

SKILL BIAS AND EMPLOYMENT FRICTIONS IN THE U.S. LABOR MARKET 1970–1990*

BY XAVIER CUADRAS-MORATÓ AND XAVIER MATEOS-PLANAS¹

Universitat Pompeu Fabra, Spain; University of Southampton, U.K.

This article studies simultaneous changes in four labor market variables: the unemployment rates for college and high-school graduates, the education wage premium, and the level of college participation. It develops an equilibrium search and matching model of the labor market where education is endogenously determined. Then the model is used to investigate quantitatively whether the change in the above labor market variables from 1970 to 1990 in the United States can be traced to changes in the environment. A skill-biased change in technology together with an increase in employment frictions can explain much of the observed variation in these variables.

1. INTRODUCTION

In the United States, over the period spanned by the years 1970 and 1990, unemployment rates for both high-school graduates and college graduates nearly doubled, the wage differential (or premium) between these two education groups widened considerably, and, at the same time, the increase in college participation among high-school graduates accelerated markedly.² Table 1 reports figures on these four variables for the male population in 1970 and 1990.³ This article addresses the following questions: Which factors have caused the changes in these

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² It is well known, though, that the increase in the wage premium was nonmonotonic. In the 1970s the premium actually decreased. This article will not deal with this particular issue.

³ This choice is not completely arbitrary since the two years correspond to the same phase of the cycle in terms of unemployment. For unemployment and education, the figures are calculated from the Statistical Abstracts of the United States, U.S. Census Bureau (1995), Tables 662 and 629, respectively. The unemployment rates refer to male civilian noninstitutional population aged 25–64. The college participation of the male labor force aged 25 and older is the percentage with 4 or more years of college divided by the percentage with 4 or more years of high school. The wage premium is the average wage of college white male workers over the average wage for high-school white male workers aged 18–64 as reported in Murphy and Welch (1992) for the years 1969 and 1989 (see also Katz and Murphy, 1992, and Card and Di Nardo, 2002).

TABLE 1
U.S. LABOR MARKET 1970–1990

Year	College Unemployment Rate	High-School Unemployment Rate	Wage Premium	College Participation
1970	1.1	2.4	1.44	0.25
1990	2.1	5.3	1.58	0.33

four variables? Is the change experienced by any one variable connected with the changes in the other variables, or is instead the observed association largely accidental?

To seek an answer to these questions, the article proceeds in two steps. In the first step, it sets up a simple theoretical model where the four key labor-market variables—unemployment rates for college and high-school graduates, wage premium, and college participation rate—are jointly determined. This is a significant contribution to the existing literature since the relation between endogenous wage inequality, unemployment, and education has not yet been articulated. In the second step, the model is used to investigate whether the U.S. changes documented in Table 1 can be traced to certain plausible changes in the environment. It is found that a skill-biased shift in technology that improves the relative productivity of the skilled workers together with a general increase in the level of employment frictions in the labor market—for example, a rise in the rate of job separation—can explain nearly all of the variation in noncollege unemployment, college participation, and the wage premium, and over half of the rise in the educated unemployment rate. In this explanation, the two shocks prove to be important for the wage premium, unemployment, and education; hence the joint analysis of these variables is warranted.

In previous studies, either of the two types of disturbances considered here has been emphasized to analyze a particular subset, but not all, of the four variables under study. For example, there seems to be compelling evidence—such as Krusell et al. (2000)—of a skill-biased change in technology that may have driven the rise in wage inequality. On the other hand, changes in frictions have been considered in the existing literature as an explanation for rising unemployment associated with more turbulent economic conditions.⁴ The present article considers the two factors jointly to explain a broader set of variables that, besides the wage premium, includes also college participation and education-specific unemployment rates.

To meet its goals, this article proposes and analyzes an equilibrium matching-search model of a labor market that is segmented in two education levels. Workers

⁴ Works that study one or more of these factors to explain one or more, but not all, of the variables of interest are the following. Skill-biased change: Acemoglu (1999), Mortensen and Pissarides (1999), Saint-Paul (1996), Caselli (1999), Albrecht and Vroman (2002), Shi (2002), Wong (2003). Friction shock: Ljungqvist and Sargent (1998), Marimon and Zilibotti (1999).

are differentiated by innate productive skills and take forward-looking decisions about acquiring education and which labor-market segment (or career) to participate in. Firms decide which education segment to participate in, and which types of workers to hire taking into account the skill composition of the labor force in the different segments. Workers and firms interact in this economy in an otherwise standard framework with search and matching frictions to determine wages and vacancy/unemployment ratios in the two segments.

This model brings two key features into the analysis. The first is a labor market that is segmented. It presupposes that a worker's education is observable *ex ante* and that firms post job vacancies that specify the minimal attainment required on prospective candidates to filling the post. More specifically, vacancies in the "educated" segment require a degree and vacancies in the "noneducated" segment do not. Segmentation means that both vacant firms and unemployed workers confine their search efforts to the particular segment they choose, not both. The second key feature is the assumption that there is an imperfect correlation between skill and education status. That is, the model recognizes that some of the workers in possession of a degree may have a low skill, whereas some workers without a degree may have a high skill.⁵ Because of the imperfect skill–education correlation, by altering the skill composition of the population by education level and the labor force participating in the different segments, endogenous changes in education and career (or segment) choices may have implications for equilibrium unemployment and wages across education groups.

Numerical experiments are conducted to study quantitatively the joint effects in the model of a skill-biased shock in technology and a shock to employment frictions. The latter will be modeled as either a higher rate of job separation or a lower efficiency in the matching process. The benchmark parameters are set to match some long-run observations for the U.S. economy and the 1970 values of key endogenous variables. The two exogenous shocks are measured to match the observed changes in the wage premium and the unemployment rate for noncollege workers between 1970 and 1990. In this measurement exercise the changes in education are taken as exogenous. The measured shocks are then used to produce steady-state equilibrium outcomes for the entire range of labor market variables of interest, including now the endogenous education choices, which are to be compared with the 1990 observations in the data.

The findings are as follows. The two measured shocks in the technology skill bias and the employment frictions explain all of the variation in the noncollege unemployment rate and the wage premium, about three quarters of the rise in college participation, and over half of the rise in the college unemployment rate. The skill-biased change in technology is responsible for part of the 1970–1990 increases in the wage premium, the noneducated unemployment rate, and college education, but fails to bring about any increase in the educated unemployment

⁵ This requires that a person's skill is subject to influences other than college education. Haveman and Wolf (1995), for example, report that achievement is highly correlated with parents' income and education. Thanks to one of the referees for pointing this out.

rate. On the other hand, the friction shock accounts for part of the variation in all the U.S. labor market variables under study, including the wage premium. A substantial fraction of the increases in the wage premium and unemployment rates caused by the two shocks occurs through the endogenous changes in education.⁶ This is because the shifts in college participation choices lead to a change in the composition of the labor force that improves the average skill of educated workers and deteriorates that of noneducated workers. In sum, in this analysis unemployment rates, wage inequality, and education are closely related endogenous variables. The integrated approach adopted in this article uncovers the nature of these connections.

A number of papers have also studied the implications of shocks for the distribution of wages and unemployment in a search-matching setup. The literature dealing with the wage premium and unemployment includes Mortensen and Pissarides (1999), Acemoglu (1999), and Saint-Paul (1996). Albrecht and Vroman (2002) study in addition residual inequality. Hornstein et al. (2002) focus on residual inequality along with general unemployment. Shi (2002) and Wong (2003) restrict their attention to the study of the wage premium and residual inequality. The present article differs from these works in a number of aspects. First, here education is endogenized and thus the merit of different explanations can be evaluated against their implications for changes in educational attainment. In the rest of papers, education changes are either treated parametrically or simply absent. The second difference is that those works dealing with the wage premium treat education as equivalent to skill. In the current article, the imperfect correlation between education and skill has an important role and is determined as part of the equilibrium outcomes through the education and career choices of workers with different characteristics. These choices are absent from the papers cited. Third, and related to the previous point, the present model assumes that the labor market is segmented in terms of jobs with different observable education requirements. Technically, there is a matching function for each segment. This is also a feature of Saint-Paul (1996). The assumption in Mortensen and Pissarides (1999) is similar but, there, a different segment is associated with each different productivity-skill level, and outcomes in each segment are determined independently of changes that affect other segments such as shifts in the skill composition of the labor force. This is why that paper does not explain rising unemployment among the educated workers, which will precisely be one of the subjects of the present article. Shi (2002) has a model of directed search where the endogenous nature of the matching process leads to segmentation. In Acemoglu (1999), Albrecht and Vroman (2002), and Wong (2003) there is no segmentation in the sense of the present article, so education is not used to sort applicants into job categories through differentiated

⁶ Some authors, like Acemoglu (1999), consider that key determinants of college enrollment in the early 1970s were mainly the baby boom and the Vietnam war. The data in Topel (1997, Figure 4) suggests a strong correspondence between the college/high school wage premium and the enrollment ratio as the fraction of men 20–24 with some college for the period 1963–1995. The present article sets aside the factors emphasized by Acemoglu (1999) and focuses instead on the possibilities of an explanation consistent with Topel's (1997) findings.

matching processes.⁷ Finally, the present article uses data on several variables to evaluate the implications of changes in both the technology skill bias and the unemployment conditions. The rest of studies do not analyze the friction shock. Changes in the skill supply and/or the general level of technology not studied in this article have also been considered in the other papers. Hornstein et al. (2002) turn their attention instead to the effects of independently measurable embodied technological change in a model with an endogenous vintage capital structure. It can be viewed as a noteworthy step toward a more fundamental identification of the ultimate source of one of the changes under study here. Only Mortensen and Pissarides (1999), Hornstein et al. (2002), and Wong (2003) share with the present article a quantitative approach to evaluating the implications of the theory.

The rest of the article is organized as follows. Section 2 presents the model and basic behavior relations. Section 3 characterizes the equilibrium. Section 4 describes the baseline calibration. Section 5 reports the results of the numerical analysis. Section 6 concludes the paper with an overview of results and final remarks.

