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Gravitational waves from neutrino mass and dark matter genesis

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Prologue

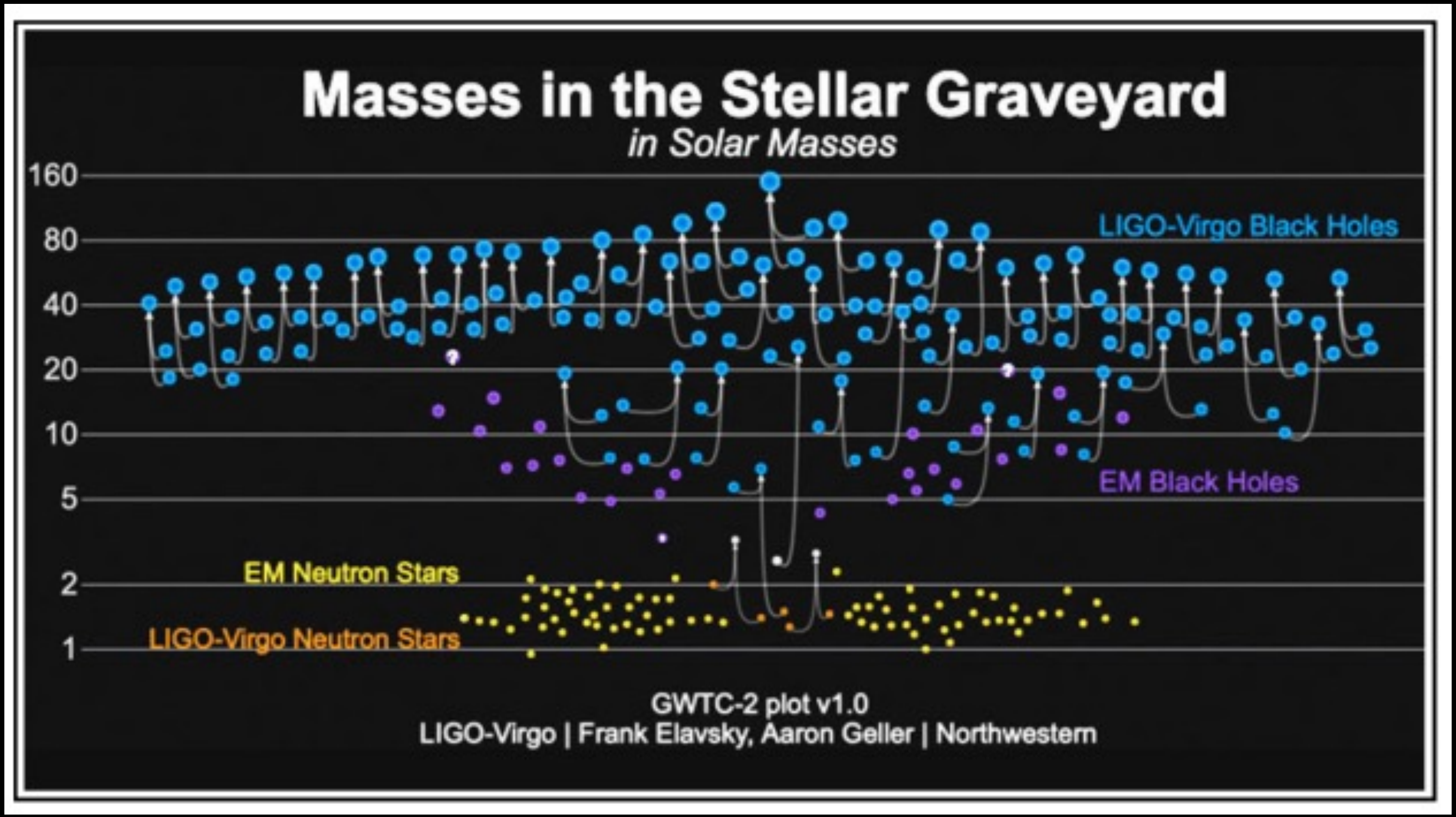
- "….if this approach to phsyics is right we should surely find the Higgs boson, because GUT theory does not work without it, and SUSY particles should be in the reach of LHC because the SUSY approach to the gauge hierarchy problem does not make sense if they are too heavy" (Witten Hertz Lecture 2002 on "Quest for unification", DESY, Hamburg, hep-ph 0207124)

- Q.: "What if we do not see anything at the LHC, maybe just the Higgs?" A. : "younger physicists should figure out what to do".

- What young physicists should figure out then?

- Fortunately, despite no new physics at the LHC (not so far at least), Nature might be suggesting….

….an alternative route to new physics?



- Are some of them primordial black holes?
- GWs as messengers of very early universe physics?

Stochastic background of primordial GWs

- Vibration of cosmic strings;
  (Vachaspati, Vilenkin '85; two recent papers addressing NANOGrav: King, Pascoli,Turner,Zhou 2005.13549; Ellis, Lewicki, 2009.06555)
- During inflation; (Grishchuck '75, Starobinsky '79)
- At preheating; (Khlebnikov, Tkachev '97)
- From the dynamics of extra-dimensions; (Hogan 2000)
- ....
- From strongly first order phase transitions (SFOPT)
  (Witten '84, Hogan '86, Turner and Wilczek '90)

GWs from SFOPTs as signature of new physics

- Though first studied in the SM, both from a QCD phase transition associated to chiral symmetry breaking (Witten '84) and from EWSB (Kamionkowski,Kosowsky,Turner '94), we know today that both SBs would occur as a smooth crossover.

- Most of the attention has then focussed on EWSB in extensions of the SM, especially in connection with EW baryogenesis and especially in SUSY models: in the MSSM and NMSSM (Apreda,Maggiore,Nicolis,Riotto 2002), in generic extensions of the SM with gauge singlets (Kehayias, Profumo 2009): together with the WIMP miracle a very attractive package.

- However, with the strong LHC RUN 2constraints on new physics at the EW scale, a SFOPT associated to EWSB, though not excluded, does not seem as compelling as in the pre-LHC era: we are in a kind of "nothing is impossible" phase on the scale (or scales) of new physics, driven more by experimental anomalies and hints rather than theoretical arguments.

Scale of new physics from GW stochastic backgrounds

- Energy density parameter spectrum:

$$\Omega_{GW0}h^2(f) = \frac{1}{\rho_{c0}h^{-2}}\frac{d\rho_{GW0}}{d\ln f}$$

- Typically the the spectrum of a stochastic background has some characteristic frequency f* at the emission.
  At the detection (assuming a standard cosmological history):

$$\bar{f}_0 = \frac{a_*}{a_0}\bar{f}_* = \left(\frac{g_{S0}}{g_{S*}}\right)^{1/3}\frac{T_0}{T_*}\bar{f}_* \simeq 6\times 10^{-3}\,\mathrm{mHz}\left(\frac{g_*}{106.75}\right)^{1/3}\frac{T_*}{100\,\mathrm{GeV}}\frac{\bar{f}_*}{H_*}$$

In this way we can probe the scale of new physics ~T_* that can be extracted from the signal knowing $\bar{f}_*/H_*$.

