out is how the economy is set on a new saddle path after the initial changes. It is instructive to summarize the equilibrium given by Eqs. (15)–(17) as a second-order difference equation $\Lambda(\cdots|\cdots)=0$. For example, at time t, and under perfect foresight, $\hat{k}_{t+1}$ is determined by

$$\Lambda\big(\hat{k}_{t-1}, \hat{k}_t, \hat{k}_{t+1} | \pi_{t-1}, \gamma_{t-1}, x_{t-1}, \eta_t, \eta_{t-1}, \pi_t, x_t\big) = 0, \tag{B1}$$

and consumption is determined by

$$\hat{c}_{t+1} = \hat{c}\big(\hat{k}_t, \hat{k}_{t+1} | \pi_t, x_t, \eta_t\big). \tag{B2}$$

With this, the procedure is as follows:

1. At time $t = T-1$, $\hat{k}_{T-1} = \hat{k}_i$, $\hat{c}_{T-1} = \hat{c}_i$. Then, by (B1) and (B2), $\hat{k}_T = \hat{k}_i$ and $\hat{c}_T = \hat{c}_i$.

2. At time $t = T$, using again (B1) and (B2), $\hat{k}_{T+1} \neq \hat{k}_i$ and is determined by

$$\Lambda\big(\hat{k}_{T-1}, \hat{k}_T, \hat{k}_{T+1} | \pi_{T-1}, \gamma_{T-1}, x_{T-1}, \eta_T, \eta_{T-1}, \pi_T, x_T\big) = 0.$$

Correspondingly,

$$\hat{c}_{T+1} = \hat{c}\big(\hat{k}_T, \hat{k}_{T+1} | \pi_T, x_T, \eta_T\big) \neq \hat{c}_i.$$

3. At time $t = T+1$, since the initial $\hat{k}_{T+1} \neq \hat{k}_i$, the value $\hat{k}_{T+2}$ must be the one that places the economy on its new stable saddle path. The next step shows how this is done. Consumption $\hat{c}_{T+2}$ is then determined by (B2).

4. Choose a $\hat{k}_{T+2}$. At time $t = T+2$, find $\hat{k}_{T+3}$ through (B1) by solving

$$\Lambda\big(\hat{k}_{T+1}, \hat{k}_{T+2}, \hat{k}_{T+3} | \pi_{T+1}, \gamma_{T+1}, x_{T+1}, \eta_{T+2}, \eta_{T+1}, \pi_{T+2}, x_{T+2}\big) = 0.$$

Iterate from $t = T+3$ onward using (B1). If the path $\hat{k}_t$ converges to the steady state, then this is the equilibrium. Otherwise, a new value for $\hat{k}_{T+2}$ must be selected and the iteration repeated.

The procedure to implement step 4 consists of specifying a large time horizon so that the economy must have reached the steady state by then. The chosen path is the one that gets $\hat{k}$ close to the steady-state value within an accuracy criterion. To speed up computations, 20 periods were selected first. The paths reported in the paper are not sensitive to increases in the number of periods.

The solution for $\hat{k}_{t+1}$ in each period through the non-linear equation implied by (B1) uses a Newton–Raphson algorithm. The choice of the rule to select the initial values that lead to the solution of this equation has proved to be an issue.

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