2. THE MODEL

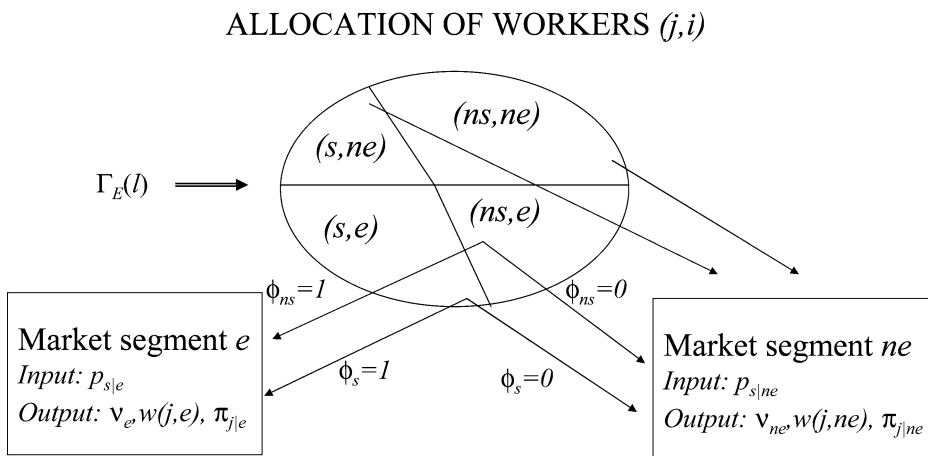
Agents in the model consist of workers and firms. Workers can be either acquiring education or actively participating in the labor market. An active worker's type is defined by two characteristics: skill (indexed by j) and education (indexed by i). An active worker can be skilled ($j = s$) or nonskilled ($j = ns$), and educated ($i = e$) or noneducated ($i = ne$). Active workers can be employed or unemployed and searching for a job. Firms can be either posting one job vacancy and searching or producing output with one worker. Workers and firms that are searching meet through a matching process. Firms observe the education status of a worker at any time but only observe her skill level after being matched. It is assumed that a posted vacancy must be directed to workers of a specific education group (further details below).⁸ More specifically, a firm can be of one of two types, educated or noneducated $i \in \{e, ne\}$, according to whether it targets educated workers or not. This implies that the labor market is segmented by education requirements, with a matching process taking place in each segment separately. The rest of this section describes in detail the model and the decision problems faced by workers and firms, and characterizes the optimal choices. Attention is restricted to stationary situations with constant decision rules.

⁷ In the equilibria studied in the present article, all firms within any single segment will be of the same type. This rules out situations with two types of firms catering for workers with different skill level in the same segment, which is precisely the type of situation that corresponds to a *separating* equilibrium in Acemoglu's (1999) model of a single-segment market. The present article assumes instead two segments and only considers *pooling* equilibria within each segment. This seems consistent with the focus of the article on intergroup differences, whereas the approach in Acemoglu (1999) looks more appropriate to study within-group differences. Similar comments carry over to Albrecht and Vroman (2002) and Wong (2003).

⁸ Saint-Paul (1996) makes a similar assumption.

2.1. *Workers.* There is a continuum of workers who are born with an innate type attribute, l , uniformly distributed on the interval $[0, 1]$. The type is distributed so that, ordering agents by increasing l , the measure h_0 at the lower end are born skilled or high-ability (i.e., if $l < h_0$, then $j = s$). The remaining fraction $1 - h_0$ are born nonskilled. Workers may acquire education. A worker who is born nonskilled (i.e., with $l \geq h_0$) may become skilled with probability μ if she acquires education. Workers who are born skilled cannot improve their skills through education. There are two components to the cost of acquiring education. There is a time requirement, T_e , which is the length of the period needed to become educated, and a component in terms of disutility, $c_e(l)$, which is increasing in the worker's type l . Thus skilled-born workers find it less costly (or more rewarding) to get education. This disutility term $c_e(l)$ can have a negative value, in which case education is interpreted to bring positive utility. Workers face a constant probability of leaving the labor force, ρ . When a worker leaves, another worker of exactly the same type replaces her.

The timing of events for a worker is as follows. First, she decides whether to acquire education. Second, the worker enters the labor force and decides which segment (or career) of the market, $i \in \{e, ne\}$, to participate in. A noneducated worker only searches for jobs in the noneducated segment. This follows because, as described below in Subsection 2.2, firms posting vacancies in the educated segment will consider contacts only with workers holding proof of education. Educated workers can instead search in either segment. Let $\phi_j \in \{0, 1\}$ represent the decision of an educated worker with skill j whether to participate (value 1) or not in the educated segment, so $1 - \phi_j$ is her decision to participate in the noneducated segment. Figure 1 is introduced to visualize the type of configurations that may arise.



The circle represents the composition of the total labor force in terms of workers' types. Given the distribution of skills before education, h_0 , the size of the different slices is entirely determined by the education choices. The bottom and top sections on the circle account for the skill composition of the educated and noneducated groups of population, respectively. Education and skill are imperfectly correlated when the sections labeled (s, ne) and (ns, e) do exist. The boxes represent the two market segments, and the arrows show the possible assignments of the labor force of different types to the market segments. The career decision, ϕ_j , determines the direction of this flow for the educated workers in the two bottom sections of the circle. Thus education (the slices in the circle) and career decisions (the arrows) will jointly determine the skill composition of the labor force that participates in each segment, p_{ji} : the proportion of workers in segment $i \in \{e, ne\}$ with skill $j \in \{s, ns\}$. The skill composition of the educated population (i.e., the relative size of the two bottom slices of the circle) and $p_{j|e}$ will coincide if $\phi_s = \phi_{ns} = 1$. Figure 1 will be repeatedly used and fully explained throughout the rest of the article.

Third, the worker starts searching for a job. Workers and firms are matched randomly. The probability that a worker searching in segment i makes contact with a suitable firm is v_i . The value of being unemployed and searching to a worker with skill j and seeking employment in segment i is denoted by $U(j, i)$. In a steady state, one can argue that the Bellman values are not indexed by the worker's education type since the worker, even if educated, will never want to exercise the option to switch segment at a later date. Fourth, on contact with a firm, the skill of the worker j is disclosed. The unemployed worker must agree with the vacant firm on whether to create the job or continue searching. The decision of the firm whether to hire the worker is denoted by the indicator $\pi_{ji} \in \{0, 1\}$, with value 1 if the decision is positive. If the job is created, the wage to the worker is $w(j, i)$, and the value of the match is $W(j, i)$. The job is terminated exogeneously with a Poisson probability λ . In this event, the agent becomes unemployed and searches for a new job. There is a flow value to the unemployed worker that depends on the wage:

$$(1) \quad b(w(j, i)) = b_0 + b_1 w(j, i)$$

The coefficient on the wage can be interpreted as the unemployment benefit replacement rate. The fixed component may include the value of leisure.

The worker seeks to maximize the expected present value of utility minus the utility component of the education cost, c_e , and discounts the future at the constant rate r . The instantaneous utility is given by the value of consumption. Free borrowing and lending is assumed so that in equilibrium, the interest rate equals r and hence the worker maximizes the present value of wages plus unemployment compensation. With the notation introduced, the Bellman equation for the value of a job to a worker with skill j and matched with a firm in segment i is

$$(2) \quad (r + \rho)W(j, i) = w(j, i) + \lambda(U(j, i) - W(j, i))$$

The value of unemployment for a worker with skill j in segment i is

$$(3) \quad (r + \rho)U(j, i) = b(w(j, i)) + v_i \pi_{ji} \max\{W(j, i) - U(j, i), 0\}$$

for $i = e, ne$ and $j = s, ns$. Concerning career (or segment) choices, for a noneducated worker the only segment available is $i = ne$. The choice of segment by an educated worker with skill j can be represented by

$$(4) \quad \phi_j = \begin{cases} 1 & U(j, e) - U(j, ne) > 0 \\ 0 & \text{otherwise} \end{cases}$$

A worker of type $l \in [0, 1]$ decides to go to college if

$$(5a) \quad \Gamma_E(l) > 0$$

where, assuming the specification for the disutility of education

$$(5b) \quad c_e(l) = c_e l - \bar{c}_e, \quad c_e > 0$$

$$(5c) \quad \Gamma_E(l) \equiv \begin{cases} e^{-(r+\rho)T_e} [\phi_s U(s, e) + (1 - \phi_s)U(s, ne) - c_e(l)] \\ \quad - U(s, ne) & l \in [0, h_0] \\ e^{-(r+\rho)T_e} \{\mu[\phi_s U(s, e) + (1 - \phi_s)U(s, ne)] \\ \quad + (1 - \mu)[\phi_{ns} U(ns, e) + (1 - \phi_{ns})U(ns, ne)] - c_e(l)\} \\ \quad - U(ns, ne) & l \in (h_0, 1] \end{cases}$$

Returning to Figure 1, one can now outline the interactions that in the complete model will be key to understanding the determination of the variables object of this article: education-specific unemployment rates and wages, and educational attainment. The boxes show that, once the skill composition of the labor force in a particular segment i , p_{ji} , has been determined, the hiring policy of firms π_{ji} , the wage structure $w(j, i)$, and the probability of contact for an unemployed worker v_i will all be fully determined within the segment. The direct implications of these segment-specific variables for the wages and unemployment rates across education groups will depend on the skill and career composition of these groups (determined by Γ_E and the ϕ_{js} , respectively). On the other hand, when deciding whether to educate and which segment to join, a worker compares the returns to the different options, which will be largely governed by the segment-specific variables in the different segments. Shifts in education and career choices have in turn two types of implications for education-specific wages and unemployment rates. The first is a *direct* effect: by altering p_{ji} and thereby the segment-specific outputs as just described. The second is a *composition* effect: changes in the relative proportion of agents with different skill and career within each education group for given segment-specific outputs. The rest of this section lays out the remaining

elements of the model that will be used to assess the role of these interacting mechanisms.

2.2. *Firms.* The timing of events is as follows. First, an inactive firm creates a job vacancy that specifies the education requirement on the worker sought, i , or the market segment. When making this choice, it is assumed that a firm in the educated segment commits to hiring only educated workers.⁹ The value of such a vacancy is $V(i)$. A vacancy is posted and there is contact with a suitable job seeker with probability ξ_i . The skill status, j , of the worker met in segment i is not observed by the firm at this stage. The firm holds instead a rational belief about the probability that a matched worker in that segment has skill of type j . This coincides with the equilibrium fraction of workers with skill j within the pool of unemployed workers participating in the market segment i , z_{ji} . Posting a vacancy has a flow recruiting cost c_R .

Second, on contact, the firm observes the worker's skill, and the firm and the worker agree on whether to create the job. As before, $\pi_{ji} \in \{0, 1\}$ denotes the decision by the firm of type i whether to hire a worker with skill j .

Third, the firm starts operating and the flow of output it receives is $y(j, i)$. The value of the existing job match for the firm is $J(j, i)$. The match terminates as the consequence of an exogenous job failure, which occurs with Poisson probability λ . The job can also break down by the worker leaving the labor force, which occurs at rate ρ . When the job is terminated, the firm will seek to open a new vacancy type i of the highest value. Like the worker, the firm discounts future values at the constant interest rate r . Formally, the value of a job of type (j, i) obeys the Bellman equation

$$(6) \quad rJ(j, i) = y(j, i) + (\lambda + \rho) \left[\max_{i' \in \{e, ne\}} V(i') - J(j, i) \right]$$

and the value of a vacancy of type i satisfies

$$(7a) \quad rV(i) = -c_R + \xi_i \sum_{j=s, ns} z_{ji} \pi_{ji} [J(j, i) - V(i)]$$

$$(7b) \quad \pi_{ji} = \begin{cases} 0 & J(j, i) - V(i) \leq 0 \\ 1 & J(j, i) - V(i) > 0 \end{cases}$$

⁹ One concern is that there might be situations where an "educated" firm would be happy to create a job with a "noneducated" worker if they were to meet. Under the present assumption, such profitable deviations never take place since noneducated workers will never search among educated vacancies. Firms commit not to hire any worker without proof of a degree so any such workers are dissuaded from trying the educated segment in the first place. Thanks to one referee for suggesting this interpretation. This rests on a strong form of ex ante commitment on hiring policies though. To the same effect, an alternative assumption would be the existence of a technology that prevents noneducated workers from ever meeting educated vacancies (e.g., a degree is an entry ticket into the pool where educated vacancies look for workers). This would involve a rational decision to adopt this technology when posting a vacancy in the educated segment.