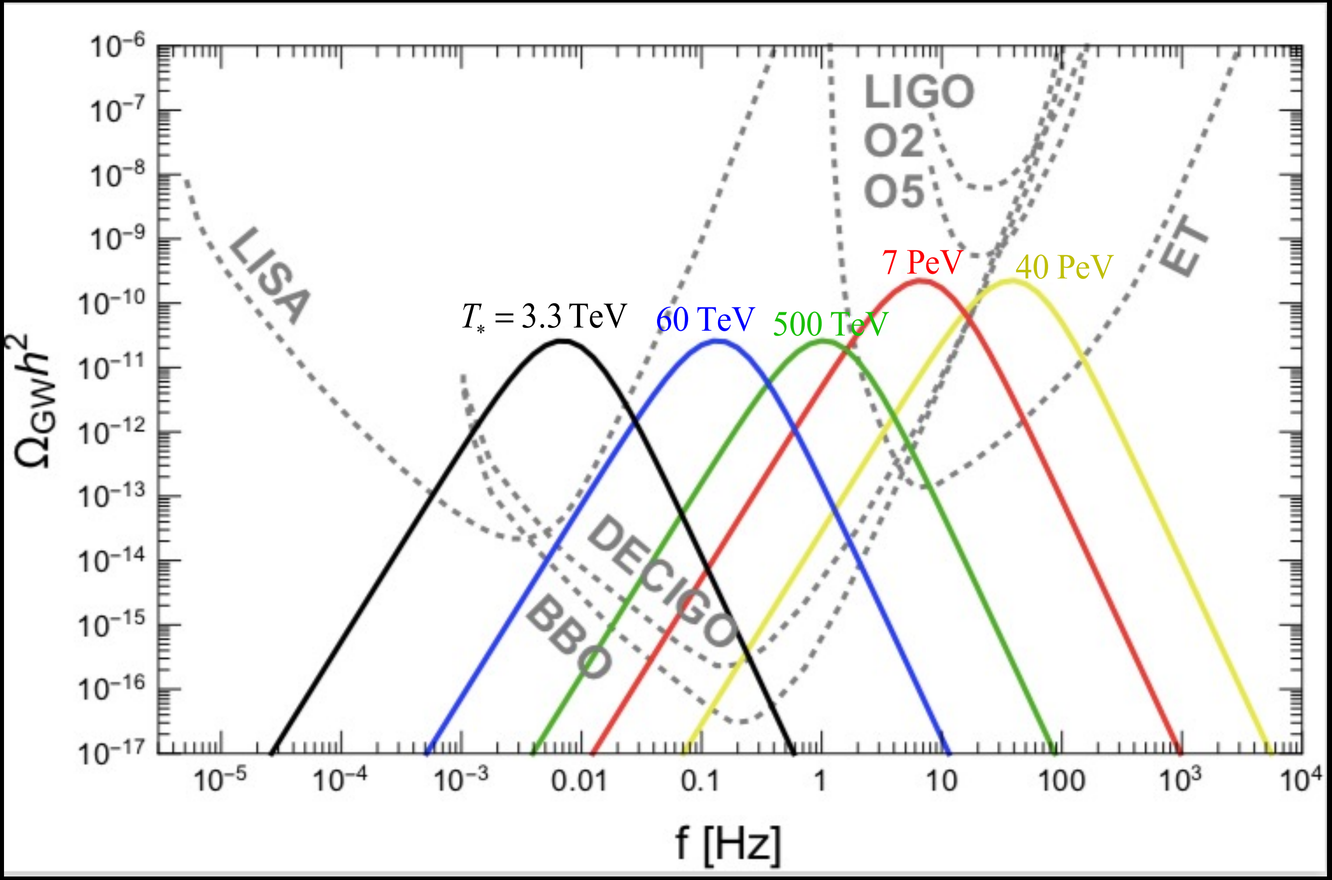
Example: $T_* \sim 100$ GeV, $f_*/H_* \sim 100$

⟹ one expects a signal in the mHZ range that will be tested by LISA

GWs from SFOPTs: tuning the knob

$$\Omega^{\rm PT}_{\rm GW} h^2(f) \simeq \omega(f; f_{\rm peak})$$

(from PDB, D. Marfatia, YL. Zhou 2001.07637)



How to calculate Ω_{GW}? Which scale and what new physics ?

Why going beyond the SM?

Even ignoring:

- (more or less) compelling theoretical motivations
(quantum gravity theory, flavour problem, hierarchy and naturalness problems,…).

- Experimental anomalies (e.g., $(g-2)_\mu$, R_K, R_K^*,…)

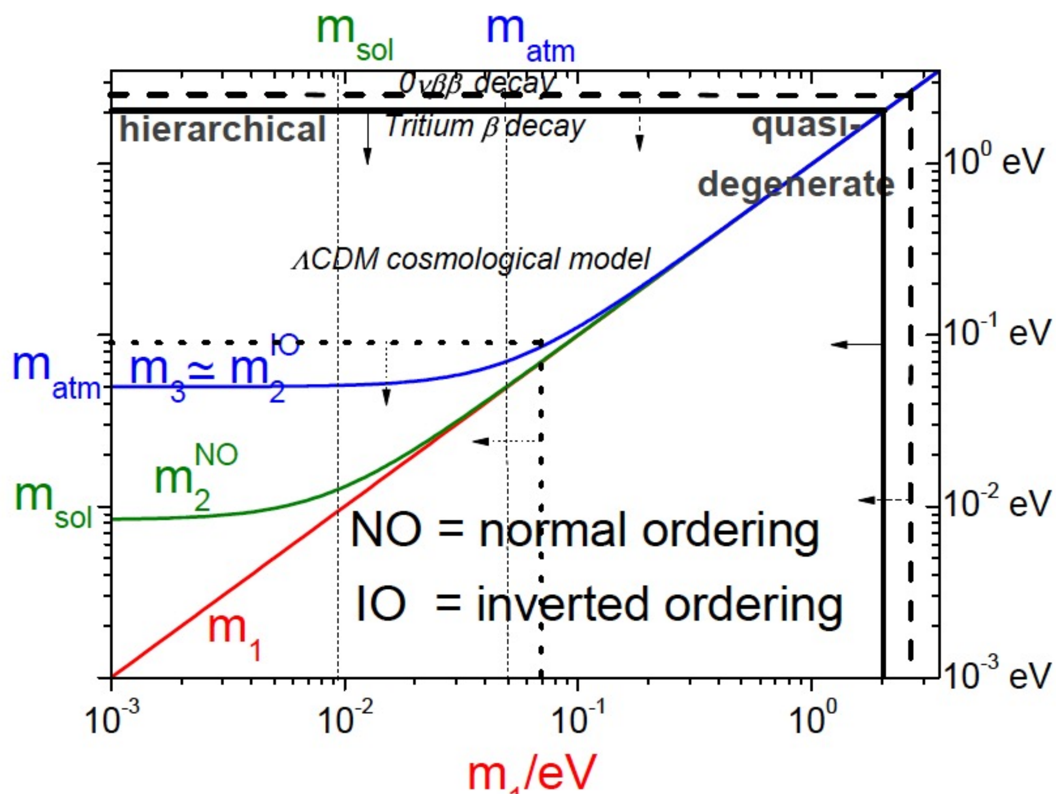
The SM cannot explain:

- *Cosmological Puzzles* :

1. Dark matter
2. Matter - antimatter asymmetry
3. Inflation
4. Accelerating Universe

- *Neutrino masses and mixing*

[redacted]



m_sol
m_atm
0νββ decay
Tritium β decay
hierarchical
quasi-
degenerate
ΛCDM cosmological model
m_atm
m_3 ≃ m_2^IO
m_2^NO
m_sol
NO = normal ordering
IO = inverted ordering
m_1
10^0 eV
10^-1 eV
10^-2 eV
10^-3 eV
10^-3
10^-2
10^-1
10^0
m_1/eV

