Holographic Colour Superconductivity and Neutron Stars

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SHEP PGR Seminar

4 December
1. Introduction
2. CFL quark matter and AdS Superconductors
3. QCD Interactions and Phase diagram
4. Neutron Stars
5. Overview and future work
A fermionic system at finite density is expected to develop a Fermi surface.

- RG flow analysis shows quarks condense in high density QCD.
- Quark condensation break the gauge group $\Rightarrow$ phenomena is referred to as colour superconductivity (CSC).
- The cores of neutron stars are too dense so cannot be described in terms of weakly interacting baryons, but not dense enough for perturbative QCD.
- Since we lack of first principles results for ultradense nuclear matter a reliable relation between the variables of state of a neutron star has been elusive.
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Holographic description

- Holography has emerged as a new tool to study strongly coupled gauge theories.
- It provides the ability to rigorously work in theories close to large $N_c$ $\mathcal{N} = 4$ SYM.
- CSC effect is suppressed at large $N_c$ due to the fact that the quark pair is not a color singlet.
- The condensate that breaks the gauge group has dimension 3, but on the gravity side only gauge invariant operators are manifest, $qq$ gets a vev but the gauge invariant is dimension 6.

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For intermediate values of density in QCD the quark-gluon plasma is strongly coupled and full of charged quarks and magnetically charged scalars.

- These generate a mass $\sim g\sqrt{T^2 + \mu^2}$
- Since $g$ is large, there can be an order of magnitude gap between the gluon mass and the $\mu/T$ scale. We squeeze a holographic description into this energy scale.
- Since gluons are gapped we avoid treating $SU(3)_c$ as a gauge symmetry and impose it as a flavour symmetry.
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The most symmetric option is the color-flavor locked phase (CFL),

- Quarks of all 3 colors and all 3 flavors form conventional zero-momentum spinless Cooper pairs.

$$\langle \psi_i^\alpha C \gamma_5 \psi_j^\beta \rangle \propto \Delta_{CFL} \epsilon^{\alpha\beta A} \epsilon_{ijA} + \Delta_{CFL} \kappa (\delta_i^\alpha \delta_j^\beta + \delta_j^\alpha \delta_i^\beta)$$

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In the three flavour massless limit

$$\begin{align*}
\bar{u} & \quad sd - ds & \Delta_1 \\
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In the holographic model we will describe an AdS-scalar $\psi$ dual to an element of this matrix which acquires a vev
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\begin{array}{ccc}
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AdS Superconductors

We will use an AdS space to phenomenologically describe the conformal symmetries of the free fermions below the gap scale, which are then broken by the operators and sources in the bulk.

We set up the AdS description of superconductivity:

$$ds^2 = r^2 (-f dt^2 + dx^2) + \frac{1}{r^2 f} dr^2, \quad f = 1 - \frac{r^4_H}{r^4}.$$  

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Scalar $\psi$ represent the quark bilinear (in the $\bar{3}$ of colour) with $B = 2/3$ and dimension 3; gauge field associated with $\text{U}(1)_B$ whose $A_t$ describe $\mu$.

We use the action

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - |\partial\psi - iBA\psi|^2 + 3\psi^2,$$

Solve E.O.M. For regularity we require $A_t = 0$ at the horizon, which implies $\psi' = -\frac{3}{4r_H}\psi$. In the UV:

$$\psi = \frac{J_c}{r} + \frac{c}{r^3} + ... \quad A_t = \mu + \frac{d}{r^2} + ...$$
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$$\psi = \frac{J_c}{r} + \frac{c}{r^3} + ... \quad A_t = \mu + \frac{d}{r^2} + ...$$
The $\psi$ functions in the unbroken and broken phase at $T = 0.1$, $\mu = 1$ (left) and $\mu = 5$ (right)
We plot the value of the condensate against $\mu$ for the $J_c = 0$ embeddings at fixed $T = 0.1$ and found there is a second order transition.
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Tune intrinsic interaction

We know that the intrinsic attractive interaction in the AdS / superconductor model is presumably more subtle than a NJL operator (four fermion terms).

To model the interaction we simply modify the strength of the $\psi A_t$ interaction to reflect the QCD interaction strength.

