

D'oh! Fourier

Theory, applications, and derivatives

Preface	vii
Style	vii
Target audience	vii
Overview of structure	vii
In gratitude	viii
Keypoints	x
1 Basic Notions and the Nature of the Fourier Transform	1
1.1 Why read this book?	1
1.2 Software and reproducibility	2
1.3 Notation.....	3
1.4 Basic functions.....	4
1.5 Analysing signals by their components: approximating functions by mathematical series.	7
1.5.1 Taylor series	7
1.5.2 Fourier series	8
1.6 What is the Fourier transform, and what can it do?	11
1.7 Everyday use of the Fourier transform.....	13
1.7.1 Transforms and speech recognition	13
1.7.2 Transforms and image compression.....	15
1.7.3 Human hearing and a transform.....	15
1.7.4 Light and frequency	16
1.8 Summary and further reading	16
2 The Continuous Fourier Transform	19
2.1 Continuous Fourier transform basis	19
2.1.1 Continuous signals and their Fourier transform.....	19
2.1.2 Magnitude and phase	22
2.1.3 Inverse Fourier transform.....	23
2.1.4 Fourier transform in Matlab	25
2.1.5 Fourier transform pairs.....	26
2.1.5.1 <i>Delta function</i>	26
2.1.5.2 <i>Sinewave</i>	26
2.1.5.3 <i>Gaussian function</i>	29
2.2 Properties of the continuous Fourier transform	29
2.2.1 Superposition	29
2.2.2 Time shift	29
2.2.3 Scaling in time	30
2.2.4 Parseval's theorem (Rayleigh's theorem)	31
2.2.5 Symmetry	31
2.2.6 Differentiation	32
2.2.7 Uncertainty principle.....	32
2.2.8 Modulation	30
2.3 Processing signals using the Fourier transform	34

Contents

2.3.1 Convolution	34
2.3.2 Correlation	37
2.4 What is the importance of phase?	39
2.4.1 Phase in signal reconstruction	40
2.4.2 Phase in shift invariance.....	41
2.5 Windowing the FT data.....	41
2.5.1 Basic windowing.....	41
2.5.2 Hanning and Hamming window operators	42
2.5.3 Window duration	44
2.5.4 Other windowing functions	47
2.6 Filtering the FT data.....	48
2.6.1 Basic filters and signal processing	48
2.6.1.1 <i>Low-pass, high-pass and band-pass filters</i>	48
2.6.1.2 <i>RC networks and transfer functions: low-pass filters</i>	49
2.6.1.3 <i>CR networks and theory: high-pass filters</i>	52
2.6.2 Bessel filters	54
2.7 Summary	54

3 The Discrete Fourier Transform 55

3.1 The sampling theorem.....	55
3.1.1 Sampling signals	55
3.1.2 Sampling process in the frequency domain	57
3.2 The Discrete Fourier Transform (DFT)	60
3.2.1 Basic DFT.....	60
3.2.2 Inverse DFT	62
3.2.3 Visualising the discrete Fourier transform data	64
3.2.4 DFT in Matlab	68
3.2.5 Discrete Fourier transform pairs	68
3.2.5.1 <i>Pulse</i>	68
3.2.5.2 <i>Gaussian</i>	69
3.3 Properties of the DFT.....	70
3.3.1 Basic considerations.....	70
3.3.2 Linearity/ Superposition	70
3.3.3 Time shift	70
3.3.4 Time scaling	71
3.3.5 Parseval's theorem (Rayleigh's theorem)	71
3.3.6 Symmetry	71
3.3.7 Differentiation	72
3.3.8 Importance of phase - DFT.....	72
3.3.9 Discrete data windowing functions	74
3.4 Discrete Convolution and Correlation	75
3.4.1 Discrete convolution	75
3.4.2 Discrete correlation	80
3.5 Digital filters; averaging and differencing samples	83
3.6 The Fast Fourier transform (FFT).....	84
3.6.1 The butterfly operation and basic components of the FFT	84
3.6.1.1 <i>FFT basis</i>	84
3.6.1.2 <i>FFT computation and speed</i>	88
3.6.1.3 <i>Extending the FFT</i>	89
3.6.2 Decimation in time.....	91
3.6.3 Radix 2 FFT.....	94
3.6.4 Computational time for FFT compared with DFT	96

3.6.4.1 Improvement in speed vs DFT	96
3.6.4.2 Speeding convolution via the convolution	97
3.6.5 Optimising the FFT	98
3.6.6 Even faster FFT algorithms	99
3.7 Summary	100