Minimally extended SM

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_Y^\nu$$

$$-\mathcal{L}_Y^\nu = \overline{\nu_L} h \nu_R \phi \Rightarrow -\mathcal{L}_{Dirac}^\nu = \overline{\nu_L} m_D \nu_R$$ Dirac Mass

(in a basis where charged lepton mass matrix is diagonal)

diagonalising m_D : $$m_D = V_L^\dagger D_{m_D} U_R \qquad D_{m_D} \equiv \begin{pmatrix} m_{D1} & 0 & 0 \\ 0 & m_{D2} & 0 \\ 0 & 0 & m_{D3} \end{pmatrix}$$

$\Rightarrow$ neutrino masses: $m_i = m_{Di}$

leptonic mixing matrix: $U = V_L^\dagger$

<u>But many unanswered questions:</u>

- Why neutrinos are much lighter than all other fermions?
- Why large mixing angles (differently from CKM angles)?
- Cosmological puzzles?
- Why not a Majorana mass term as well?

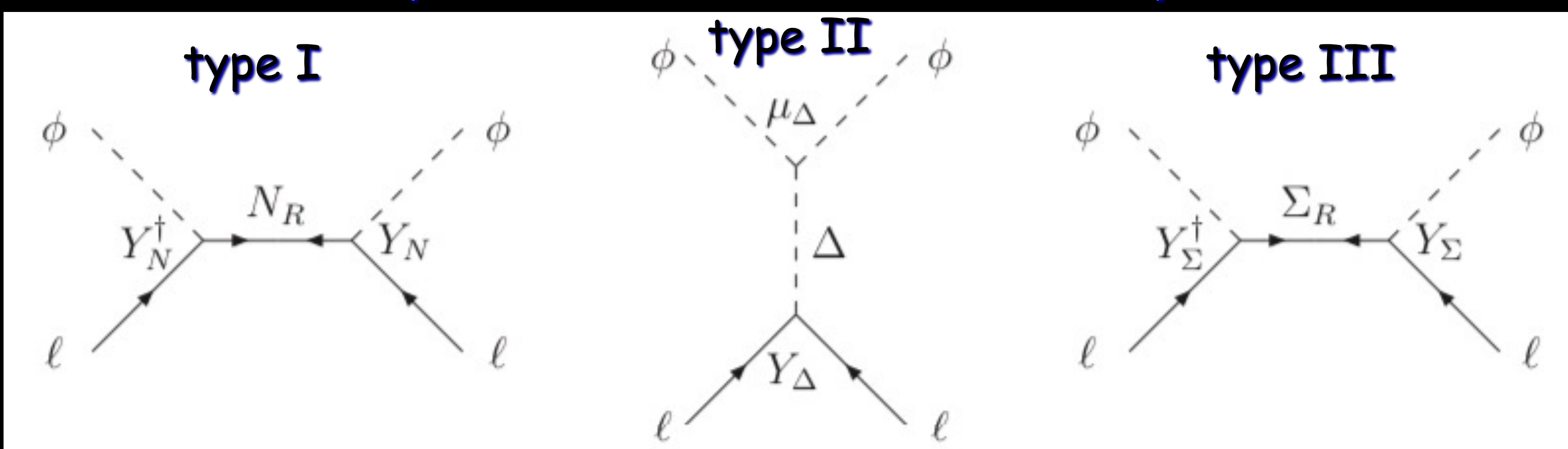
Majorana mass from new physics: effective theory approach

(Weinberg 1979)

$$\mathcal{L}_{\text{Weinberg}} = (\overline{L^c_\alpha}\,\tilde{\phi}^*)\frac{g_{\alpha\beta}}{\Lambda}(\tilde{\phi}^\dagger L_\beta) + h.c.$$

$$\Rightarrow -\mathcal{L}^{\nu}_{\text{Mayorana}} = \overline{\nu^c_{L\alpha}}\, m_{\nu\alpha\beta}\, \nu_{L\beta} \quad \text{with} \quad m_{\nu\alpha\beta} = \frac{g_{\alpha\beta} v^2}{\Lambda} \propto \frac{m_D^2}{\Lambda}$$

At tree level (see 0707.4058 for a review):



Minimal seesaw mechanism (type I)

- Dirac + (right-right) Majorana mass terms

(Minkowski '77; Gell-mann,Ramond,Slansky; Yanagida; Mohapatra,Senjanovic '79)

violates lepton number

$$-\mathcal{L}^{\nu}_{mass} = \overline{\nu}_L m_D \nu_R + \frac{1}{2}\overline{\nu^c_R} M \nu_R + h.c. = -\frac{1}{2}(\overline{\nu_L}\ \overline{\nu^c_R})\begin{pmatrix} 0 & m_D^T \\ m_D & M \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix} + h.c.$$

In the see-saw limit ($M \gg m_D$) the mass spectrum splits into 2 sets:

- 3 light **Majorana neutrinos** with masses (seesaw formula):

$$\mathrm{diag}(m_1, m_2, m_3) = -U^{\dagger}\, m_D\, \frac{1}{M}\, m_D^T\, U^{\star}$$

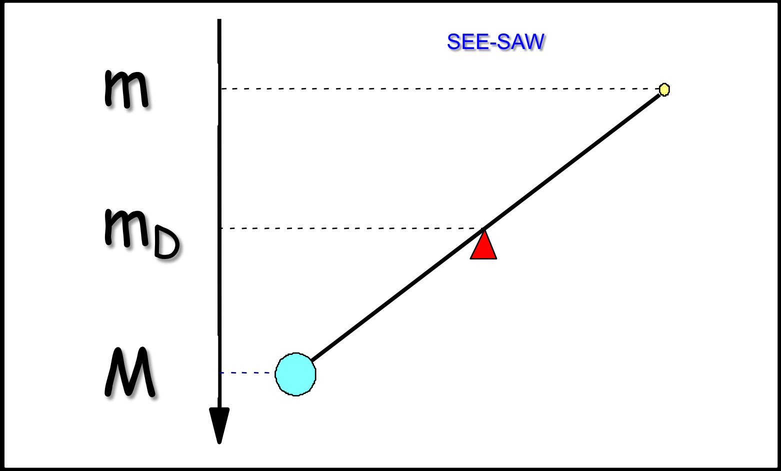
- 3(?) very heavy Majorana neutrinos N_1, N_2, N_3 with $M_3 > M_2 > M_1 \gg m_D$

1 generation toy model :