2.3. *Technology.* The flow of output to a match depends on the worker's productivity. Let η_j denote the productivity of a worker with skill j and assume that $\eta_s > \eta_{ns}$. The output of a match involving a worker with skill j equals the value of the gross income flows

$$(8) \quad \eta_j = w(j, i) + y(j, i)$$

There is a homogenous-of-degree-one matching function that gives the number of matches per period in segment i , $m_i = m(v_i, u_i)$, where v_i is the mass of vacant firms and u_i is the number of unemployed workers in this segment. This matching technology is specified as

$$m(v, u) = m_0 v^{1-\theta} u^\theta, \quad \theta \in [0, 1]$$

and $m_0 > 0$ characterizes the efficiency of the matching process. Then the probabilities of contact that are relevant to firms and workers are

$$(9) \quad \begin{aligned} \xi_i &= m_i/v_i = \xi(v_i/u_i) = m_0(v_i/u_i)^{-\theta} \\ v_i &= m_i/u_i = v(v_i/u_i) = m_0(v_i/u_i)^{1-\theta} \end{aligned}$$

so $\xi'(\cdot) < 0$ and $v'(\cdot) > 0$.

2.4. *Bargaining, Free-Entry, and Skills of the Unemployed.* The wage is determined at each instant of time through bargaining over the surplus of a match between the firm and the worker that have agreed to create a job. The solution to the corresponding generalized Nash bargaining problem is

$$w(j, i) = \arg \max \{ \beta \log S_W(j, i) + (1 - \beta) \log S_F(j, i) \}$$

where S_W and S_F represent the match surplus to the worker and the firm, respectively, and β represents the workers' bargaining power. For the worker, $S_W(j, i) \equiv W(j, i) - U(j, i)$. For the firm, $S_F(j, i) \equiv J(j, i) - V(i)$. Using Equations (1)–(3) and (6)–(7), the necessary first-order condition for this problem is

$$(10) \quad \frac{1 - \beta}{\beta(1 - b_1)} (W(j, i) - U(j, i)) = J(j, i) - V(i)$$

There is free entry in vacancies, which leads to the exhaustion of pure rents from vacancy creation in both segments $i \in \{e, ne\}$:

$$(11) \quad V(i) = 0$$

The firms in a segment i take as given the skill composition of the pool of unemployed workers from which matches are drawn, z_{ji} . This depends on the skill composition of the labor force in this segment, p_{ji} , and the forces determining

unemployment, including matching and hiring rates, v_i and $\pi_{j|i}$. Supposing that workers are always willing to accept the job offers made by firms, then in a steady state, the equalization of the flows in and out of employment will lead to the following expression (see Section A of the appendix):

$$(12) \quad z_{s|i} = 1 - z_{ns|i} = p_{s|i} \left[\frac{v_i \pi_{s|i} + \lambda + \rho}{v_i \pi_{ns|i} + \lambda + \rho} (1 - p_{s|i}) + p_{s|i} \right]^{-1}$$

3. EQUILIBRIUM

An equilibrium is a situation consistent with Equations (1)–(11) above and, then, also Equation (12).¹⁰ Given the technology described in (8) and (9), the fundamental endogenous variables in this system are, for $i = e, ne$ and $j = s, ns$, the market tightness v_i/u_i (or, by (11), the contact probability v_i), wages $w(j, i)$, hiring decisions $\pi_{j|i}$, labor participation (or career) choices ϕ_j , and the distribution of the value of education $\Gamma_E(l)$ on $l \in [0, 1]$. These are just the variables displayed in Figure 1. As indicated in that figure, the significance of ϕ_j and $\Gamma_E(l)$ for the outcomes of any segment i will be conveniently summarized by the skill composition of the labor force $p_{j|i}$ in that segment.

The focus of the analysis will be restricted to equilibrium situations where both the educated and the noneducated labor-market segments are operative, and where the proportion of skilled workers is higher among the labor force that participates in the educated segment (i.e., $p_{s|e} > p_{s|ne}$). The latter is intuitive and convenient since it restricts the type of career choices that may arise in equilibrium. In effect, the skilled workers that are educated must decide to participate in the educated segment, or $\phi_s = 1$. Otherwise, existence of an active educated segment would require the nonskilled educated workers to be the only participants in the educated segment (see again Figure 1). But then nonskilled agents would be most numerous in the educated segment, which is inconsistent with the required condition on the $p_{j|i}$ s. Thus variation in labor-force participation (or career) will occur only through the choice by the educated nonskilled workers, for which the shorter notation $\phi \equiv \phi_{ns}$ is introduced.

3.1. Exogenous Skill Composition. This section studies first the equilibrium in each segment when the skill composition is exogenous. When the distribution of skills, $p_{j|i}$, is given, then an equilibrium determines the variables v_i/u_i (or, by (11), v_i), $w(j, i)$, and $\pi_{j|i}$ for $i = e, ne$ and $j = s, ns$. These are the outputs inside the boxes in Figure 1. They can be characterized by a number of relations. The derivations are in Section A of the appendix.

One type of relation comes from developing the bargaining condition (10). This is a version of the *job-destruction* curve in Mortensen and Pissarides (1994), which

¹⁰ The decision of a worker whether to accept an offer in (3) is dominated by the hiring decision of the firm $\pi_{j|i}$ in (7) since (10) must hold. Thus, unemployed workers accept equilibrium job offers made by firms and (12) holds.

gives the wage as a function of the worker's skill, the segment's tightness and the firm's hiring decisions. It reads as follows:

$$(13a) \quad w(j, i) = \omega \left(\eta_j, \Omega \left(\frac{v_i}{u_i}, \pi_{j|i} \right) \right) \equiv \frac{b_0 + \Omega(v_i/u_i, \pi_{j|i})(\eta_j - b_0)}{1 - b_1 + \Omega(v_i/u_i, \pi_{j|i})b_1}$$

where the effect of tightness on bargaining outcomes is in the term

$$(13b) \quad \Omega \left(\frac{v_i}{u_i}, \pi_{j|i} \right) \equiv \frac{\beta(1 - b_1)(r + \rho + \lambda + v_i(v_i/u_i)\pi_{j|i})}{\beta(1 - b_1)(r + \rho + \lambda + v_i(v_i/u_i)\pi_{j|i}) + (r + \rho + \lambda)(1 - \beta)}$$

Equation (13) traces out a positive relation between the wage and the equilibrium market tightness. The interpretation is as follows. A higher probability of meeting a vacancy for the worker, v_i , means that the outside option of a job is also higher. Hence the wage has to be also higher to keep the worker into the job. Since v_i depends positively of market tightness, the positive relation between v_i/u_i and $w(j, i)$ follows.

The other type of relation comes from developing the free-entry condition (11), using (13) to replace the wage terms. This delivers the following:

$$(14a) \quad \xi(v_i/u_i) \sum_{j=s,ns} z_{j|i} \pi_{j|i} \Psi(\eta_j, v_i/u_i, \pi_{j|i}) - c_R = 0$$

$$(14b) \quad \pi_{j|i} = \begin{cases} 0 & \Psi(\eta_j, v_i/u_i, 1) < 0 \\ 1 & \text{otherwise} \end{cases}$$

where the firm's net-profit term is defined as

$$(14c) \quad \Psi(\eta_j, v_i/u_i, \pi_{j|i}) \equiv \frac{1}{r + \lambda + \rho} [\eta_j - \omega(\eta_j, \Omega(v_i/u_i, \pi_{j|i}))]$$

with $\omega(\cdot)$ as defined in (13), and using (12) with (9),

$$(14d) \quad z_{s|i} = 1 - z_{ns|i} = p_{s|i} \left[\frac{v(v_i/u_i)\pi_{s|i} + \lambda + \rho}{v(v_i/u_i)\pi_{ns|i} + \lambda + \rho} (1 - p_{s|i}) + p_{s|i} \right]^{-1}$$

This expression is associated with the idea of *job creation*. A higher probability of contacting a worker for the firm, ξ_i , increases the expected profits for a given allocation of output between the worker and the firm. Then free-entry would drive the wages (i.e., the terms $\omega(\cdot)$) upward so as to restore the zero value of creating vacancies. Since ξ_i depends negatively on market tightness, a negative relation between v_i/u_i and wages follows for each $j = s, ns$. However, by (13), the wages themselves depend on market tightness, and the term $\Psi(\cdot, \cdot, \cdot)$ in (14c) picks up this dependence. For each i , the equilibrium can then be expressed as a market tightness v_i/u_i that satisfies (14a). It must be consistent with the firm's hiring policy

$\pi_{j|i}$ in (14b), which characterizes the choice of the firm whether to create the job when contacting an unemployed worker with skill j . There is the possibility that a match with a particular skill j is not profitable and the job is not created, but observe that (14a) requires that at least for one j , the job is created. Changes in the composition of the labor force $p_{s|i}$ enter this condition through changes in the probabilities that, for example, an unemployed worker is skilled in the educated segment, $z_{s|e}$, in (14d). Since the educated and the noneducated sectors are both operative, any firm must be willing to hire at least the skilled workers so that one can set $\pi_{s|e} = \pi_{s|ne} = 1$. In principle, the nonskilled workers may be hired in either sector, both sectors, or no sector. The following proposition makes this characterization more precise.

