Preface

Style

This book is aimed to help understanding of the Fourier Transform (FT): it is a guided tour, with applications, pictures and some jokes. There are three elements to this material:

i) the book itself;
ii) the website https://www.southampton.ac.uk/~msn/Doh_Fourier/; and
iii) some Matlab software.

The maths is aimed to present the material in the simplest manner. The website contains all the software, from which most of the diagrams were derived, and some other material we hope you find useful. The software has been written in Matlab so you can see working equations and it also makes the book reproducible (via open-source software) which is a welcome theme in modern research. There is colour throughout since it is targeted at electronic reading rather than print copy. An added advantage of electronic reading is its help with any obscure words. Some of this material is repeated in the obligatory 'Why was this book written' Section in the introduction, since if it was all here some might miss it. The book is certainly a bit of a romp through some pretty complex material and the style aims to make it digestible. Education should be challenging and fun. If you don't like the jokes, it's just tough ****.

Target audience

The level here is aimed at undergraduates with a mathematical background who cover Fourier as part of their undergraduate curriculum. As the book covers 1-D and 2-D signal analysis the target curricula include courses on signal processing and on communications, speech analysis and understanding, and image processing and computer vision, amongst others. Fourier has a wide set of applications indeed, as we shall find. The book is also aimed at people who are interested in furthering their knowledge, for whom Maths might less practiced, and so we include diversions around some of the maths bits (and yes, others might use them too). So there is maths in this book. That's what will put our astronauts on Mars and food on our plates, so don't knock it. This book is a guide rather than a prescription and the aim here is to be as succinct as possible, with plenty of diagrams.

I was once at a (University) cocktail party where an old dragon asked me what I did so I replied that I am a research mathematician (OK, a bit of a stretch - but not an especially big one). "Oh my" replied the dragon "I always thought maths was a waste of time". Well I didn't especially like her (or the way she avoided challenge), so I replied "if I said 'I couldn't be bothered to learn how to read or write', you'd think I was really thick". "Ooh rather" replied the absence of neurons, and then she realised what I'd said. "Ooh I say, that's rather rude" was her parting sally. That was one Christmas card less.

Overview of structure

The structure here is essentially lexical, starting with the basics and ending with some more advanced stuff and applications. The structure and background material is exposed in Chapter 1. This also describes the basic nature of the FT and some of the applications, which concurrently

describes the book's structure. You should be able to start the book anywhere: it could also be read back to front (intentionally, not just for a different market), so let us pick out some possible starting points. One could start with the applications (which contain less maths) covered in Chapter 6. There is more maths when we start with the continuous Fourier transform in Chapter 2. Given that computers are ubiquitous, some might prefer to start with sampled data which is covered in Chapter 3. If you prefer to see material covered as images, then start with Chapter 4. It is unlikely that any would prefer to start with variants and derivatives of the Fourier transform, covered in Chapter 5 though these are often used in applications, as described in Chapter 6. Chapter 5 also includes wavelets and they could subsume the more traditional windowing functions of earlier Chapters. Chapter 7 describes Fourier and his life, together with some of the context in which his transform was developed. It gives historians a place to start too. For ready reference, there are Appendices which tabulate basic material. The References are collocated at the end of the book, prior to its Index.

In gratitude

Naturally I must thank many people. I remain grateful to the many students who have survived my teaching in the past 30 odd years on BSc, BEng, MEng and MSc courses within the Department of Electronics and Computer Science at the University of Southampton, and to my many PhD students. Those who read all, or parts, of it include Dr. John Carter, Professor Kevin Bowyer, Dr. David Heathfield, Dr. Seth Nixon, Dr. Alberto Aguado, Professor Rob Maunder, Dr. Vishal Patel, Dr. Sasan Mahmoodi and Michael Beale my WSP Editor and the reviewers. Your advice was superb, and your comments were most welcome and have helped to enrich this text. It is a much better book with your help: many thanks. Naturally, living with an academic can be a bit of a pain, so I remain grateful to the forbearance, fine food and excellent company of my wife Caz, and the 'kids' Seth and Nimi.

Writing a book takes a long time and a lot of ideas, some of which might not originally be one's own. If any idea here arose from an unattributed conversation or one of the many web searches made during its writing, obviously it has been forgotten but please accept some apologies as it would have been attributed if possible. Naturally errors will appear in the final version by virtue of the production though some are invariably human. For this, apologies again and an up-to-date list will be included on the book's website https://www.southampton.ac.uk/~msn/Doh_Fourier/. If you're the first to find them, you'll be sent a pint of good English real ale for free. That's a promise in all of my books, as it actually helps to debug the book. Fourier brings a new dimension to understanding as you'll find herein. And to you dear reader, thanks for picking it up. Enjoy!!

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Key points (tldr)

Rather than bury some of the most important points within the text, they are listed separately here, and repeated later at the appropriate point. There are other points, and some of the complexities are not alluded to here (we have stated the key points as succinctly as possible, using italics for emphasis). If you are new to the subject, move straightaway to page 1 (and come back later). If you are reasonably confident, after reading this move to page 11; if you want to add another point or have a quibble with their semantics, go to Chapter 7.

# (Location)	Key point
1 (Section 1.6)	A *signal* is constructed by addition in the time domain and its *Fourier transform* is determined by separation in the frequency domain.
2 (Section 2.1.5)	A *transform pair* means that a signal which exists as signal A in the time domain with a transform signal B in the frequency domain, also exists as signal B in the time domain and signal A in the frequency domain.
3 (Section 2.3.1)	The dual of *convolution* in the time domain is multiplication in the frequency domain, and vice versa.
4 (Section 2.5.3)	For *windowing* a periodic signal, choose a window length of three times its period.
5 (Section 3.1.2)	*Sampling*: in order to be able to reconstruct a (discrete) signal from its samples we must sample at *minimum* at twice the maximum frequency in the original signal.
6 (Section 3.2.3)	*Replication*: the transform pair of the sampling function implies that the spectrum of a sampled signal repeats indefinitely in the frequency domain.
7 (Section 3.6.4)	The *Fast Fourier Transform* (FFT) gives the same result as the Discrete FT and for N points it is $\frac{N}{\log_2 N}$ times faster (and when N is large that means a lot faster).
8 (Section 5.1)	*Variants of the Fourier transform*: the Fourier transform is based on sine and cosine waves and uses only one of an infinite number of possible basis function sets.
9 (Section 5.7)	*Wavelets* allow sensitivity to frequency/ time and frequency/ space and allow for multi-resolution time-scale analysis.
10 (Section 7.1)	There is no universally accepted unique *definition* of the Fourier transform.