$m_D \sim m_{top}$,

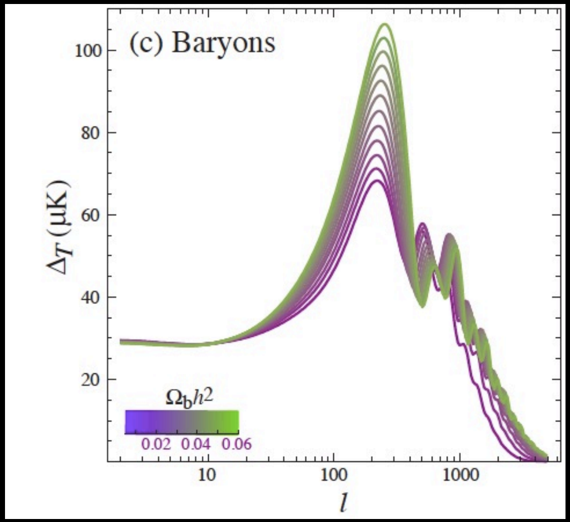
$m \sim m_{atm} \sim 50$ meV

$\Rightarrow M \sim M_{GUT} \sim 10^{15}$ GeV

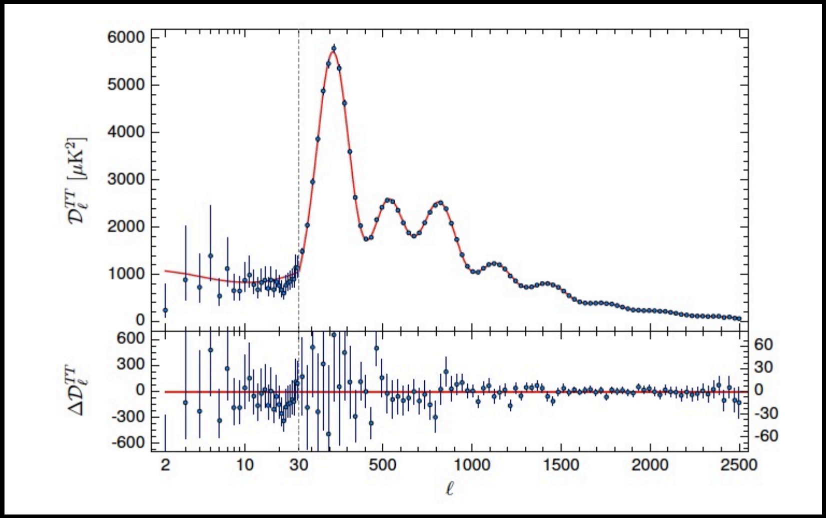


Baryon asymmetry of the universe

(Hu, Dodelson, astro-ph/0110414)



(Planck 2018, 1807.06209)



(CMB+BAO)

$$\Omega_{B0} h^2 = 0.02242 \pm 0.00014$$

$$\eta_{B0} \equiv \frac{n_{B0} - \bar{n}_{B0}}{n_{\gamma 0}} \simeq \frac{n_{B0}}{n_{\gamma 0}} \simeq 273.5\,\Omega_{B0} h^2 \times 10^{-10} = (6.12 \pm 0.04) \times 10^{-10} = \eta_{B0}^{CMB}$$

- Consistent with (older) BBN determination but more precise and accurate
- Asymmetry coincides with matter abundance since there is no evidence of primordial antimatter
- Though all 3 Sakharov conditions are satisfied in the SM, any attempt to reproduce the observed value fails by many orders of magnitude $\Longrightarrow$ it requires NEW PHYSICS!

Minimal scenario of leptogenesis

(Fukugita,Yanagida '86)

- Type I seesaw mechanism
- Thermal production of RH neutrinos: $T_{RH} \gtrsim T_{lep} \simeq M_i / (2 \div 10)$

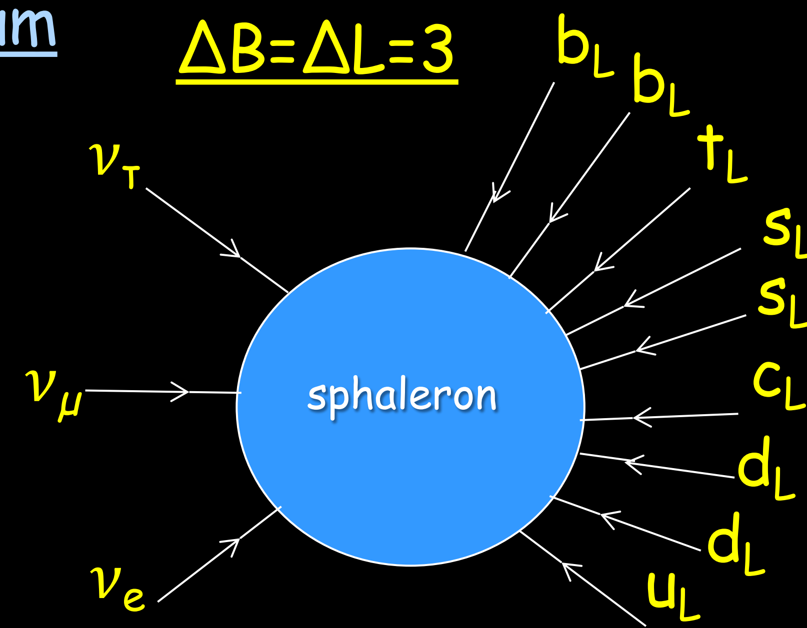
heavy neutrinos decay $N_I \xrightarrow{\Gamma_I} L_I + \phi^\dagger$ $\quad N_I \xrightarrow{\bar{\Gamma}} \bar{L}_I + \phi$

total CP asymmetries $\varepsilon_I \equiv -\dfrac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$ $\Rightarrow$ N_{B-L} production

- Sphaleron processes in equilibrium

$\Rightarrow$ $T_{lep} \gtrsim T^{off}_{sphalerons} \sim 140$ GeV

(Kuzmin,Rubakov,Shaposhnikov '85)

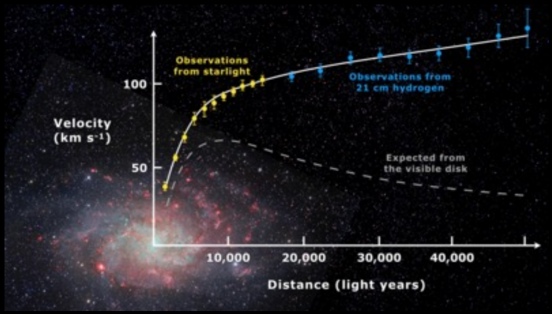


$$\Rightarrow \quad \eta_{B0}^{lep} = \frac{a_{sph} N_{B-L}^{fin}}{N_\gamma^{rec}}$$

Dark Matter

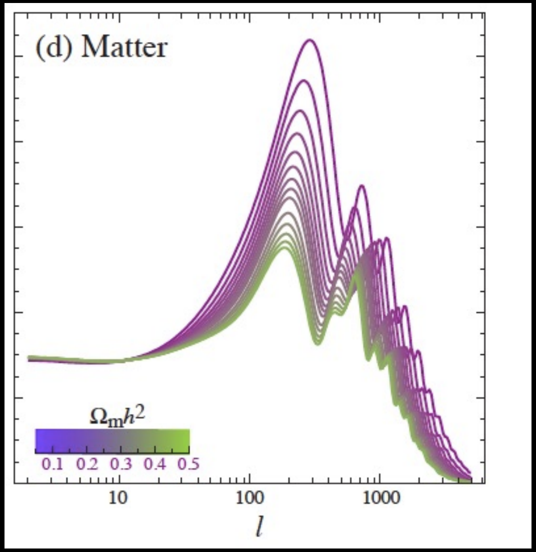
At the present time DM acts as a cosmic glue keeping together