Remember gluons are massive in this strongly coupled phase so the colour of quarks is treated as a global symmetry.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |\partial\psi - iGBA\psi|^2 + \frac{3}{L^2} \psi^2$$
We interpret the $A\psi$ term as the holographic models knowledge of the broken gauge interactions. We will relate $G$ to the QCD running coupling.

We find the phase boundary for the CSC phase as a function of $G$: for each $T$ and $G$ we make $c$ vs $\mu$ plots and find $\mu_c$, then plot $\mu_c(T)$. 
We assume the chiral phase transition occurs at $T^2 + \mu^2 = \Lambda_{QCD}^2 := 1$

In the quark gluon plasma phase we set $G$ at the cut off scale to a sensible ansatz in QCD:

$$G^2 = \frac{\kappa}{b \ln(T^2 + \mu^2)/\Lambda_{QCD}^2}, \quad b = \frac{11 N_c}{3} - \frac{2 N_f}{3}$$

$\kappa$ would represent the strength of the attraction that generates the $\bar{3}$ of colour condensate. We let it vary.
QCD phase diagram: The phase edges shows where the CFL phase is present for the choices of $\kappa = 1, 10, 20$ from bottom to top.
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Holographic thermodynamic quantities

As a first approach we worked with:

\[ \mathcal{L} = -N_f T_{D7} h(\rho) \rho^3 \sqrt{1 + (\partial_\rho L)^2 - (2\pi\alpha' \partial_\rho A_t)^2} \]

\( h(\rho) \) is an effective dilaton term that introduces chiral symmetry breaking. Here \( h(\rho) = \alpha \left( \ln(\sqrt{\rho^2 + L(\rho)^2}) \right) \).

We find the running coupling of QCD by solving the equation

\[ \frac{d\alpha}{d \ln \mu^2} = \beta(\alpha) \]

From the free Energy \( \mathcal{F} \) we can find the Pressure \( P \) and the energy density \( \mathcal{E} \) as:

\[ P = -\mathcal{F}, \quad \mathcal{E} = \mu \frac{\partial P}{\partial \mu} - P \]
We can identify the point in which chiral symmetry is restored. We argue that there is a phase of quark matter with chiral symmetry broken.
To find the equation of state (EoS) we find points of the parametric curve \((P(\mu), \mathcal{E}(\mu))\) varying \(\mu\) and then fit these points with a polynomial that will give us an analytic expression \(\mathcal{E}(P)\).
Now we can integrate the TOV equations:

\[
\frac{dP}{dr} = -G(E(P) + P) \frac{M + 4\pi r^3 P}{r(r - 2GM)},
\]

\[
\frac{dM}{dr} = 4\pi R^2 E(P)
\]

Obtain \(P(r)\) and \(M(r)\). The radius \(R\) of the star will be \(r \ni P(r) = 0\).

Varying the central pressure \(P(R = 0) = P_0\) as a parameter we can construct a curve for the mass of the star \(M(r = R)\) against its radius.
The black line represent the deconfined phase of matter inside the Neutron star, in which the Holographic model give us the initial conditions $r_n$, $m_n$ where the quark matter ends and the nuclear matter starts inside the star.
Stability of the Star

We neglect the possibility of mixed phases and consider stars made of pure phases.

A turn of the curve towards smaller masses and radii is a sign of instability with respect to radial oscillations since a necessary condition for stability is that:

$$\frac{\partial M(E_c)}{\partial E_c} > 0$$
CSC in Neutron Stars

We take a step further and add the CSC phase in the core of the Neutron Star to see if the instability of the mixed star changes. We use the Lagrangian:

\[
\mathcal{L} = -N_f T_D f(\rho) \rho^3 \sqrt{1 + (\partial_\rho L)^2 - (2\pi \alpha' \partial_\rho A_t)^2 + \lambda \chi^2 L^2} - \partial \chi - iGBA \chi^2 + 3\chi^2
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- We avoided the colour charged nature of the condensate: at strong coupling and intermediate $T$, $\mu$ gluons are gapped by the plasma $\Rightarrow$ treat colour symmetry as a global index.

- To include QCD interactions we adjust the strength of the intrinsic interaction to reflect the QCD couplings value as a function of $\mu$, $T$.

- Transition temperature near 20MeV, matches the quoted range of 10-100MeV.

- We obtained the Mass-Radius curve of a Neutron star using the D3/D7 model for the deconfined phase and are in a position to add the CSC phase.

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