PROPOSITION. *Assume that $p_{j|i}$ is given. Consider the following conditions:*

- (C1) $\Psi(\eta_s, 0, 1) > 0$
- (C2) $\Psi(\eta_{ns}, 0, 1) < 0$
- (C3) $\xi((v_i/u_i)^*) p_{s|i} \Psi(\eta_s, (v_i/u_i)^*, 1) - c_R < 0$
- (C4) $\xi((v_i/u_i)^*) z_{s|i}^* \Psi(\eta_s, (v_i/u_i)^*, 1) - c_R > 0$

where $(v_i/u_i)^*$ is such that $\Psi(\eta_{ns}, (v_i/u_i)^*, 1) = 0$ and $z_{s|i}^*$ is as in (14d) with $v_i/u_i = (v_i/u_i)^*$, $\pi_{s|i} = 1$, and $\pi_{ns|i} = 0$. The following holds:

- (a) *In equilibrium, $\pi_{s|i} = 1$ and the pair $(v_i/u_i, \pi_{ns|i})$ is unique.*
- (b) *Assume $p_{s|i} \in (0, 1)$. Then C1 is a necessary condition for existence. This condition is also sufficient if $p_{s|i} = 1$.*
- (c) *Assume $p_{s|i} \in (0, 1)$ and C1. An equilibrium does not exist if and only if none of conditions C2, C3, and C4 hold. If C2 holds, then the equilibrium has $\pi_{ns|i} = 0$. If C3 holds, then C4 does not and the equilibrium has $\pi_{ns|i} = 1$. If C3 does not hold and C4 does, the equilibrium has $\pi_{ns|i} = 0$.*
- (d) *Assume $p_{s|i} = 0$. That C2 does not hold is a necessary and sufficient condition for existence with $\pi_{ns|i} = 1$.*

The meaning and justification of the proposition is best conveyed by using Figure 2. It represents the left-hand side of Equation (14a) as a decreasing function of v_i/u_i , on account of (14b)–(14c). Thus an equilibrium must be unique (point *a*). Also existence requires that the curve lie above the horizontal zero line for low values of the market tightness. Formally, given the limiting properties of the function $\xi(\cdot)$, a necessary condition for existence is that firms be willing to hire at least skilled workers when tightness is very low (point *b*).

The left-hand side of Equation (14a) has a discontinuity at the value $(v_i/u_i)^*$ of market tightness where, according to (14b), $\pi_{ns|i}$ shifts from 1 to 0 or, in other words, the value at which it is no longer profitable for firms in segment i to hire nonskilled workers. At this point, the probability of meeting a skilled worker in the pool of unemployed, $z_{s|i}$ in (14d), drops because all the nonskilled workers

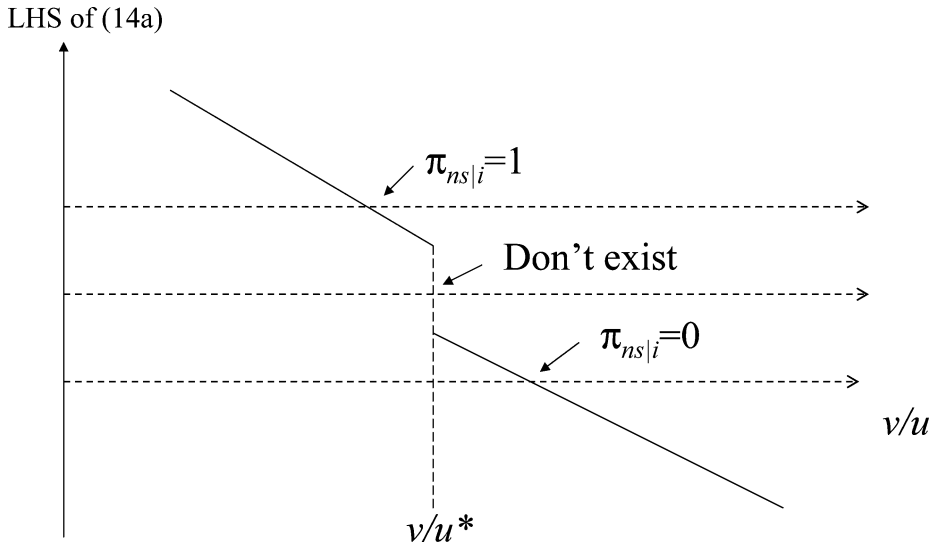


FIGURE 2
EQUILIBRIUM

in this segment become unemployed. An equilibrium may not exist owing to this discontinuity (first part of point *c*). When there is a zero of the left-hand side of (14a) to the right of $(v_i/u_i)^*$, nonskilled workers are not hired; otherwise both skill types are hired (second part of point *c*). If all the labor force are nonskilled workers, there is no issue of discontinuity (point *d*).

This characterization can also be used to derive comparative statics results. For the purpose of the present article, it is important to retain that v_i/u_i and, through (13), $w(j, i)$ increase with $p_{s|i}$ as the curve in Figure 2 shifts upward. Intuitively, a higher average quality of the workers leads to more vacancy creation and hence better employment chances for an unemployed worker in the segment. At the same time, this places employed workers in a stronger position to negotiate a higher wage.

3.2. *Endogenous Skill Composition.* The skill composition $p_{s|i}$ has been taken as given so far. In equilibrium, as indicated by the arrows in Figure 1, it will depend on career and education decisions of workers. The career choice in equilibrium is determined by (4), with the equilibrium values of searching in alternative segments determined by

$$(15) \quad (r + \rho)U(j, i) = b(w(j, i)) + v(v_i/u_i)\pi_{j|i} \frac{w(j, i) - b(w(j, i))}{r + \rho + \lambda + v(v_i/u_i)\pi_{j|i}}$$

where the equality follows from (2) and (3). Note that the equilibrium condition that the career choice of skilled educated workers must be to participate in the

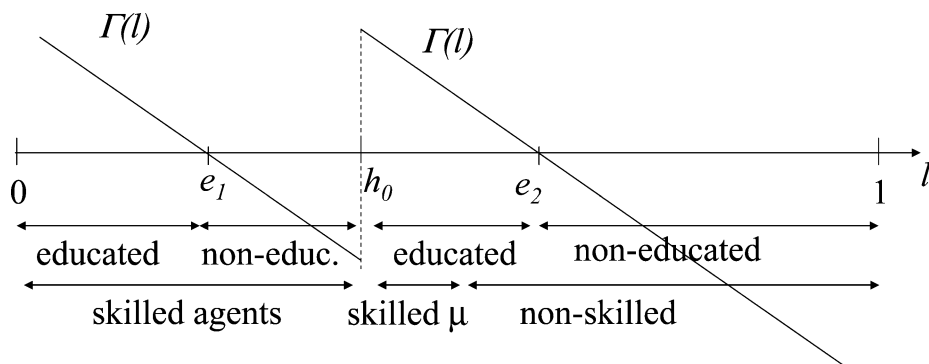


FIGURE 3

DISTRIBUTION OF TYPES

educated segment (i.e., $\phi_s = 1$) requires that $U(s, e) > U(s, ne)$. In general, one has to account for the possibility that two equilibria exist for given education, one with $\phi = \phi_{ns} = 0$ and the other with $\phi = \phi_{ns} = 1$.

Endogenizing education choices will complete the characterization of the equilibrium. An equilibrium requires that the education decisions satisfy (5). The assumption in (5c) implies that the curve $\Gamma_E(l)$ is piecewise decreasing in the regions of values for l below and above h_0 . Therefore, the distribution of education over the labor force can be characterized by two numbers, $e_1 \in [0, h_0]$ and $e_2 \in (h_0, 1]$, such that the individuals with type $l \in [0, e_1] \cup [h_0, e_2]$ are educated. Figure 3 depicts a typical configuration.

Provided that in an equilibrium skilled workers that are educated will decide to participate in the educated segment, the distribution of skills within the labor force in each segment can be written explicitly as follows:

$$(16a) \quad p_{s|e} = \frac{e_1 + \mu(e_2 - h_0)}{e_1 + (e_2 - h_0)[\mu + \phi(1 - \mu)]}$$

and

$$(16b) \quad p_{s|ne} = \frac{h_0 - e_1}{h_0 - e_1 + 1 - e_2 + (e_2 - h_0)(1 - \phi)(1 - \mu)}$$

Education outcomes also determine the total measure of skilled workers, denoted h , as follows:

$$(16c) \quad h = h_0 + \mu(e_2 - h_0)$$

An equilibrium is a fixed point in the education outcome that can be visualized using Figure 1 again. A particular Γ_E —or, equivalently, (e_1, e_2) —yields equilibrium values for ϕ , and through $p_{s|i}$ s in (16), the $\{\pi_{j|i}, w(j, i), v_i\}_{i \in \{e, ne\}, j \in \{s, ns\}}$.

These values do in turn affect Γ_E and (e_1, e_2) . The equilibrium (e_1, e_2) must be consistent with the equilibrium it generates. By aggregating appropriately the segment-specific variables, such an equilibrium delivers implications for educational attainment and education-specific unemployment rates and wages. The *direct* and *composition* effects discussed at the end of Subsection 2.1 will be crucial to interpret these implications. Sections B and C in the appendix contain details on computation.

4. CALIBRATION

A model's time interval of length one is assumed to correspond to one quarter. The parameters to be determined are: $b_1, r, T_e, \rho, \lambda, \theta, \beta, \eta_s, \eta_{ns}, b_0, c_R, m_0$, and either $h, p_{s|e}$, and $p_{s|ne}$ if the skill distribution is exogenous, or \bar{c}_e, c_e, h_0 , and μ if the skill distribution is endogenous. The procedure adopted here is divided in two steps. First, the parameters are chosen assuming that education—and hence the skill distribution—is exogenous. In the second step, the parameters of the education technology, \bar{c}_e and c_e , and the original endowment of skill, h_0 , are calibrated so that the equilibrium outcomes with endogenous education are consistent with the values of $p_{s|e}, p_{s|ne}$, and h chosen in the first step. This will be done for a range of values for the the productivity of education, μ .

Seven of these parameters can be set directly. The choice of b_1 is consistent with an unemployment benefit (UI) replacement rate of 20% from OECD (1997). Stewart (2002) finds on March CPS data an average separations rate for males in 1970 of 23%. This includes employment-to-unemployment and employment-to-employment transitions. This is approximately 6% per quarter, the number used for λ in the calibration. A 5% annual interest rate in Cooley and Prescott (1995) implies the value 0.013 for r . A life expectancy in the labor market of 45 years implies $\rho = 1/(45 \times 4)$. The matching elasticity θ is set following the estimate in Blanchard and Diamond (1990). Four full years required for a college degree in Autor et al. (1998) determine T_e . Since the calibration procedure will target figures for unemployment rates, one can argue that the choice of either m_0 or c_R is a normalization and can fix m_0 .

To determine the remaining parameters with exogenous education, outcomes are restricted to be consistent with targets for the key endogenous variables corresponding to the year 1970. There are two blocks to this task. One block of the calibration procedure consists of matching the four targets for the wage premium, the unemployment rates of educated and noneducated workers, and the value of hiring cost as a proportion of wage income by choice of the four parameters η_s, b_0, β , and c_R . It is assumed that $\eta_s + \eta_{ns} = 3$, so η_s determines η_{ns} directly and changes in η_s can be regarded as skill-biased changes in technology. The targets for the wage premium and unemployment are taken from the 1970 data of Table 1. Concerning hiring costs, Hamermesh (1993) estimated that average hiring costs represent about 2% of the wage bill.

