Introduction to non-perturbative cavity quantum electrodynamics

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Quantum Theory and Technology
Fundamental interactions

Strong interaction

Mass of up quark: 2.3 MeV
Mass of down quark: 4.8 MeV
Mass of a proton: 938 MeV
99% of proton mass is due to interaction (virtual quark-gluon plasma)

Electromagnetic interaction

In light-matter interaction the dimensionless coupling constant is $\alpha \approx \frac{1}{137}$

Low order perturbation theory works well (photon absorption and emission)

The interaction strength $\Omega_R$ is much smaller than the bare frequency $\omega_0$
Ultrastrong coupling

Ultrastrong light-matter coupling regime: \( \eta = \frac{\Omega_R}{\omega_0} \) non negligible

Ultrastrong coupling between light and matter,
Non-perturbative CQED phenomenology

- Quantum phase transitions
- Quantum vacuum radiation
- Topologically protected ground states
- Increase in electrical conductivity
- Modified electroluminescent properties
- Change in chemical properties
- Change in structural molecular properties
- Modified lasing
- Vacuum nonlinear processes
- …
From weak to the ultrastrong
Purcell effect

\[ |e\rangle \quad \omega_0 \quad |g\rangle \]

\[ \Gamma_{sp} = \frac{2\pi}{\hbar} |\langle i | H_{int} | f \rangle|^2 \rho(\hbar \omega_0) \]

Photonic density of states

Free space:

\[ \rho(\omega) \]

\[ \Gamma_{sp} = \frac{\omega_0^3 d_{ge}^2}{3\pi \epsilon_0 \hbar c^3} \]

Cavity:

\[ \rho(\omega) \]

Enhancement

Suppression
Strong coupling (time domain)

Fermi golden rule: first order perturbation.

It cannot account for higher order processes, i.e. reabsorption. Valid if $\Omega_R < \Gamma$

If $\Omega_R > \Gamma$ the emitted photons is trapped long enough to be reabsorbed
Strong coupling (frequency domain)

The coupling splits the degenerate levels, creating the Jaynes-Cummings ladder.

The losses give the resonances a finite width.

Strong coupling: \( \Omega_R > \Gamma \)

Condition to spectroscopically resolve the resonant splitting.

In the strong coupling regime we cannot consider transitions between uncoupled modes, e.g., \( |0, e\rangle \rightarrow |1, g\rangle \).

We are obliged to consider the dressed states, \( |1, -\rangle, |1, +\rangle \), etc…
The Polariton

\[ p^\dagger |0\rangle = x \big| \qquad \big\rangle + y \big| \qquad \big\rangle \]

Half light and half matter excitation

Modes that are:
- easy to excite and observe
- interact strongly

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\[ \frac{\omega}{\omega_0} \]
Perturbation theory

Let us do perturbation using the full Hamiltonian

\[
H_{\text{QRM}} = \hbar \omega_0 a^\dagger a + \hbar \omega_0 |e\rangle \langle e| + \hbar \Omega_R (a + a^\dagger)(|e\rangle \langle g| + |g\rangle \langle e|)
\]

\[
H_0 \quad \quad \quad \quad \quad \quad H_{\text{int}}
\]

First order perturbation: \[\Delta E^{(1)}_\phi \propto \Omega_R \]

Second order perturbation: \[\Delta E^{(2)}_\phi = \sum_{|\psi\rangle \neq |\phi\rangle} |\langle \phi|H_{\text{int}}|\psi\rangle|^2 \propto \frac{\Omega_R^2}{\omega_0} = \Omega_R \times \frac{\Omega_R}{\omega_0} \]

Higher-order effects are observable when \[\frac{\Omega_R}{\omega_0} \] is non negligible

Ultrastrong coupling regime
Coupling regimes

Fermi Golden rule  Dressed states  New physics

Weak coupling  Strong coupling  Ultrastrong coupling

0  $\Gamma$  $\omega_0$  $\Omega_R$
Is ultrastrong coupling possible?

Hydrogen atom

\[ E_n = -\frac{\text{Ry}}{n^2} \]

Wavelength

\[ \lambda = \frac{2\pi c}{\omega_0} \]

Dimensionless volume

\[ \tilde{V} = \frac{V}{(\lambda/2)^3} \]

We end up with

\[ \frac{\Omega_R}{\omega_0} = \frac{\alpha^{3/2}}{n\pi \sqrt{\tilde{V}}} \]

Three ways to ultrastrong coupling

- Reducing \( \tilde{V} \)
- Increasing the number of dipoles
- Coupling to currents (\( \alpha^{-1/2} \))

Reducing the mode volume

Mode confinement: smaller cavity = larger coupling

Collective coupling: more dipoles = larger coupling

\[ \Omega_R \propto \frac{1}{\sqrt{V}} \]

\[ \Omega_R \propto \sqrt{N} \]

N dipoles of length \( d \) = 1 dipole of length \( \sqrt{N} d \)
Virtual photons & Decoupling
The coupled ground state

\[ |\text{G}\rangle \]

Energy

\[ \Omega_R \]

\[ \omega_0 \]

Excited states

Renormalised excited states

\[ \mathcal{G} \]

The coupled ground state \(|G\rangle\) has a population of virtual photons

\[ \langle G | a^\dagger a | G \rangle \propto \frac{\Omega_R^2}{\omega_0^2} + O\left(\frac{\Omega_R^4}{\omega_0^4}\right) \]

Stable against losses

S. De Liberato
Nature Communications 8, 1465 (2017)
Emmission of photons out of the ground state. **Wrong!**

Except that:

\[ \langle G | a^\dagger a | G \rangle \propto \frac{\Omega_R^2}{\omega_0^2} + O(\frac{\Omega_R^4}{\omega_0^4}) \]

Emission of photons out of the ground state. **Wrong!**

\[ \mathcal{L}(\rho) = \frac{\Gamma}{2} (2\rho a a^\dagger - a^\dagger a \rho - \rho a a^\dagger) \]

These are white baths

\[ n_{\text{out}} = \Gamma \langle a^\dagger a \rangle \]

No negative frequency modes

All a baths are colored


Quantum vacuum emission

The coupling changes the ground state

Free system: \( |0\rangle \) Standard vacuum

Coupled oscillators: \( |G\rangle \) Coupled vacuum

\[
\Omega(q, t)
\]

\( |G\rangle \quad \tilde{\Omega}(q) \quad \) Nonadiabatic quantum dynamics

Coupled vacuum

Standard vacuum \( |0\rangle \)

\( |G\rangle \quad \tilde{\Omega}(q) \quad \) Nonadiabatic quantum dynamics

Photon emission

\( \tau_{\text{relax}} \quad \) Time \( t \)


Purcell effect breakdown

\[ H = H_{\text{field}} + \frac{p^2}{2m} + V(r) - \frac{e p A(r)}{m} + \frac{e^2 A(r)^2}{2m} \]

Intensity of the field at the location of the dipoles

If \( \frac{\Omega_R}{\omega_0} > 1 \) the last term, always positive, becomes dominant

The low energy modes need to minimize the field location over the dipoles

Light and matter decouple in the deep strong coupling regime

Purcell effect breakdown

Example: a two-dimensional metallic cavity enclosing a wall of in-plane dipoles

The wall becomes a metallic mirror

In the non-perturbative regime the Purcell effect fails

Ultimate limit to switching frequency

Observed and exploited in microcavity fabrication

Thank you for your attention