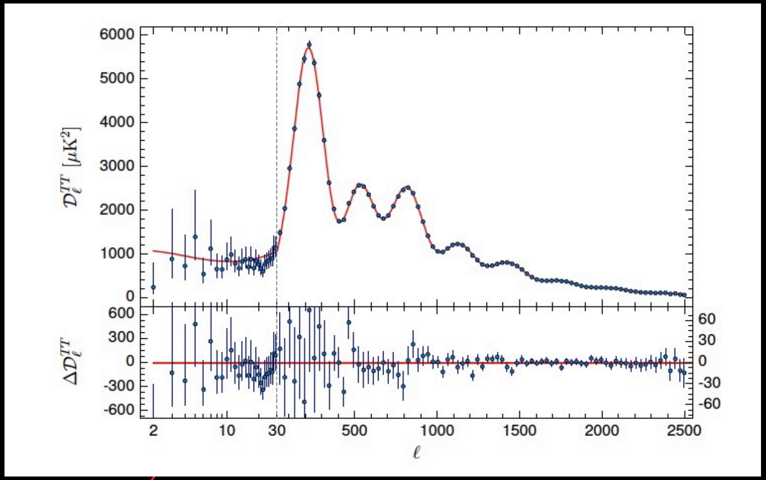
Stars in galaxies.... … and galaxies in cluseters of galaxies (such as in Coma cluster)



But it has to be primordial to understand structure formation and CMB anisotropies

(Hu, Dodelson, astro-ph/0110414) (Planck 2018, 1807.06209)





(CMB + BAO)

$$\Omega_{CDM,0}h^2 = 0.11933 \pm 0.0009 \sim 5\Omega_{B,0}h^2$$

Dark matter from LH-RH neutrino mixing

(Asaka, Blanchet, Shaposhnikov '05)

- LH-RH neutrino mixing

$$\nu = (\nu_L + \nu_L^c) + \frac{m_D}{M}(\nu_R + \nu_R^c)$$

$$N = (\nu_R + \nu_R^c) - \frac{m_D}{M}(\nu_L + \nu_L^c)$$

- For $M_1 \ll m_e \Rightarrow$ $\tau_1 = 5 \times 10^{28} s \left(\frac{M_1}{keV}\right)^{-5} \left(\frac{10^{-8}}{\theta^2}\right) \gg t_0$ $\left(|\theta|^2 \equiv \sum_\alpha |m_{D\alpha 1}/M_1|^2\right)$

- Solving Boltzmann equations abundance is produced at T~100 MeV:

(Dodelson Widrow '94)

$$\Omega_{N_1} h^2 \sim 0.1 \left(\frac{\bar{\theta}}{10^{-4}}\right)^2 \left(\frac{M_1}{keV}\right)^2 \sim \Omega_{DM,0} h^2$$

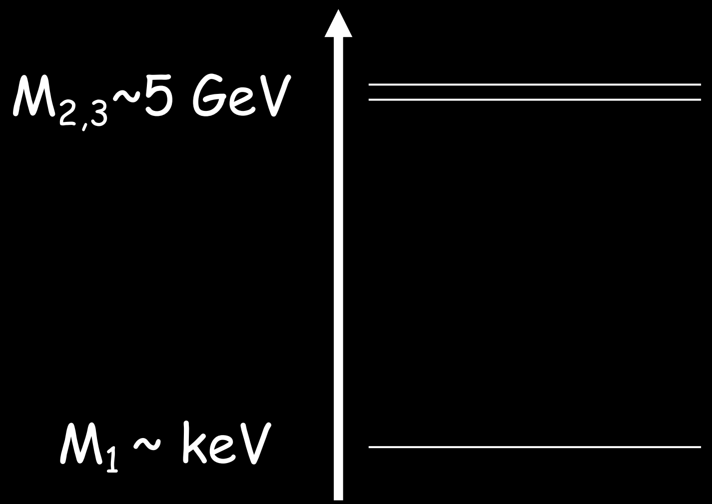
- The lightest neutrino mass is $\lesssim 10^{-5}$ eV $\Longrightarrow$ hierarchical limit
- The N_1's also radiatively decay and this produces constraints from X-ray observations (or opportunities to observe it).
- Considering also structure formation constraints, one is forced to consider a resonant production induced by a large lepton asymmetry (Shi, Fuller '99, Dolgov and Hansen '00)
- L ~10^{-4} (3.5 keV line?). (Horiuchi et al. '14; Bulbul at al. '14; Abazajian '14)

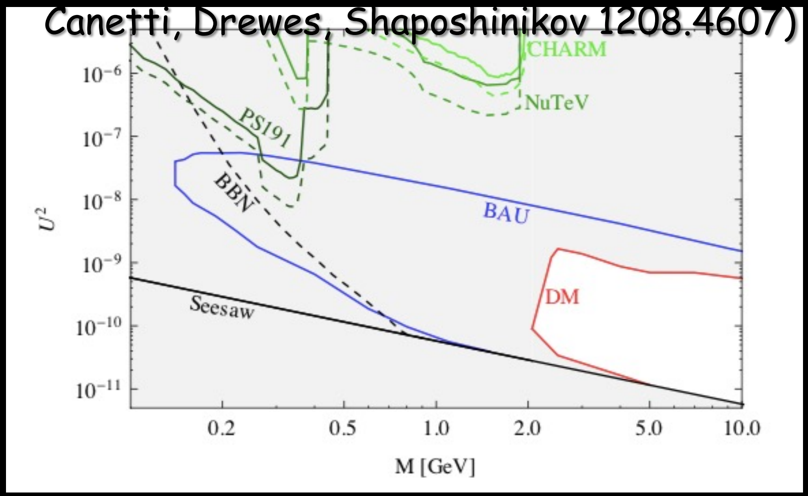
νMSM model

(Asaka,Blanchet,Shaposhnikov '05; Asaka,Shaposhinikov '06; Canetti, Drewes, Shaposhinikov 1208.4607)

- In addition to DM from resonant production in the presence of large asymmetry also the observed BAU is explained by leptogenesis from oscillations (Akhmedov, Rubakov, Smirnov '99)

- The mixing of the two heavier RH neutrinos with quasi-degenerate masses $M_{2,3}$~ 1GeV and ΔM~1eV can reproduce BAU and produce the large asymmetry after sphaleron freeze-out necessary for DM resonant production. Moreover if $M_{2,3} \lesssim 5$ GeV direct tests from meson decays are possible at SHiP .





- However, recent analyses fails to reproduce both asymmetry and DM for such low $M_{2,3}$ masses and M_1 = 7 keV (M.Laine 1905.08814)

Cold dark matter from heavy RH-RH neutrino mixing

(Anisimov,PDB '08)

1 RH neutrino has tiny Yukawa couplings (DARK RH neutrino):

flavour basis

$$m_D \simeq \begin{pmatrix} 0 & m_{De2} & m_{De3} \\ 0 & m_{D\mu 2} & m_{D\mu 3} \\ 0 & m_{D\tau 2} & m_{D\tau 3} \end{pmatrix} \text{ or } \begin{pmatrix} m_{De1} & 0 & m_{De3} \\ m_{D\mu 1} & 0 & m_{D\mu 3} \\ m_{D\tau 1} & 0 & m_{D\tau 3} \end{pmatrix} \text{ or } \begin{pmatrix} m_{De1} & m_{De2} & 0 \\ m_{D\mu 1} & m_{D\mu 2} & 0 \\ m_{D\tau 1} & m_{D\tau 2} & 0 \end{pmatrix}$$

What production mechanism? For high masses just a tiny abundance is needed:

$$N_{DM} \simeq 10^{-9} (\Omega_{DM,0} h^2) N_\gamma^{prod} \frac{\text{TeV}}{M_{DM}}$$

Can the tiny Yukawa couplings be sufficient?