In the other block, the parameters $p_{s|e}, p_{s|ne}$, and h are calibrated to match the targets of educational attainment and measures of inequality within defined occupational categories that can be associated with the two segments in the

model.¹¹ The target for education is the 1970 figure for the proportion of college participation in Table 1. Concerning inequality within job categories, Gould (2002, Figure 1b) reports the variance of the OLS log-wage residuals from uniform March CPS data for white males, after controlling for years of schooling, experience, region of residence, marital status, and living in a standard metropolitan statistical area. These measures are provided for various years within three different occupation groups: professional sector, service sector, and blue collar sector.¹² In the calibration, it is assumed that the educated segment and the non-educated segment correspond to the professional sector and the blue-collar sector, respectively. For 1970, the residual variance of log wages for the professional and blue-collar sectors are 0.18 and 0.12, respectively. The target to match will be the 0.06 differential residual variance of educated over noneducated jobs in 1970.¹³ Since only the differential in residual variance is targeted, instead of the two absolute values, this block would leave one degree of freedom in the choice of h . To deal with this, the model has been calibrated for alternative values of h around the target for education 0.25. With a large value such as $h = 0.30$, it was found that the calibration procedure reaches a point where existence fails before meeting all the targets. With $h = 0.25$, the model can be calibrated, but then for small increases in η_s , an equilibrium fails to exist, which precludes conducting some of the experiments of interest. For h below 0.17, the difference in residual variance across groups is too big to match the corresponding target. The benchmark calibration features $h = 0.20$.

Table 2 displays the benchmark calibration with exogenous education and summarizes the procedure. All targets set out are matched exactly, with the educated and noneducated residual variances being 0.07 and 0.01.

The next step consists of calibrating the parameters of the cost of education and skill endowment, \bar{c}_e , c_e , and h_0 , so that the specific values for h , $p_{s|e}$, and $p_{s|ne}$ calibrated in Table 2 obtain as the outcome of endogenous education choices. This is done for a range of values for μ .¹⁴ The results along with the implied education choices e_1 and e_2 in each case are in Table 3.

5. NUMERICAL EXPERIMENTS

This section studies the response of unemployment rates, the wage premium, and educational attainment to shifts in various exogenous factors within the model.

¹¹ The education target, $e_1 + e_2 - h_0$, can be written in terms of these parameters as $1 - (h - p_{s|e}) / (p_{s|ne} - p_{s|e})$.

¹² The professional sector includes all workers in the professional, technical, managerial, and academic occupations; the service sector, all service workers as well as clerical and sales workers; the blue collar sector, all construction workers, craftsmen, machinists, operatives, and laborers.

¹³ Hence there must exist positive residual inequality in the educated segment, so the calibrated equilibrium must necessarily imply that $e_2 > h_0$ and $\phi = 1$. In the model, one can calculate the variance within a segment i as $(\log w(s, i) - \log w(ns, i))^2 p_{s|i} (1 - p_{s|i})$, where $p_{s|i}$ is the share of skilled in the segment.

¹⁴ Note that the 1970 calibration target $p_{s|e} = 0.66$ is the upper bound for μ . A value exceeding 0.66 would require $e_1 < 0$.

TABLE 2
CALIBRATION WITH EXOGENOUS EDUCATION

Parameter	Value	Target to Match	Source
b_1	0.2	UI replacement 20%	OECD (1997)
r	0.013	Annual interest 5%	Cooley et al. (1995)
T_e	16	Time in college 4 years	Autor et al. (1998)
ρ	0.0055	Working life 45 years	
λ	0.06	Annual separation rate 25%	Stewart (2002)
θ	0.5	Matching elast. 0.5	Blanchard and Diamond (1990)
m_0	1	Normalize to unity	
h	0.20	College participation 25%	U.S. Census Bureau (1995)
$p_{s e}$	0.6600	Residual ineq. diff. 0.06	Gould (2002)
$p_{s ne}$	0.0467		
$\eta_s = 3 - \eta_{ns}$	1.915	Wage premium 1.44	Murphy and Welch (1992)
b_0	0.775	Unemp. educated 1.1%	U.S. Census Bureau (1995)
β	0.15	Unemp. non-educated 2.4%	U.S. Census Bureau (1995)
c_R	0.10	Hiring costs 2%	Hamermesh (1993)

The purpose is to assess the role of these different factors in explaining the 1970–1990 changes in the U.S. labor market reported in Table 1. The two explanatory factors considered will be a skill-biased change in technology and a shock to the level of employment frictions. Changes in education decisions will constitute a third explanatory factor when taken as exogenous, but will become one of the endogenous variables otherwise. This section is divided in two parts accordingly.

The first part, which consists of Subsections 5.1, 5.2, and 5.3, takes education decisions as exogenous. The analysis builds on two steps. The first step (Subsection 5.1) considers the role of a skill-biased technological change, either alone or along with a change in education. This type of technical change has been commonly singled out as the main cause of rising wage premia. It will be shown, however, that this type of shock will not be able to account for the generalized rise in unemployment rates. Thus, other sources of shocks bearing on unemployment rates are needed to explain the complete set of facts. The second step (Subsection 5.2) intends precisely to measure the mix of employment friction shocks, skill-biased shocks, and education shifts that best matches the 1970–1990 observed changes in the endogenous variables. Since the consequences of given education choices

TABLE 3
CALIBRATION WITH ENDOGENOUS EDUCATION

μ	Parameters Calibrated			Education Implied	
	\bar{c}_e	c_e	h_0	e_1	e_2
0.00	42.100	79.698	0.2000	0.165	0.285
0.30	46.966	140.121	0.1636	0.129	0.285
0.50	40.718	147.098	0.1150	0.080	0.285
0.65	29.880	130.745	0.0421	0.007	0.285

TABLE 4
SKILL BIAS AND SKILLS DISTRIBUTION

	Parameters			Education Parameters			Endogenous Variables			
	η_s	$p_{s e}$	$p_{s ne}$	μ	e_1	e_2	$w p$	un_e	un_{ne}	edu
(1)	1.915	0.660	0.047				1.44	0.011	0.024	0.25
(2)							1.58	0.021	0.053	0.33
(3)	2.014	0.660	0.047				1.58	0.011	0.038	0.25
(4)	2.005	0.606	0.000	0.00	0.200	0.33	1.58	0.011	0.062	0.33
(5)	1.980	0.647	0.000	0.30	0.164	0.33	1.58	0.011	0.043	0.33
(6)	1.955	0.695	0.000	0.65	0.042	0.33	1.58	0.011	0.035	0.33

for unemployment and wages depend on the technology of education as characterized by the efficiency of education, μ , this exercise will also help measure this parameter. Finally, Subsection 5.3 studies separately the response of the economy to each of the shocks considered in the previous section. This exercise will permit us to insulate the effects of each shock and explain in detail the economic forces that drive the results.

The second part, which consists of Subsection 5.4, takes education decisions as endogenous and seeks to judge the model against its implications for the wage premium, education-specific unemployment rates, and educational attainment. To this end, it studies the response of these four endogenous variables to the skill-biased and friction shocks measured in the first part. The interest of this exercise is to assess the model's ability to produce an endogenous response of education consistent with the change observed in this variable. It will also disentangle the specific contribution of each shock to the changes in education.

5.1. Skill-Biased Technology and Exogenous Education. The skill-biased shock can be represented by a rise in the productivity of the skilled workers, η_s , and a corresponding decline in the productivity of the nonskilled workers, $\eta_{ns} = 3 - \eta_s$.¹⁵ An exogenous change in education can be represented by a shift in the values of the education critical points e_1 and e_2 for given μ or, equivalently, in the associated skill composition of the two segments, $p_{s|e}$ and $p_{s|ne}$.¹⁶ This section reports the effect of these shocks on the calibrated benchmark economy.

The main results of the experiments conducted are contained in Table 4. Each row shows first the value of the parameters η_s , $p_{s|e}$, $p_{s|ne}$, along with corresponding values for e_1 , e_2 , and μ . The rest of entries on each row contain the key endogenous variables: the wage premium $w p$, the unemployment rate of the educated un_e , the

¹⁵ Equivalently, one might postulate a widening gap with positive rises in both productivities, whereas the economy-wide parameters increase at an average rate that exceeds the rise in nonskilled productivity.

¹⁶ The nonskilled educated worker chooses invariably the educated segment (i.e., $\phi = 1$) throughout all the experiments of this article. Thus, (16) implies the correspondence between (μ, e_1, e_2) and $(p_{s|e}, p_{s|ne})$. Given the target for education, $edu \equiv e_1 + e_2 - h_0$, and using (16), the skill compositions of the two groups can be written as $p_{s|e} = [e_1 + \mu(e_2 - h_0)]/edu$ and $p_{s|ne} = (h_0 - e_1)/(1 - edu)$.

unemployment rate of the noneducated un_{ne} , and education attainment edu . The first row shows the figures corresponding to the benchmark 1970 equilibrium, with the empty cells corresponding to any of the combinations of μ , e_1 , and e_2 from Table 3. The second row reproduces the 1990 data on the observable variables to facilitate the comparison with the implications of the numerical experiments.

The first experiment focuses on the skill-biased shock only, holding education variables at their 1970 values. The shift in η_s from 1.915 to 2.014 has been calibrated so that, after the change, the model produces a wage premium comparable with the 1990 value of 1.58. The third row in Table 4 shows this new value and those of the endogenous variables. This skill-biased shock produces a rise of the unemployment rate for the noneducated from 0.024 to 0.038, which still falls short of the U.S. 1990 figure of 0.053. On the other hand, the unemployment rate for the educated does not undergo any visible change, thus failing to reproduce the observed rise.

The second type of experiment introduces changes in education along with the skill-biased shock. For a given μ (and its associated h_0 as calibrated in Table 3), the shifts in technology, η_s , and education, (e_1, e_2) , are calibrated to jointly match the 1990 observed wage premium and educational attainment. Since the education target can be written as $e_1 + e_2 - h_0$, there will generally be various pairs of the education parameters for which a valid calibration can be found.¹⁷ To narrow down the possibilities, the choices of parameters are restricted so that $e_1 = h_0$, which means that all the skilled-born individuals will be educated in 1990.¹⁸ The cases reported in the fourth to sixth rows of Table 4 approximately span the range of variation in the skill composition that various choices of μ and e_1 can produce. In the fourth row, the change in education damages the quality of both the educated group and the noneducated group of workers. The skill-biased shock required to match the wage premium is comparable to that in the third row for the case without changes in education. These changes have little impact on the unemployment rate of the educated, yet the unemployment rate of the noneducated rises substantially above the 1990 value. This follows as a response to the drop in the proportion of skills in the noneducated segment. The fifth and sixth rows increase the proportion of skills in the educated segment. The composition effect tends to raise the wage premium so that the skill-biased shock required now is smaller than in the fourth row. The unemployment rate among the educated hardly changes, whereas the unemployment rate of the noneducated increases by less.

The main message of this section can be summarized as follows. Consider a skill-biased shock in technology and a change in the education choices consistent with the observed rises in both the wage premium and educational attainment.

