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Longer lives, fertility, and accumulation

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Abstract

In the neoclassical growth model with dynastic households, a reduction of mortality may lead to a steady-state with higher income and lower fertility and population growth rates. This requires that reductions in mortality have a sufficiently larger relative impact at younger ages.

Keywords: Neoclassical growth model; Fertility; Mortality; Demographic transition

JEL classification: J1; O0

1. Introduction

An exogenous decline in mortality rates is considered within a neoclassical growth model with purely dynastic preferences. The main finding is that such a change can trigger a process of rising productivity and reduced fertility. This is in contrast to the previous studies of Barro and Becker (1989) and Becker et al. (1990) which, within the same type of setting, argue the opposite effect must follow. This paper clarifies the apparent conflict by identifying the age-distribution of mortality changes as an important determinant of their demographic and economic consequences. A necessary condition for the result is that the reductions in mortality are sufficiently greater at younger ages.

The literature on the effects of mortality changes has departed from the assumptions of purely dynastic preferences and/or neoclassical technology (Ehrlich and Lui, 1991; Barro and Sala-i-Martin, 1995; Dahan and Tsiddon, 1985; Kalemli-Ozcan et al., 2000; Zhang and Zhang, 2001) or have taken fertility as exogenous (Boucekkine et al., 1999, 2001).

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2. The model

2.1. Demographics

The agents in the economy can live for up to two periods: youth and adulthood. The economy is populated by a continuum of households, each consisting of (possibly) an adult and her heirs. At time $t-1$ a number of children N_{t-1} are born. A member of generation $t-1$ survives to period t with probability π_{t-1}. The adult population growth factor between two consecutive generations is thus $g_t \equiv (\pi_t N_t)/(\pi_{t-1}N_{t-1})$. A member of generation $t-1$ plans to have born n_t children by the end of her first life-time period $t-1$. If she survives to adulthood, with probability π_{t-1}, then the plan will be effectively implemented. However, if she dies before completing adulthood, which occurs with probability $1-\pi_{t-1}$, the fertility plan will be carried out with probability γ. This probability is intended to characterize the age distribution of mortality rates: the higher γ, the higher the incidence on the older. It follows that $N_t = N_{t-1}n_t[\pi_{t-1} + \gamma(1-\pi_{t-1})]$. As a consequence, the adult population growth factor between t and $t+1$ equals:

$$g_t = \frac{\pi_t}{\pi_{t-1}} n_t[\pi_{t-1} + \gamma(1-\pi_{t-1})] \tag{1}$$

When mortality and planned fertility have constant values, the growth rate of total population is the same as that of adult population. Note the direct impact of changes in π on population growth is negatively related to γ.

2.2. Technology

A single sector produces final output. Capital K_t and labor L_t are the inputs used to produce total output through a Cobb–Douglas neoclassical production function $F(K_t, L_t) = K_t^{\theta} L_t^{1-\theta}$. Output produced at t can be used for consumption C_t, for accumulation of next-period capital K_{t+1} or for producing children. Capital depreciates at the rate $\delta < 1$. Each birth is assumed to imply a goods-cost. This cost amounts to η_t for every born child and is assumed to depend on the economy-wide level of capital per worker. As in Barro and Sala-i-Martin (1995), I consider the following simple linear specification:

$$\eta_t = \eta(\hat{k}_t) = \eta_0 + \eta_1 \hat{k}_t, \quad \eta_0 \geq 0, \eta_1 > 0,$$

with $\hat{k}_t$ being capital per-worker.

2.3. Households

The utility of an individual of cohort $t-1$, V_{t-1}, depends on own adult consumption, c_t, and on the utility of children born, with decreasing utility to the number of children:

$$V_{t-1} = E\left[c_t^{\sigma} + \beta n_t^{1-\epsilon} V_t | t-1\right], \quad \sigma \in (0,1), \;\; \epsilon < 1, \;\; \beta < 1$$