from the flavour to the Yukawa basis

$$m_D = V_L^\dagger D_{m_D} U_R \qquad D_{m_D} \equiv v\, diag(h_A, h_B, h_C) \text{ with } h_A \le h_B \le h_C$$

$$\boxed{\tau_{DM} = \frac{4\pi}{h_A^2\, M_{DM}} = 0.87 h_A^2\, 10^{-23} \frac{\text{GeV}}{M_{DM}} s} \Rightarrow \boxed{\tau_{DM} > \tau_{DM}^{\min} \simeq 10^{28} s \Rightarrow h_A < 3\times 10^{-26} \sqrt{\frac{GeV}{M_{DM}} \times \frac{10^{28} s}{\tau_{DM}^{\min}}}}$$

Too small to reproduce the correct abundance with any production mechanism, so we can simply assume h_A=0 and U_R=I in the following (we can focus on two RH neutrino mixing assuming no mixing with a third RH neutrino

Higgs portal RH neutrino mixing DM

(Anisimov '06, Anisimov,PDB '08)

Assume new interactions with the standard Higgs:

$$\mathcal{L}_{5-\mathrm{dim}} = \frac{\lambda_{IJ}}{\Lambda}\,\phi^\dagger\phi\,\overline{N_J^c}N_I \qquad (I,J=A,B,C)$$

In general they are non-diagonal in the Yukawa basis: this generates a RH neutrino mixing. Consider two RH neutrino mixing between a dark and a source RH neutrino and consider medium effects: these can be described by effective potentials into the Hamiltonian:

from the Yukawa interactions: $V_{IJ}^{Y} = \frac{T^2}{8E_J}\,h_J^2\,\delta_{IJ}$

from the new interactions: $V_{IJ}^{\Lambda} = \frac{T^2}{12\Lambda}\,\lambda_{IJ}$

Effective mixing angle from misalignment: $\sin 2\theta_\Lambda(T) = T^3/(\tilde{\Lambda}\,\Delta M^2)$ $\quad \tilde{\Lambda} = \Lambda/\lambda_{mix}$ $\quad \Delta M^2 = M_S^2 - M_{DM}^2$

effective mixing Hamiltonian (in the monocromatic approximation: p≃3T):

$$\Delta H_{IJ} \simeq \begin{pmatrix} -\frac{\Delta M^2}{12T} - \frac{T^2}{48T}h_S^2 & \frac{T^2}{12\tilde{\Lambda}} \\ \frac{T^2}{12\tilde{\Lambda}} & \frac{\Delta M^2}{12T} + \frac{T^2}{48T}h_S^2 \end{pmatrix} \Rightarrow \sin 2\theta_\Lambda^{\rm m} = \frac{\sin 2\theta_\Lambda}{\sqrt{(1+v_{\rm S}^{Y})^2 + \sin^2 2\theta_\Lambda}}$$

$$v_{\rm S}^{Y} \equiv T^2\,h_{\rm S}^2/(4\,\Delta M^2)$$

If $\Delta M^2 < 0$ ($M_{DM} > M_S$) there is a resonance for $v_S^Y = -1$ at: $\quad T_{res} = \frac{2\sqrt{|\Delta M^2|}}{h_S}$

Non-adiabatic conversion

(Anisimov,PDB '08; P.Ludl.PDB,S.Palomarez-Ruiz '16)

Adiabaticity parameter at the resonance

$$\gamma_{\rm res} \equiv \left.\frac{|E^{\rm m}_{\rm DM} - E^{\rm m}_{\rm S}|}{2\,|\dot{\theta}_m|}\right|_{\rm res} = \sin^2 2\theta_\Lambda(T_{\rm res})\,\frac{|\Delta M^2|}{12\,T_{\rm res}\,H_{\rm res}}\,,$$

Can we use a Landau-Zener formula ?

$$\left.\frac{N_{N_{\rm DM}}}{N_{N_{\rm S}}}\right|_{\rm res} \simeq \frac{\pi}{2}\,\gamma_{\rm res}$$

(remember that we need only a small fraction to be converted so necessarily $\gamma_{res} \lll 1$)

$$\Rightarrow \quad \Omega_{\rm DM}\,h^2 \simeq \frac{0.15}{\alpha_{\rm S}\,z_{\rm res}}\left(\frac{M_{\rm DM}}{M_{\rm S}}\right)\left(\frac{10^{20}\,{\rm GeV}}{\widetilde{\Lambda}}\right)^2\left(\frac{M_{\rm DM}}{\rm GeV}\right)$$

For successful dark-matter genesis $\Rightarrow$

$$\widetilde{\Lambda}_{\rm DM} \simeq 10^{20}\sqrt{\frac{1.5}{\alpha_{\rm S}\,z_{\rm res}}\,\frac{M_{\rm DM}}{M_{\rm S}}\,\frac{M_{\rm DM}}{\rm GeV}}\,{\rm GeV}$$

2 options: either $\Lambda < M_{Pl}$ and $\lambda_{AS} \lll 1$ or $\lambda_{AS} \sim 1$ and $\Lambda \ggg M_{Pl}$:

it is possible to think of models in both cases.

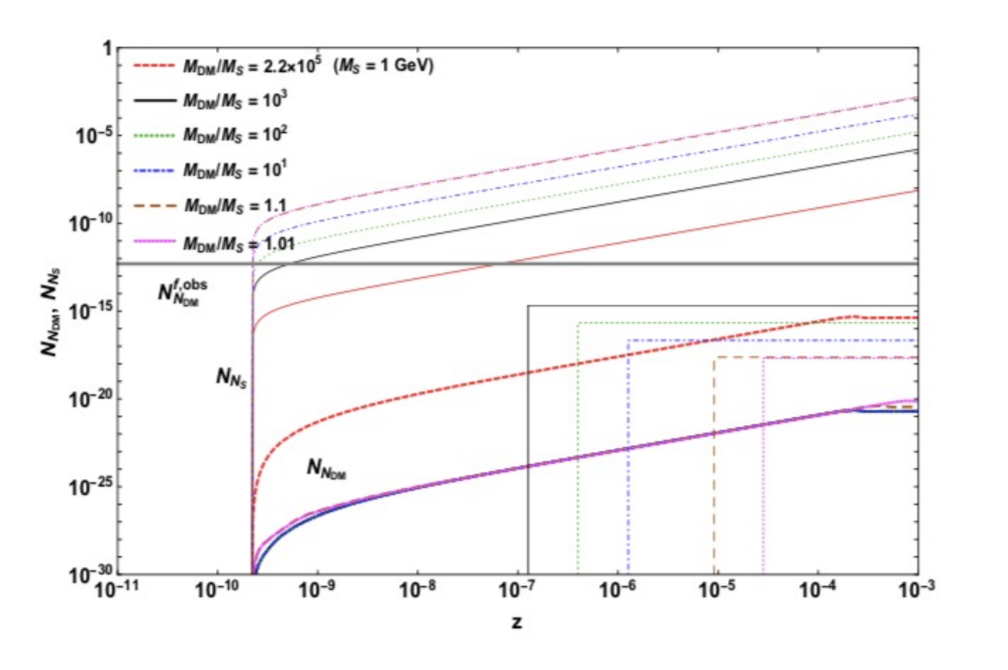
Density matrix calculation of the relic abundance

(PDB, K. Farrag, R. Samanta, Y. Zhou, 1908.00521)