¹⁷ There are bounds to the choices available though. In effect, since a lower proportion of the skilled in the educated group, $p_{s|e}$, induces a reduction of the wage premium, matching the wage premium may require a large skill-biased shock. However, existence of an equilibrium may fail before meeting the target in the sense that (4) fails to hold (i.e., $U(s, e) < U(s, ne)$ or $U(ns, e) < U(ns, ne)$). This is the case when μ is small and e_1 is not chosen close enough to its upper limit h_0 . For a relatively larger μ , a calibration exists as long as e_1 is not much smaller than h_0 .

¹⁸ This restriction proves inconsequential for the main conclusions of Subsections 5.1, 5.2, and 5.3 where education is viewed as exogenous. When education will be endogenous in Subsection 5.4, the result will be consistent with this restriction.

Although it can produce a realistic upward shift in the noneducated unemployment rate, no such a combination of exogenous shocks can cause a visible rise in the educated unemployment rate. To explain the full set of facts, at least one new source of changes must be considered in addition to the technology skill bias and the schooling decisions.

5.2. *Adding a Shock to Employment Frictions.* The additional shock required will be generically referred to as a general labor-market (or employment) *friction* shock since it manifests itself to exactly the same degree in the two segments of the labor market. Changes in two parameters of the model that fit this description will be studied separately: the job separation rate λ , and the efficiency of the matching process m_0 .

This subsection and the next consider three exogenous sources of changes: the just introduced friction shock, along with the change in schooling and the skill-biased technology shock already studied in the previous subsection. The first task will be to measure these three shocks so that they jointly match the changes in three endogenous variables consisting of educational attainment, the wage premium, and the noneducated unemployment rate. As in the previous subsection, this is done for alternative admissible settings for μ and e_1 . These settings have different implications for the educated unemployment rate, the fourth and most elusive variable, which will be used to assess the model's explanatory power. As a second task, for the case that most closely matches the observations, the contribution of the various shocks to the changes in the different variables will be analyzed in the next subsection.

The first entries of a row in Table 5 correspond to the values of the parameters η_s , $p_{s|e}$, $p_{s|ne}$, m_0 , and λ along with corresponding e_1 , e_2 , and μ . The rest of the entries contain the key endogenous variables. The first and second rows are the same as in Table 4, with the addition of the calibrated values of m_0 and λ . The results of the experiments are displayed in the remainder of the table. Each of the rows (3)–(5) calibrates, for a given degree of education efficiency μ , the parameters η_s , $p_{s|e}$,

TABLE 5
SKILL BIAS, SKILLS DISTRIBUTION, AND EMPLOYMENT FRICTION

	Parameters					Education Parameters			Endogenous Variables			
	η_s	$p_{s e}$	$p_{s ne}$	m_0	λ	μ	e_1	e_2	wp	un_e	un_{ne}	edu
(1)	1.915	0.660	0.047	1.000	0.060				1.44	0.011	0.024	0.25
(2)									1.58	0.021	0.053	0.33
Friction parameter is λ :												
(3)	2.000	0.606	0.000	1.000	0.056	0.00	0.200	0.330	1.58	0.011	0.053	0.33
(4)	1.980	0.647	0.000	1.000	0.074	0.30	0.164	0.330	1.58	0.013	0.053	0.33
(5)	1.960	0.695	0.000	1.000	0.089	0.65	0.042	0.330	1.58	0.016	0.053	0.33
Friction parameter is m_0 :												
(6)	2.000	0.606	0.000	1.060	0.060	0.00	0.200	0.330	1.58	0.011	0.053	0.33
(7)	1.985	0.647	0.000	0.865	0.060	0.30	0.164	0.330	1.58	0.013	0.053	0.33
(8)	1.959	0.695	0.000	0.691	0.060	0.65	0.042	0.330	1.58	0.016	0.053	0.33

$p_{s|ne}$, and λ to match the 1990 targets for the wage premium, the noneducated unemployment rate and the education participation.¹⁹ As before, for each μ there is a degree of freedom in the value of e_1 , and the choice shown here corresponds to the one that leads to an educated unemployment rate closest to its 1990 target. This invariably implies that schooling of the born skilled, e_1 , must take on its highest feasible value, or $e_1 = h_0$. In the experiments reported in rows (6)–(8), it is m_0 instead of λ , the parameter that characterizes the friction shock but the results, to which the discussion turns next, are practically identical.

The ability of the model to explain the observed rise in the educated unemployment rate improves as higher values of the schooling efficiency parameter μ are considered. The reason is as follows. With a higher μ , the calibrated shift in schooling decisions has a larger impact on the share of skilled workers in the educated group $p_{s|e}$. For a large enough μ , this impact will in fact be positive (compare rows 5 and 8 against row 1). This causes a positive response of the wage premium through a composition effect of the type advanced toward the end of Subsection 2.1. Therefore, the larger μ the lower the measured skill-biased shock η_s needed to match the increase in the wage premium. However, along the lines of the discussion about Table 3 earlier, this will reduce the noneducated unemployment rate. Matching the latter thus calls for a larger friction shock, whether in the form of separation λ or mismatch m_0 . This causes the sought-after increase in the educated unemployment rate associated with a larger school efficiency μ . In quantitative terms, the measured shocks can explain up to about 50% of the observed rise in the educated unemployment rate. This is demonstrated in the fifth and eighth rows of Table 5, for λ and m_0 representing the unemployment shock, respectively.²⁰ This contrasts favorably with the previous experiments without friction shocks where the educated unemployment rate never rose. It is true nonetheless that this requires a high school efficiency parameter such as $\mu = 0.65$.

The remainder of the article will focus on one of the two sets of shocks associated with $\mu = 0.65$. The choice made here is the setting displayed in the fifth row of Table 5, where the friction shock consists of a change in the job separation rate λ . The alternative choice, shown in the eighth row, of m_0 as the friction-shock parameter would lead to virtually identical results. This choice about the friction shock can nonetheless be justified on empirical grounds. Although a skill-biased change in technology has been widely recognized as the main source of the changes in the U.S. wage structure, the evidence on the type of friction shocks dealt with here is less well known. On one hand, there is evidence that the job loss rate among men has increased over the period under investigation and about as much for college-educated and older workers as for less educated and less experienced workers (see Boisjoly et al., 1998; Monks and Pizer, 1998; Polsky, 1999; Valletta,

¹⁹ An outline of the calibration procedure is as follows: (i) Set a value for λ , (ii) set a value for η_s that matches the wage premium, (iii) check the noneducated unemployment rate, update λ , back to ii. As seen earlier, matching the schooling target simply requires e_1 and e_2 to satisfy $edu = e_1 + e_2 - h_0$.

²⁰ There is a skill composition for which the match is exact. Nonetheless, this would be inconsistent with the 1970 education parameters. In this article, variation in these parameters is assumed away.

TABLE 6
CONTRIBUTION OF EACH FACTOR

	1970	1990	Constant $p_{s i}$	Constant η_s	Constant λ
un_e	0.011	0.016	0.016	0.016	0.011
un_{ne}	0.024	0.053	0.041	0.041	0.036
wp	1.44	1.58	1.49	1.51	1.59
v_e/u_e	34.54	35.96	34.24	35.02	37.07
v_{ne}/u_{ne}	7.37	2.83	4.89	4.99	3.11
$w(s, e)$	1.85	1.87	1.87	1.83	1.89
$w(s, ne)$	1.78	1.70	1.75	1.72	1.77
$w(ns, e)$	1.08	1.03	1.03	1.07	1.04
$w(ns, ne)$	1.07	1.02	1.02	1.06	1.03

1999). This can be interpreted as the rise in λ considered in this article.²¹ On the other hand, the plausibility of a matching shock supporting a fall in the model's parameter m_0 is more tenuous. Although Blanchard and Diamond (1989) find evidence of a decline in the effectiveness of matching over the period 1968–1981, Bleakey and Fuhrer (1997) study the period 1979–1993 and find that the efficiency with which unemployed workers are matched with job vacancies has increased in the late 1980s and early 1990s. The net effect over the period 1970–1990 is far from clear.²²

5.3. How Does Each Shock Affect the Wage Premium and Unemployment?

This section studies the response of the economy to each shock separately. The objective is to identify their specific contribution to the changes in the different variables. The upper part of Table 6 displays the unemployment rates and the wage premium corresponding to different settings for the shock parameters. The bottom part of the table shows the values for the market tightness, v_i/u_i , in the two segments and the wage structure across segments and skill status. These variables will be useful in explaining the intuition behind the results along the lines of Figure 1. The first two columns show the 1970 and 1990 benchmark outcomes. The former consists of the 1970 calibration in Table 3, and the latter corresponds to the 1990 benchmark in the fifth row of Table 5. The experiments reported in each of the remaining columns of Table 6 hold one of the three shifting parameters at its 1970 level whereas the other two take on their measured 1990 values. The difference between the values of the endogenous variables thus obtained and the corresponding 1990 reference values in the second column will measure the contribution of the factor held fixed.

The third column holds the skill-composition parameters, $p_{s|e}$ and $p_{s|ne}$, at their 1970 values, and is to be compared with the second column. This comparison

²¹ Stewart (2002), however, finds that overall job separation rates changed very little over the period.

²² An alternative source of unemployment shock in the model would be a rise in b_0 (the fixed part of the unemployment benefit). However, this shock does not help explain the rise in the educated unemployment rate.

shows that the shift in schooling choices and in the associated skill composition contribute to the rise in the noneducated unemployment, as indicated by the positive difference between 0.053 and 0.041. This is accounted for by the fall in the noneducated segment market tightness from 4.89 to 2.83. The mechanism rests on the deterioration of the share of skilled workers in the noneducated segment $p_{s|ne}$ (from 0.047 to 0 as shown in the fifth row of Table 5) that follows directly from the change in education. The lower average productivity of workers reduces job profitability, vacancy creation, and thus raises unemployment as the market tightness declines. Regarding the educated unemployment rate, the improvement in the skill composition $p_{s|e}$ (from 0.660 to 0.695 as shown in Table 5) tends to reduce it by an analogous argument. This is reflected in the market tightness that increases from 34.24 to 35.96. This change in the educated unemployment rate is quantitatively negligible though. As for the wage premium, the positive difference between 1.58 and 1.49 points that the education shock contributes to the rise in this variable. There are two types of effects. First, there is a standard direct effect on wages since the rise in market tightness in the educated segment raises wages in this segment, whereas the fall in tightness in the noneducated segment lowers bargained wages there. Second, there is a composition effect because the proportion of high-wage skilled workers increases in the educated group, whereas it decreases in the noneducated group. Quantitatively, the direct effect on the wages, $w(j, i)$, is small and hence virtually all of the increase in the wage premium accounted for by education is explained by the composition effect.²³

The fourth column holds the skill-bias parameter, η_s , constant at its 1970 level, and is also to be compared with the second column. This shows that the skill-biased change in technology contributes to the rise in the noneducated unemployment rate, as indicated by the difference between 0.053 and 0.041. This is accounted for by the fall in the noneducated segment market tightness from 4.99 to 2.83. This is in turn explained by the relative reduction in the productivity of the nonskilled workers that reduces job vacancy creation in the nonskill-abundant noneducated segment. By an analogous argument, the skill-biased shock makes no contribution to the rise in the educated unemployment rate since it increases (from 35.02 to 35.96) rather than decreases the market tightness in the skill-abundant educated segment.²⁴ The fall in the educated unemployment rate is again negligible. Concerning the wage premium, the positive difference between 1.58 and 1.51 indicates that the skill-biased shock contributes to the rise in this variable. This follows from the direct impact of this shock on the wage structure. For the skill-abundant educated group, the combined effect of the increase in the wage of the skilled workers $w(s, e)$ outweighs the relative fall in the wage on the nonskilled workers $w(ns, e)$ and causes a net wage increase. For the nonskill-abundant noneducated group, the wage response is driven by the fall in the relevant wage $w(ns, ne)$.