An agent of generation $t-1$ receives at $t-1$ a claim on the amount of wealth k_t from her parent. In the first period of her life, this wealth is invested in a portfolio of one-period assets. If she survives to period t, then she receives the returns from her portfolio and labor income and spends the revenues on own consumption of the single homogeneous consumption good, child-rearing, and wealth accumulation in the form of *voluntary* bequests. If the individual dies before t and leaves descendants, the portfolio returns is devoted to cover the rearing cost of her heirs, $n_t\eta_t$, and to provide *accidental* bequests. I will assume that there are competitive insurance markets. With actuarially fair prices, agents will fully insure bequests, k_{t+1}, against the mortality risk and the budget constraint can be written:

$$c_t = w_t + R_t \frac{k_t}{\pi_{t-1}} - \left(\frac{\eta_t}{\pi_t} + \frac{k_{t+1}}{\pi_t} \right) \frac{\pi_t}{\pi_{t-1}} n_t(\pi_{t-1} + (1-\pi_{t-1})\,\gamma)$$

where R_t is the return on riskless bonds, and w_t is the wage rate paid on the one unit of labor supplied.[1] Households have perfect foresight and take as given the paths for the wage rate, asset returns and the per-child cost. Since there is full insurance, the household seeks to solve the following recursive problem:

$$V_{t-1}(k_t) = \max_{n_t, k_{t+1}} \left\{ \pi_{t-1} c_t^{\sigma} + \beta n_t^{1-\epsilon} (\pi_{t-1} + (1-\pi_{t-1})\,\gamma)\, V_t(k_{t+1}) \right\}$$

the maximization being subject to the constraint above. It can be shown that the solution is characterized by a version of the standard Euler equation for intergenerational transfers, $n_t c_t^{\sigma-1} = \beta n_t^{1-\epsilon} R_{t+1} c_{t+1}^{\sigma-1}$, and a condition for fertility, $[R_{t+1}\eta_t + \pi_t(c_{t+1} - w_{t+1})]\,\sigma c_{t+1}^{\sigma-1} = (1-\epsilon)\,\pi_t c_{t+1}^{\sigma}$, which balances the marginal cost (LHS) and the marginal dynastic utility (RHS) of children.

3. Equilibrium

Given $F(.,.)$, $\eta(.)$, δ, β, σ, ϵ, γ, $\{\pi_t\}_{t=0}^{\infty}$ and $\hat{k}_0$, a *competitive equilibrium* consists of sequences of quantities for capital per worker $\hat{k}_t$, consumption per worker c_t, labor-force growth g_t, and for prices w_t, r_t, R_t for $t = 0,1,2,\ldots$ such that: taking r_t and w_t as given firms maximize profits; taking R_t and w_t as given, households maximize their dynastic utility; returns on capital and bonds are equalized; all markets clear. Equivalently, one could have replaced g_t by n_t by virtue of Eq. (1).

Market clearing implies that total wealth equals the aggregate capital stock. Since all households are assumed to be identical, $K_t = N_{t-1} k_t$. Each adult provides one unit of labor so that $L_t = N_{t-1}\pi_{t-1}$. Then $\hat{k}_t = k_t/\pi_{t-1}$. Output per unit of labor is then $f(\hat{k}_t) \equiv F(\hat{k}_t, 1)$. The no-arbitrage condition implies that $R_t = 1 - \delta + r_t$. Maximization by competitive firms leads to $r_t = f'(\hat{k}_t)$ and $w_t = f(\hat{k}_t) - \hat{k}_t f'(\hat{k}_t)$. Thus we can represent equilibrium prices as functions of $\hat{k}$, $w(\hat{k})$ and $r(\hat{k})$. I will focus on steady-state equilibrium situations with constant mortality rates, population growth, prices, and per-worker variables. A steady-state is described by the following set of equations in $\hat{k}$, $\hat{c}$, and g:

$$f(\hat{k}) + (1-\delta)\,\hat{k} = c + \left(\hat{k} + \frac{\eta(\hat{k})}{\pi} \right) g \tag{2}$$

$$c = \frac{\sigma}{1-\epsilon-\sigma} \left[(1 - \delta + r(\hat{k}))\, \frac{\eta(\hat{k})}{\pi} - w(\hat{k}) \right] \tag{3}$$

[1] Details on this and other claims throughout can be found in the working paper Mateos-Planas (1998).

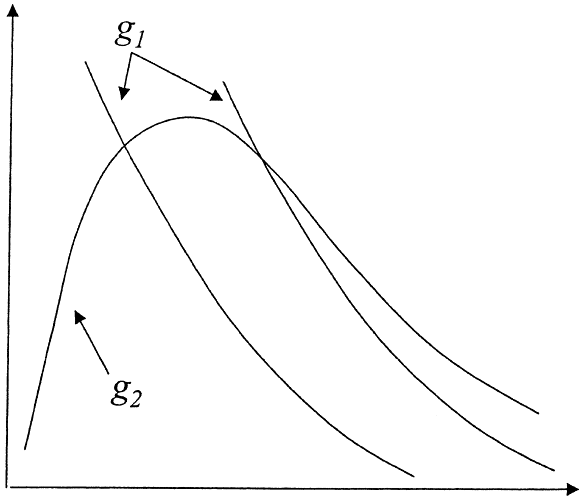

Fig. 1. Steady-state: g (vertical axis) and $\hat{k}$ (horizontal axis).