4 The Two-dimensional Fourier Transform 101

4.1 2-D functions and images.....	101
4.1.1 Image formation.....	101
4.1.2 Human vision.....	101
4.1.3 Sampling images.....	103
4.1.4 Discrete images.....	105
4.1.5 Discrete image frequency components.....	107
4.2 2-D Fourier Transform and its inverse.....	108
4.2.1 2-D continuous Fourier transform, and separability	108
4.2.2 2-D discrete Fourier transform	109
4.3 Properties of the 2-D Discrete Fourier transform	112
4.3.1 Displaying images.....	112
4.3.1.1 Transforms and their repetition properties	112
4.3.1.2 Intensity normalisation	113
4.3.2 Rotation.....	113
4.3.3 Scaling.....	115
4.3.4 Shift invariance.....	115
4.3.5 The importance of phase	116
4.3.6 Computational cost of 2-D DFT and FFT	117
4.4 Image Processing via the Fourier Transform	117
4.4.1 Convolution	117
4.4.1.1 Image convolution	117
4.4.1.2 Template convolution.....	118
4.4.1.3 Filtering an image via convolution.....	120
4.4.2 Computational considerations of image convolution and template convolution	121
4.4.3 Correlation	122
4.4.3.1 Image correlation	122
4.4.3.2 Template correlation/ template matching	123
4.4.3.3 Finding objects by template correlation/ matching	124
4.4.4 Filtering	126
4.4.4.1 Low- and high-pass filtering	126
4.4.4.2 Unsharp masking.....	127
4.5 Summary	128

5 Variants of the Fourier Transform 129

5.1 Cosine and Sine Transforms, including the Discrete Cosine Transform (DCT)	129
5.1.1 1-D Continuous transforms	129
5.1.2 1-D Discrete cosine and sine transforms	129
5.1.2.1 Discrete cosine transform and compression	129
5.1.2.2 Basic coding	133
5.1.2.3 Relationship between the DCT and the DFT.....	133
5.1.2.4 Other properties of the DCT	135
5.1.2.5 Discrete sine transform.....	135
5.1.3 2-D Discrete cosine transform	136
5.2 Walsh–Hadamard transform.....	137

Contents

5.2.1 Walsh transform.....	137
5.2.1.1 <i>The 1-D transform</i>	137
5.2.1.2 <i>The 2-D Walsh transform</i>	140
5.2.2 Walsh-Hadamard transform	140
5.3 Hartley Transform.....	141
5.4 Image compression properties of Fourier, DCT, Walsh and Hartley transforms	143
5.5 Laplace, Mellin and Fourier Mellin	145
5.5.1 Laplace and Mellin transforms	145
5.5.1.1 <i>Laplace transform and basic systems analysis</i>	145
5.5.1.2 <i>Mellin transform for scale invariance</i>	147
5.5.2 Fourier-Mellin transform.....	148
5.6 z transform.....	149
5.7 Wavelets	150
5.7.1 Filter banks and signal analysis	150
5.7.2 Gabor Wavelets	151
5.8 Summary	155

6 Applications of the Fourier Transform 157

6.1 Overview	157
6.2 Fourier transforms.....	157
6.2.1 The continuous Fourier transform and Fourier optics	157
6.2.2 Magnitude and phase, and beamforming.....	159
6.3 Properties of the Fourier transform.....	160
6.3.1 Superposition and fingerprint analysis	160
6.3.2 Invariance and image texture analysis	161
6.3.3 Invariance and image registration	164
6.3.4 Differentiation and image feature extraction	165
6.3.4.1 <i>Template convolution and edge detection</i>	165
6.3.4.2 <i>z-transform and Fourier analysis of edge detection operators</i>	167
6.4 Processing signals using the Fourier transform	168
6.4.1 Convolution theorem and ear biometrics	168
6.4.2 Deconvolution	169
6.4.3 Speech recognition and correlation.....	170
6.5 The importance of phase and phase congruency	173
6.6 Image enhancement, filtering and denoising	178
6.7 Variants of the Fourier transform and coding	180
6.8 Summary	182

7 Who and What was Fourier? 183

7.1 Nature and origins of the Fourier transform.....	183
7.1.1 The basic nature and definitions of the Fourier transform	183
7.1.2 On the development of the Fourier transform	185
7.2 Baron Jean Baptiste Joseph Fourier	186
7.3 Final summary	187

8 Appendices 189

8.1 Fourier series of a more realistic signal	189
8.2 Fourier transform of a triangle waveform	191
8.3 Fourier transform of a discrete triangle waveform	192
8.4 Summary of Fourier transforms and their variants	193

8.5 Summary of properties of the continuous Fourier transform	194
8.6 Continuous Fourier transform pairs	195
8.7 Summary of properties of the discrete Fourier transform	197
8.8 Discrete Fourier transform pairs	198

References	201
-------------------	------------

Index	207
--------------	------------