²³ Note that, since in the 1990 benchmark $p_{s|ne} = 0$, the most sensitive wage entry $w(s, ne)$ does not enter the calculations.

²⁴ This result is reminiscent of the finding in Mortensen and Pissarides (1999) of a convex relation between productivity and unemployment, which explains why the skill-biased shock leads to a higher average unemployment rate. However, this does not explain why unemployment also rises among the skilled/educated, which is one subject of the present article.

The fifth column holds the friction parameter, λ , at its 1970 level. The comparison with the second column shows that the friction shock causes a considerable rise in the noneducated unemployment rate as indicated by the difference between 0.053 and 0.036, and fully accounts for the increase in the educated unemployment rate that the model can possibly explain as indicated by the difference between 0.016 and 0.011. The fall in market tightness in the two segments—from 37.07 to 35.96 in the educated segment and from 3.11 to 2.83 in the noneducated segment—encapsulates the underlying mechanism. These effects can be traced to the decline that this shock causes in job duration, which lowers job profitability and thus vacancy creation. On the other hand, this friction shock does not contribute to the wage premium. The displayed changes in the structure of $w(j, i)$ s do in effect indicate that the friction shock has a fairly uniform impact on wages throughout. The quantitative net affect is actually a slight decrease in the wage premium, as indicated by the difference between 1.59 and 1.58.

Summing up, all three exogenous factors are important in explaining the facts. The friction shock is the only significant source of variation of the unemployment rate for educated workers, and the shocks in education and the technology skill bias are necessary for the change in the wage premium. The three shocks contribute to the change in the noneducated unemployment rate.

5.4. Endogenous Education. The preceding analysis has established that the measured exogenous shift in education choices, along with the measured friction shock and the skill-biased technology shock, is a significant factor behind the 1970–1990 labor market changes in the United States. This section will instead treat the choice of education as endogenous and study its response to the same technology and friction shocks measured when education was exogenous. The objective is to assess the ability of the model to explain the changes in education together with the rest of the labor market variables.

The schooling choices, e_1 and e_2 , and hence the associated distribution of skills, $p_{s|e}$ and $p_{s|ne}$, are now determined in equilibrium given the values of the parameters \bar{c}_e , c_e , and h_0 calibrated to 1970 outcomes in the bottom row of Table 3 for the chosen $\mu = 0.65$. The exogenous shocks consist of the same shifts in the skill-bias parameter η_s and the separation rate λ considered in the previous sections, which are displayed in Table 5 as the change in their values between the first and fifth rows.

The upper section of Table 7 displays the unemployment rates, the wage premium, the level of education attainment $e_1 - h_0 + e_2$, and the associated schooling variables e_1 and e_2 corresponding to different settings. The bottom section provides the values of variables that help interpret the results. These include the skill composition $p_{s|i}$, and the option values of a working career across education and skill categories expressed in (15). The latter will be useful to understand schooling decisions according to (5). The first column shows the 1970 benchmark outcomes. The second column contains the response of the variables to the combined skill-biased and friction shocks. The third and fourth columns report the experiments that hold either exogenous parameter at its 1970 level, whereas the other parameter takes on its measured 1990 value.

TABLE 7
ENDOGENOUS EDUCATION

	1970	1990	Constant η_s	Constant λ
un_e	0.011	0.016	0.016	0.011
un_{ne}	0.024	0.053	0.039	0.032
w_p	1.44	1.58	1.49	1.55
$educ$	0.250	0.310	0.276	0.299
e_1	0.007	0.042	0.035	0.028
e_2	0.285	0.310	0.283	0.313
$p_{s e}$	0.660	0.698	0.694	0.682
$p_{s ne}$	0.047	0.000	0.010	0.021
$(r + \rho)U(s, e)$	1.84	1.86	1.82	1.88
$(r + \rho)U(s, ne)$	1.77	1.67	1.69	1.76
$(r + \rho)U(ns, e)$	1.08	1.03	1.07	1.03
$(r + \rho)U(ns, ne)$	1.07	1.02	1.06	1.03

The second column shows the main result that education attainment increases from 0.250 to 0.310, thus reproducing about 75% of the increase to 0.330 documented in the data for 1990. The change in the rest of the endogenous variables is practically the same as in the case of exogenous education, and therefore the same interpretation developed in Subsection 5.3 carries over. As for education, the interpretation is based on the option values. For the skilled-born workers (i.e., those with $l \leq h_0$), the return to schooling $(r + \rho)U(s, e)$ rises from 1.84 to 1.86, whereas the value of not getting education $(r + \rho)U(s, ne)$ falls from 1.77 to 1.67. This raises school participation within this group, e_1 . For the nonskilled-born workers (i.e., those with $l > h_0$), the value of education is given by $(r + \rho)[\mu U(s, e) + (1 - \mu)U(ns, e)]$, which remains fairly constant at about 1.57 since the impact of the gain in the value as skilled $U(s, e)$ and of the loss in the value as unskilled $U(ns, e)$ nearly balance each other given the efficiency of schooling μ . The loss from 1.07 to 1.02 in the value of not getting education for the nonskilled, $(r + \rho)U(ns, ne)$, then leads a larger number of these workers to decide to attend college. This shows as the increase in e_2 displayed. The rest of this section turns to the detailed interpretation of the responses just described.

The third column holds the skill-bias parameter, η_s , at its 1970 level. It is to be compared with the second column. It shows that the technological shock accounts for part of the rise in education as indicated by the positive difference between 0.310 and 0.276. For the skilled-born workers, the skill-biased change shifts upward the value of participating in education $(r + \rho)U(s, e)$ from 1.82 to 1.86, and reduces the value of staying out of education $(r + \rho)U(s, ne)$ from 1.69 to 1.67. This explains why the technological change causes an increase in e_1 , as indicated by the difference between 0.042 and 0.035. For the nonskilled-born workers, the skill-biased change also causes the value of education $(r + \rho)[\mu U(s, e) + (1 - \mu)U(ns, e)]$ to go up, which follows from the positive impact on the value if skill is gained $(r + \rho)U(s, e)$ outweighing the negative impact on the value if skill is not gained $(r + \rho)U(ns, e)$. The value of not participating in education

$(r + \rho)U(ns, ne)$ falls, as indicated by the negative difference between 1.02 and 1.06 owing to the technology shock. Both effects explain the positive effect of this shock on e_2 expressed in the difference between 0.310 and 0.283. Intuitively, the skill-biased change in technology increases the returns to education generally since it raises the wage premium, lowers unemployment for the educated, and increases unemployment for the noneducated. Regarding the changes in variables other than education, the technology shock appears to explain more of the change in the wage premium and the noneducated unemployment rate than when education was assumed exogenous earlier in Table 6.²⁵ The impact on education and thereby on the skill composition, shown by the changes from 0.694 to 0.698 in $p_{s|e}$ and from 0.010 to 0.0 in $p_{s|ne}$, explains this.

The fourth column holds the friction parameter, λ , at its 1970 level. It is also to be compared with the second column. It shows that the friction shock accounts for part of the rise in education attainment, as indicated by the positive difference between 0.310 and 0.299. A rise in the parameter λ causes a generalized decrease in all values of $(r + \rho)U(j, i)$ because now separation from any profitable match is more likely. This affects education differently for skilled-born and nonskilled-born workers. For the skilled-born workers, the friction shock increases their college attendance e_1 because the fall from 1.76 to 1.67 in the opportunity cost of going to college $(r + \rho)U(s, ne)$ is sharper than the fall from 1.88 to 1.86 in the return to education $(r + \rho)U(s, e)$.²⁶ This explains why the friction shock pushes this group's college participation e_1 up, as indicated by the positive difference between 0.042 and 0.028. For the nonskilled-born workers, the friction shock causes a fall in value of the return to education $(r + \rho)[\mu U(s, e) + (1 - \mu)U(ns, e)]$ slightly bigger than the fall from 1.03 to 1.02 in the opportunity cost $(r + \rho)U(ns, ne)$. This explains the slight reduction in e_2 brought about by the friction shock and indicated by the negative difference between 0.310 and 0.313. The friction shock accounts for most of the increase in e_1 and thus the change in the skill composition of the educated segment $p_{s|e}$, which proves a critical factor in explaining variables other than education. Turning precisely now to those variables, compared with the exogenous-education exercises in Table 6, the friction shock explains substantially more of the noneducated unemployment and, remarkably, the wage premium.²⁷ This has to be attributed to the sizable impact on education decisions and thereby on the skill composition shown by the changes from 0.682 to 0.698 in $p_{s|e}$ and from 0.021 to 0.0 in $p_{s|ne}$. This brings about the composition effects that are key to understanding part of the rise in the wage premium.

²⁵ To see this, consider the fourth column of Table 6 and the third column of Table 7, and compare the figures 0.041 and 1.51 against 0.039 and 1.49, respectively.

²⁶ Furthermore, in the relevant calculations in (5), $(r + \rho)U(s, e)$ is discounted, whereas, $(r + \rho)U(s, ne)$ is not.

²⁷ To see this, consider the fifth column of Table 6 and the fourth column of Table 7, and compare the figures 0.036 and 1.59 against 0.032 and 1.55, respectively.

6. CONCLUSION

This article extends a standard search-matching model by introducing education choices in a segmented labor market where education and skill are not equivalent attributes of a worker. The endogeneity of education is one major contribution of this article to a growing body of literature that studies wage inequality in related models. Equilibrium properties of the model are characterized. Then a calibrated setting is used to study a skill-biased change in technology and a rise in employment frictions as causes of the changes in education-specific unemployment rates, the wage premium, and college education attainment in the U.S. economy between 1970 and 1990. The two shocks can account for most of the observed changes in these four variables.