$$g = (\pi + \gamma(1-\pi))\left[\beta(1-\delta + r(\hat{k}))\right]^{1/\epsilon} \tag{4}$$

plus the condition that dynastic utility be bounded $g < 1 - \delta + r(\hat{k})$. Under the functional forms assumed, $\eta(\hat{k}) = \eta_0 + \eta_1 \hat{k}$, $f(\hat{k}) = \hat{k}^{\theta}$, $r(\hat{k}) = \theta \hat{k}^{\theta-1}$, and $w(\hat{k}) = (1-\theta)\hat{k}^{\theta}$. Eq. (2) is clearing in output market or, equivalently, the household's budget constraint; Eq. (3) and (4) correspond to optimal fertility and capital transfers, respectively. This equilibrium is similar to the one studied in Barro and Becker (1989). The key difference is the presence of the life-uncertainty terms, π and γ. Another, inconsequential, difference is the specification of the child-cost as a goods cost only.

Graphically, an equilibrium can be regarded as the intersection of two curves in the $g - \hat{k}$ space. The $g_1(\hat{k})$ schedule is provided by Eq. (4). The function $g_1(\hat{k})$ is monotonically decreasing as a higher interest rate, which increases the return to investing in children, must be matched by an increase in their cost through larger fertility. The second locus, $g_2(\hat{k})$, is given by Eq. (2) with consumption as determined by Eq. (3). This curve is in general hump-shaped, reflecting the substitution and income effects on fertility choices.[2] A steady-state occurs at a $\hat{k}$ such that $g_1(\hat{k}) = g_2(\hat{k})$ and $g_1(\hat{k}) < 1 - \delta + r(\hat{k})$. Fig. 1 shows some typical settings. None of the numerical setups considered in this research has been found to have more than two steady-states. Concerning stability, the roots of the linearized second-order system have been investigated numerically. For all the economies considered in this research, the condition $g_2'(\hat{k}) - g_1'(\hat{k}) > 0$ characterizes saddle-path stable steady-states.[3] This property will be used to establish the following result.

4. Results

Proposition 1. *Suppose a local increase in the probability of survival π occurs in an initial stable steady-state (i.e. $g_2'(\hat{k}) - g_1'(\hat{k}) > 0$).*

[2] If $\eta_0 = 0$ then g_2 is monotonically decreasing. If $\eta_1 = 0$ then g_2 is monotonically increasing.

[3] This is also consistent with the conjecture in Barro and Becker (1989).

(a) *If* $\gamma = 1$ *then* $\hat{k}$ *falls. If* $\gamma < 1$ *the effect is ambiguous. In particular, if* $\gamma = 0$ *a sufficient condition for* $\hat{k}$ *to increase is that*:

$$\frac{\sigma}{1-\epsilon-\sigma}\,\frac{\frac{\eta_0}{\hat{k}}+\eta_1}{\pi} < \frac{\pi(\beta(1-\delta))^{1/\epsilon}}{1-\delta}$$