Density matrix equation for the DM-source RH neutrino system

$$\frac{dN_{IJ}}{dt} = -i\,[\mathcal{H}, N]_{IJ} - \begin{pmatrix} 0 & \frac{1}{2}(\Gamma_D + \Gamma_S)\,N_{\rm DM-S} \\ \frac{1}{2}(\Gamma_D + \Gamma_S)\,N_{\rm S-DM} & (\Gamma_D + \Gamma_S)\,(N_{N_S} - N_{N_S}^{\rm eq}) \end{pmatrix}$$

A numerical solution shows that a Landau-Zener overestimated the relic Abundance by a few orders of magnitude (especially in the hierarchical case)

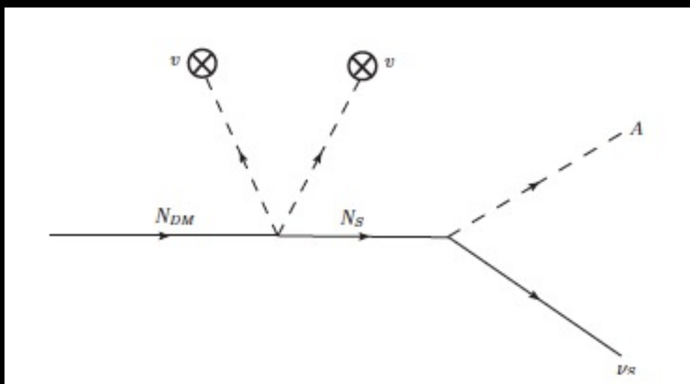


Constraints from decays

(Anisimov,PDB '08; Anisimov,PDB'10; P.Ludl.PDB,S.Palomarez-Ruiz'16)

2 body decays

DM neutrinos unavoidably decay today into A+leptons (A=H,Z,W) through the same mixing that produced them in the very early Universe



$$\theta_{\Lambda 0} = \frac{2\, v^2/\widetilde{\Lambda}}{M_{\rm DM}\,(1 - M_{\rm S}/M_{\rm DM})}.$$

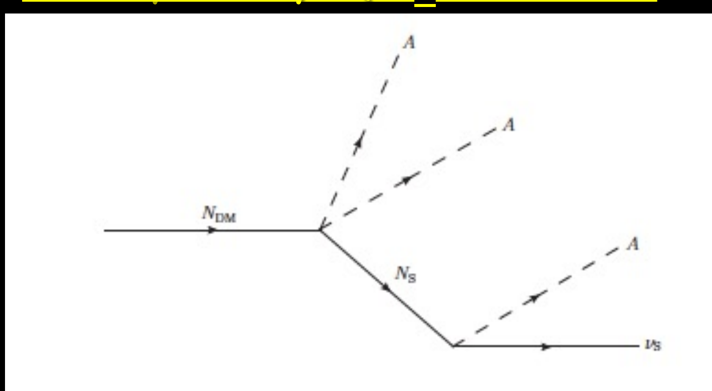
mixing angle today

Lower bound on M_{DM} ($\tau_{28} \equiv \tau_{DM}^{min}/10^{28}$s)

$$M_{\rm DM} \geq M_{\rm DM}^{\rm min} \simeq 54\,{\rm TeV}\,\alpha_{\rm S}\,\tau_{28}\left(\frac{M_{\rm S}}{M_{\rm DM}}\right)$$

4 body decays (M_S>100 GeV)

$$N_{\rm DM} \to 2\,A + N_{\rm S} \to 3\,A + \nu_{\rm S}\ (A = W^{\pm}, Z, H).$$



Upper bound on M_{DM} ($\tau_{28} \equiv \tau_{DM}^{min}/10^{28}$s)

$$M_{\rm DM} \lesssim 5.3\,{\rm TeV}\,\alpha_{\rm S}^{-\frac{2}{3}}\,z_{\rm res}^{-\frac{1}{3}}\,\tau_{28}^{-\frac{1}{3}}\left(\frac{N_{N_{\rm S}}}{N_\gamma}\right)_{\rm res}^{\frac{1}{3}}\left(\frac{M_{\rm DM}}{M_{\rm S}}\right)^{\frac{2}{3}}$$

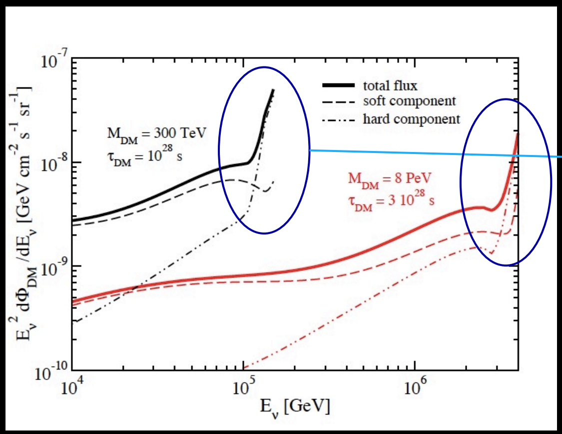
3 body decays and annihilations also can occur but yield weaker constraints

It nicely predicts a signal at IceCube

(Anisimov,PDB,0812.5085;PDB, P.Ludl,S. Palomarez-Ruiz 1606.06238)

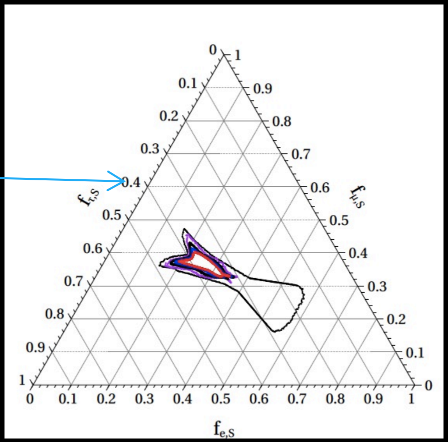
- DM neutrinos unavoidably decay today into A+leptons (A=H,Z,W) through the same mixing that produced them in the very early Universe
- Potentially testable high energy neutrino contribution