The skill-biased change in technology contributes to the changes in the wage premium, the college participation rate, and the unemployment rate of the non-educated labor force. However, the rise in the unemployment rate of the educated cannot be accounted for by this shock alone. On its part, the shock to employment frictions explains part of the changes in the wage premium, the unemployment rate for the noneducated labor force, and the college participation rate, and virtually all of the rise in the unemployment rate among educated workers. The endogenous change in education choices plays an essential part as it explains a substantial fraction of the consequences of the two shocks for the wage premium and the non-educated unemployment rate. The reason is that the response of education causes the skill composition of the labor force to improve within the educated group and to deteriorate within the noneducated group. The labor market outcomes, in turn, govern the response of education. The increase in education occurs because the skill-biased shock raises the return to acquiring skills and, on the other hand, the higher unemployment rates reduce the opportunity cost of the time spent in college.

This article demonstrates that it may be useful to analyze unemployment rates, wage inequality, and education jointly instead of in isolation. It shows, for example, that a sizable part of the rise in the wage premium can be traced down to an increase in employment frictions and its implications for education decisions and the makeup of the labor force. This is remarkable given the emphasis generally placed on the skill-biased interpretation of the transformations in the U.S. wage structure.

This article is based on a highly stylized model though. Although quantitatively consistent with most of the changes in three of the four variables under study, the measurement exercise still leaves unexplained a sizable portion of the rise in the educated unemployment rate. On the other hand, the article has not dealt with residual wage inequality, an important dimension of the observed shifts in wage dispersion. There are at least two features of the present analysis that deserve consideration in this regard. First, the quantitative exercises always imply equilibria where educated workers, regardless of their skills, invariably choose to participate in the segment of jobs that require a degree. This precludes the existence of over-education and its role in residual wage inequality and unemployment. Second, by assumption all jobs have exactly the same technology irrespective of the segment

they belong to. Decisions on equipment investment on the part of the firms would cause heterogeneity across job types with consequences for wage disparities and unemployment rates. Future work will also extend the approach of this article to explore the ability of different hypotheses to explain how the institutions and policies in European countries may have led to labor-market outcomes that differ from the U.S. experience. This type of analysis may shed new light on the possible differential role of education policies across countries.

APPENDIX

A. *Equations (12), (13), and (14).* Denote by $U_{j|i}$ the mass of unemployed workers with skill j in segment i . By definition, the mass of employed workers with these characteristics is $p_{j|i} - U_{j|i}$. The flow equation is $\dot{U}_{j|i} = (p_{j|i} - U_{j|i})(\lambda + \rho) - v_i \pi_{j|i} U_{j|i}$. In a steady state, $p_{j|i}(\lambda + \rho) = (v_i \pi_{j|i} + \lambda + \rho) U_{j|i}$. Then the definition of $z_{s|i} \equiv U_{s|i} / (U_{s|i} + U_{ns|i})$ leads to Equation (12).

The decision of a worker whether to accept an offer in Equation (3) is dominated by the hiring decision of the firm $\pi_{j|i}$ in Equation (7) since Equation (10) must hold. This permits us to remove the max term in Equation (3) and simplify the algebra in what follows. Then, Equation (13) results from developing the surplus-sharing bargaining Equation (10) using the optimal behavior of firms Equations (6)–(7) and workers Equations (2)–(3), and the technology relations Equations (8)–(9), with Equation (11) to pin down the value of a vacancy in equilibrium. On the other hand, Equations (14) comes from the free-entry condition Equation (11) on using the equilibrium firm's Bellman Equations (6)–(7), the technology relations (8)–(9), and (13) to substitute the wage terms $w(j, i)$.

B. *Computation of Equilibrium.* An outline of the main steps involved in the computation follows:

1. Set a pair $(e_1, e_2) \in [0, h_0] \times [h_0, 1]$.
2. Use (16) to find $p_{j|i}$ s. This must be done for both $\phi \in \{0, 1\}$ if the region (ns, e) in Figure 1 exists, that is, if $e_2 > h_0$.
3. Use the proposition to establish existence and the $\pi_{ns|i}$ s within each segment. Do so for each candidate ϕ , if pertinent.
4. Use (4) to verify that $\phi = 1$ and, if needed, determine existence and value(s) of ϕ . This requires computing the equilibrium for given e_1, e_2 , and $\pi_{ns|i}$ s for all the relevant alternative values of ϕ .
5. Compute equilibrium outcomes for all surviving candidate ϕ s. Update the pair (e_1, e_2) using (5) and go back to step 2 until convergence.

In step 3, if $p_{s|i} = 1$, by the proposition, $\pi_{ns|i}$ remains indeterminate and must be found through (14b) in equilibrium. This is not a problem, as the equilibrium can be computed independently of $\pi_{ns|i}$ in this case. In steps 4 and 5, the equilibrium is computed by solving (14) using, for each segment, a Newton–Raphson iteration on market tightness

C. *Unemployment Rates and Wage Premium.* The calibration and the derivation of results requires computing education-specific unemployment rates and wages.

C.1. *Unemployment rates.* Let U_e denote the mass of educated workers that are unemployed, and L_e the size of the educated labor force. Also let $U_{j|e}$ denote the mass of workers with skill j within the unemployed pool of educated workers. The flow equation is

$$\begin{aligned}\dot{U}_e &= (\lambda + \rho)(L_e - U_e) - U_{s|e}v_e\pi_{s|e} \\ &\quad - U_{ns|e}[\phi v_e\pi_{ns|e} + (1 - \phi)v_{ne}\pi_{ns|ne}]\end{aligned}$$

In the steady state, the unemployment rate is

$$\left(\frac{U_e}{L_e}\right) = \frac{\lambda + \rho}{\lambda + \rho + \left(\frac{U_{s|e}}{U_e}\right)v_e\pi_{s|e} + \left(\frac{U_{ns|e}}{U_e}\right)[\phi v_e\pi_{ns|e} + (1 - \phi)v_{ne}\pi_{ns|ne}]}$$

It remains to calculate the proportions $U_{j|e}/U_e$ involved. Use the equation $\dot{U}_{s|e} = (\lambda + \rho)[e_1 + \mu(e_2 - h_0) - U_{s|e}] - U_{s|e}v_e\pi_{s|e}$ and $\dot{U}_{ns|e} = (\lambda + \rho)[e_2 - h_0 - \mu(e_2 - h_0) - U_{ns|e}] - U_{ns|e}[\phi v_e\pi_{ns|e} + (1 - \phi)v_{ne}\pi_{ns|ne}]$. In a steady state, $\dot{U}_{s|e} = \dot{U}_{ns|e} = 0$ so

$$\begin{aligned}U_{s|e} &= \frac{(\lambda + \rho)[e_1 + \mu(e_2 - h_0)]}{v_e\pi_{s|e} + \lambda + \rho} \\ U_{ns|e} &= \frac{(\lambda + \rho)(1 - \mu)(e_2 - h_0)}{\phi v_e\pi_{ns|e} + (1 - \phi)v_{ne}\pi_{ns|ne} + \lambda + \rho}\end{aligned}$$

Since $U_e = U_{s|e} + U_{ns|e}$,

$$\begin{aligned}\left(\frac{U_{s|e}}{U_e}\right) &= \{[e_1 + \mu(e_2 - h_0)][\phi v_e\pi_{ns|e} + (1 - \phi)v_{ne}\pi_{ns|ne} + \lambda + \rho]\} \\ &\quad \times \{p_{e1}[\phi v_e\pi_{ns|e} + (1 - \phi)v_{ne}\pi_{ns|ne} + \lambda + \rho] \\ &\quad + (1 - \mu)(e_2 - h_0)(v_e\pi_{s|e} + \lambda + \rho)\}^{-1}\end{aligned}$$

Similarly, for noneducated workers, the unemployment rate is

$$\left(\frac{U_{ne}}{L_{ne}}\right) = \frac{\lambda + \rho}{\lambda + \rho + \left(\frac{U_{s|ne}}{U_{ne}}\right)v_{ne}\pi_{s|ne} + \left(\frac{U_{ns|ne}}{U_{ne}}\right)v_{ne}\pi_{ns|ne}}$$

with

$$\left(\frac{U_{s|ne}}{U_{ne}}\right) = \frac{(h_0 - e_1)(v_{ne}\pi_{ns|ne} + \lambda + \rho)}{(h_0 - e_1)(v_{ne}\pi_{ns|ne} + \lambda + \rho) + (1 - e_2)(v_{ne}\pi_{s|ne} + \lambda + \rho)}$$

C.2. *Wages.* For the educated workers, the total wage bill is

$$\begin{aligned}\tilde{w}_e &= w(s, e) \frac{e_1 + \mu(e_2 - h_0)}{e_1 - h_0 + e_2} \left(1 - \frac{\lambda + \rho}{\lambda + \rho + \pi_{s|e} \nu_e}\right) \\ &\quad + [\phi w(ns, e) + (1 - \phi)w(ns, ne)] \frac{(1 - \mu)(e_2 - h_0)}{e_1 - h_0 + e_2} \\ &\quad \times \left[1 - \frac{\lambda + \rho}{\lambda + \rho + \phi \pi_{ns|e} \nu_e + (1 - \phi) \pi_{ns|ne} \nu_{ne}}\right]\end{aligned}$$

and then the average wage for this group is

$$\begin{aligned}w_e &= \tilde{w}_e \left\{ \frac{e_1 + \mu(e_2 - h_0)}{e_1 - h_0 + e_2} \left(1 - \frac{\lambda + \rho}{\lambda + \rho + \pi_{s|e} \nu_e}\right) \right. \\ &\quad \left. + \frac{(1 - \mu)(e_2 - h_0)}{e_1 - h_0 + e_2} \left[1 - \frac{\lambda + \rho}{\lambda + \rho + \phi \pi_{ns|e} \nu_e + (1 - \phi) \pi_{ns|ne} \nu_{ne}}\right] \right\}^{-1}\end{aligned}$$

For the noneducated workers,

$$\begin{aligned}\tilde{w}_{ne} &= w(s, ne) \frac{h_0 - e_1}{h_0 - e_1 + (1 - e_2)} \left(1 - \frac{\lambda + \rho}{\lambda + \rho + \pi_{s|ne} \nu_{ne}}\right) \\ &\quad + w(ns, ne) \frac{1 - e_2}{h_0 - e_1 + (1 - e_2)} \left(1 - \frac{\lambda + \rho}{\lambda + \rho + \pi_{ns|ne} \nu_{ne}}\right)\end{aligned}$$

And so

$$\begin{aligned}w_{ne} &= \tilde{w}_{ne} \left[\frac{h_0 - e_1}{h_0 - e_1 + (1 - e_2)} \left(1 - \frac{\lambda + \rho}{\lambda + \rho + \pi_{s|ne} \nu_{ne}}\right) \right. \\ &\quad \left. + \frac{1 - e_2}{h_0 - e_1 + (1 - e_2)} \left(1 - \frac{\lambda + \rho}{\lambda + \rho + \pi_{ns|ne} \nu_{ne}}\right) \right]^{-1}\end{aligned}$$

These calculations are written for certain values for e_1 , e_2 , and h_0 , which determine the composition of skill and education groups. If, as when the skill composition is exogenous, one is instead given values for $p_{s|e}$, $p_{s|ne}$, and h , Equation (16) must be used to perform these calculations.

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