(b) *That* $\hat{k}$ *increases is necessary for* g *to fall.*

Proof. (a) Using implicit differentiation and the stability condition, $\mathrm{sgn}(\mathrm{d}\hat{k}/\mathrm{d}\pi) = \mathrm{sgn}\big(\mathrm{d}g_1(\hat{k})/\mathrm{d}\pi - \mathrm{d}g_2(\hat{k})/\mathrm{d}\pi\big)$. At the steady-state, $\mathrm{d}g_1(\hat{k})/\mathrm{d}\pi = (1-\gamma)\, g/(\pi + \gamma(1-\pi))$ and $\mathrm{d}g_2(\hat{k})/\mathrm{d}\pi = \big(g\eta(\hat{k})\,\pi^{-2} - \mathrm{d}c/\mathrm{d}\pi\big)\big(\hat{k} + \eta(\hat{k})/\pi\big)^{-1}$. Since, by Eq. (3), $\mathrm{d}c/\mathrm{d}\pi < 0$, $\mathrm{d}g_2(\hat{k})/\mathrm{d}\pi > 0$. Thus, if $\gamma = 1$ it follows that $\mathrm{d}\hat{k}/\mathrm{d}\pi < 0$. If $\gamma = 0$, using that $\mathrm{d}c/\mathrm{d}\pi = -(\sigma/(1-\epsilon-\sigma))\big(1-\delta + r(\hat{k})\,\eta(\hat{k})\,\pi^{-2}\big)$, it follows that $\mathrm{sgn}\big(\mathrm{d}g_1(\hat{k})/\mathrm{d}\pi - \mathrm{d}g_2(\hat{k})/\mathrm{d}\pi\big) = \mathrm{sgn}\big(g\pi^{-1}\hat{k} - (\sigma/(1-\epsilon-\sigma))\big(1-\delta + r(\hat{k})\,\eta(\hat{k})\,\pi^{-2}\big)$. The sufficient condition uses the fact that $g_1(\hat{k})/\big(1-\delta + r(\hat{k})\big)$ decreases with $\hat{k}$ and has a lower bound given by $\lim_{\hat{k}\to\infty}\big[g_1(\hat{k})/(1-\delta+r(\hat{k}))\big] = \big[\pi(\beta(1-\delta))^{1/\epsilon}\big]/[1-\delta]$. (b) One can calculate $\mathrm{d}g/\mathrm{d}\pi = \big(\mathrm{d}g_1(\hat{k})/\mathrm{d}\hat{k}\big)\big(\mathrm{d}\hat{k}/\mathrm{d}\pi\big) + \mathrm{d}g_1(\hat{k})/\mathrm{d}\pi$. Clearly, if $\mathrm{d}\hat{k}/\mathrm{d}\pi < 0$ then $\mathrm{d}g/\mathrm{d}\pi > 0$. Q.E.D.

Graphically, when $\gamma = 1$ a higher π shifts the $g_2(\hat{k})$ curve upwards as higher survival lowers the perceived cost of rearing children. On a stable steady-state with $g_2'(\hat{k}) > g_1'(\hat{k})$, a fall in $\hat{k}$ must follow. This conforms exactly the argument in Barro and Becker (1989). When $\gamma < 1$, however, the other curve $g_1(\hat{k})$ also shifts upward since a larger π means a higher population growth rate on impact. On a stable steady-state, the substitution of quality for quantity of children is favorable to a positive adjustment of $\hat{k}$ following a drop in mortality. The net effect depends on the relative scale of these two opposing forces. The condition in Proposition 1 shows that for some parameters and a small enough γ the response of $\hat{k}$ will be positive. Part (b) establishes that this is necessary for a fall in population growth g, but by no means sufficient. Graphically, another necessary condition for lower g is that the slope of $g_2(\hat{k})$ be negative at the steady-state. Intuitively, in this region the substitution effect of a rise in capital on fertility decisions dominates the income effect. That g can effectively decrease with π is demonstrated with a numerical example. Parameter values are: $\theta = 0.75$, $\delta = 0.3$, $\beta = 0.9$, $\epsilon = 0.4$, $\sigma = 0.1$, $\eta_0 = 0.01$, $\eta_1 = 0.2$, and $\gamma = 0.0$. An increase in π from 0.5 to 0.7 is considered to occur. The table below displays stable steady-state values of capital per-worker, population growth factor, and planned fertility, as well as the local comparative-statics change in population growth to a local positive increase in π.

π	$\hat{k}$	g	n	$\mathrm{d}g/\mathrm{d}\pi$
0.5	0.4534	1.2715	2.5430	0.13
0.7	1.4173	1.2195	1.7422	−0.5

Finally, if rises in survival rates are greater at young ages, they should be accompanied also by a shift in the age-distribution of mortality rates that increases γ. The following establishes the consequences of this.

Proposition 2. *Suppose a local increase in γ occurs on an initial stable steady-state. (a) $\hat{k}$ increases unambiguously. (b) Planned fertility n declines. g declines if and only if $g_2'(\hat{k}) < 0$.*

Proof. (a) Graphically, $g_1(\hat{k})$ shifts upward. (b) Then, note that, with Eq. (1), Eq. (4) implies $n = (\beta(1 - \delta + r(\hat{k}))^{1/\epsilon}$. Q.E.D.

5. Concluding remarks

Ehrlich and Lui (1991) found that, in a cross-section of countries, growth is negatively related to mortality, and particularly to mortality rates at young ages. This is interpreted as evidence against the model with purely dynastic preferences. This paper shows that the dynastic model cannot be dismissed on these grounds alone. Mateos-Planas (2002) assesses quantitatively a model similar to the one used in this paper against data on early demographic transitions.

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