Energy neutrino flux



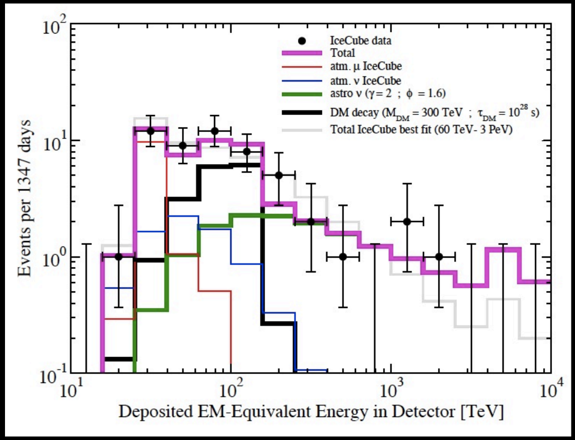
Hard component

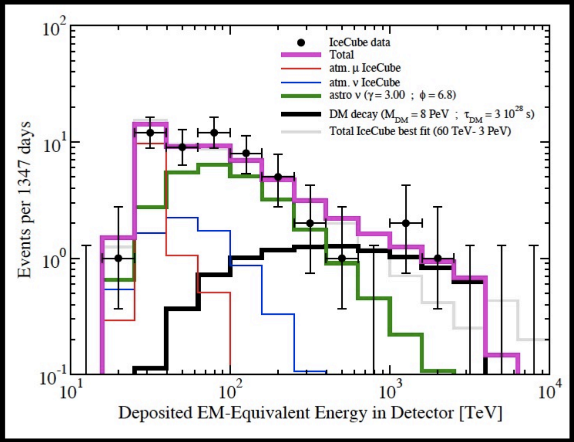
Flavour composition at the detector



Neutrino events at IceCube: 2 examples

M_{DM}=300TeV





M_{DM}=8 PeV

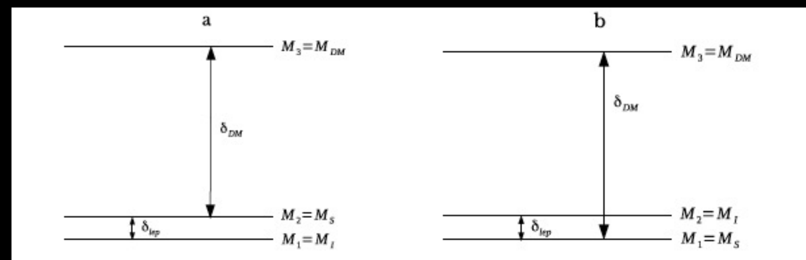
Unifying Leptogenesis and Dark Matter

(PDB, NOW 2006;Anisimov,PDB,0812.5085;PDB, P.Ludl,S. Palomarez-Ruiz 1606.06238)

- Interference between N_A and N_B can give sizeable CP decaying asymmetries able to produce a matter-antimatter asymmetry but since $M_{DM}>M_S$ necessarily $N_{DM}=N_3$ and $M_1 \simeq M_2 \Rightarrow$ leptogenesis with quasi-degenerate neutrino masses

$$\delta_{DM} \equiv (M_3 - M_S)/M_S$$

$$\delta_{lep} \equiv (M_2 - M_1)/M_1$$



$$\varepsilon_{i\alpha} \simeq \frac{\overline{\varepsilon}(M_i)}{K_i} \left\{ \mathcal{I}_{ij}^{\alpha}\, \xi(M_j^2/M_i^2) + \mathcal{J}_{ij}^{\alpha}\, \frac{2}{3(1 - M_i^2/M_j^2)} \right\}$$

(Covi,Roulet,Visssani '96)

$$\overline{\varepsilon}(M_i) \equiv \frac{3}{16\,\pi} \left(\frac{M_i\, m_{\rm atm}}{v^2} \right) \simeq 1.0 \times 10^{-6} \left(\frac{M_i}{10^{10}\,{\rm GeV}} \right),$$

$$\xi(x) = \frac{2}{3} x \left[(1+x) \ln\left(\frac{1+x}{x} \right) - \frac{2-x}{1-x} \right],$$

Analytical expression for the asymmetry:

$$\eta_B \simeq 0.01\, \frac{\overline{\varepsilon}(M_1)}{\delta_{\rm lep}}\, f(m_\nu, \Omega),$$

Efficiency factor

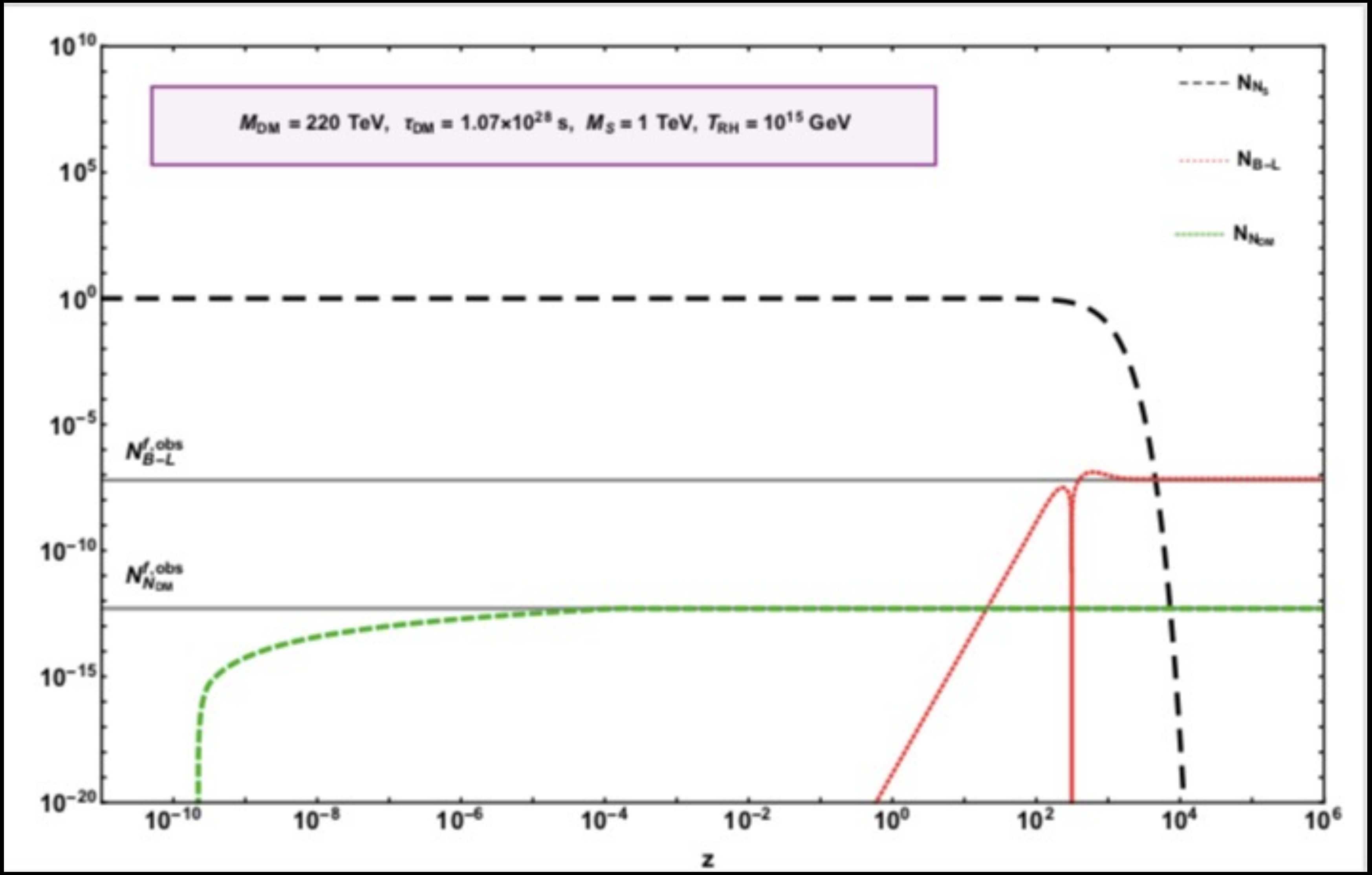
$$f(m_\nu, \Omega) \equiv \frac{1}{3} \left(\frac{1}{K_1} + \frac{1}{K_2} \right) \sum_{\alpha} \kappa(K_{1\alpha} + K_{2\alpha}) \left[\mathcal{I}_{12}^{\alpha} + \mathcal{J}_{12}^{\alpha} \right],$$

- $M_S \gtrsim 2\, T_{sph} \simeq 300$ GeV $\Rightarrow$ 10 TeV $\lesssim M_{DM} \lesssim$ 1 PeV
- $M_S \lesssim$ 10 TeV
- $\delta_{lep} \sim 10^{-5} \Rightarrow$ leptogenesis is not fully resonant

Unifying Leptogenesis and Dark Matter

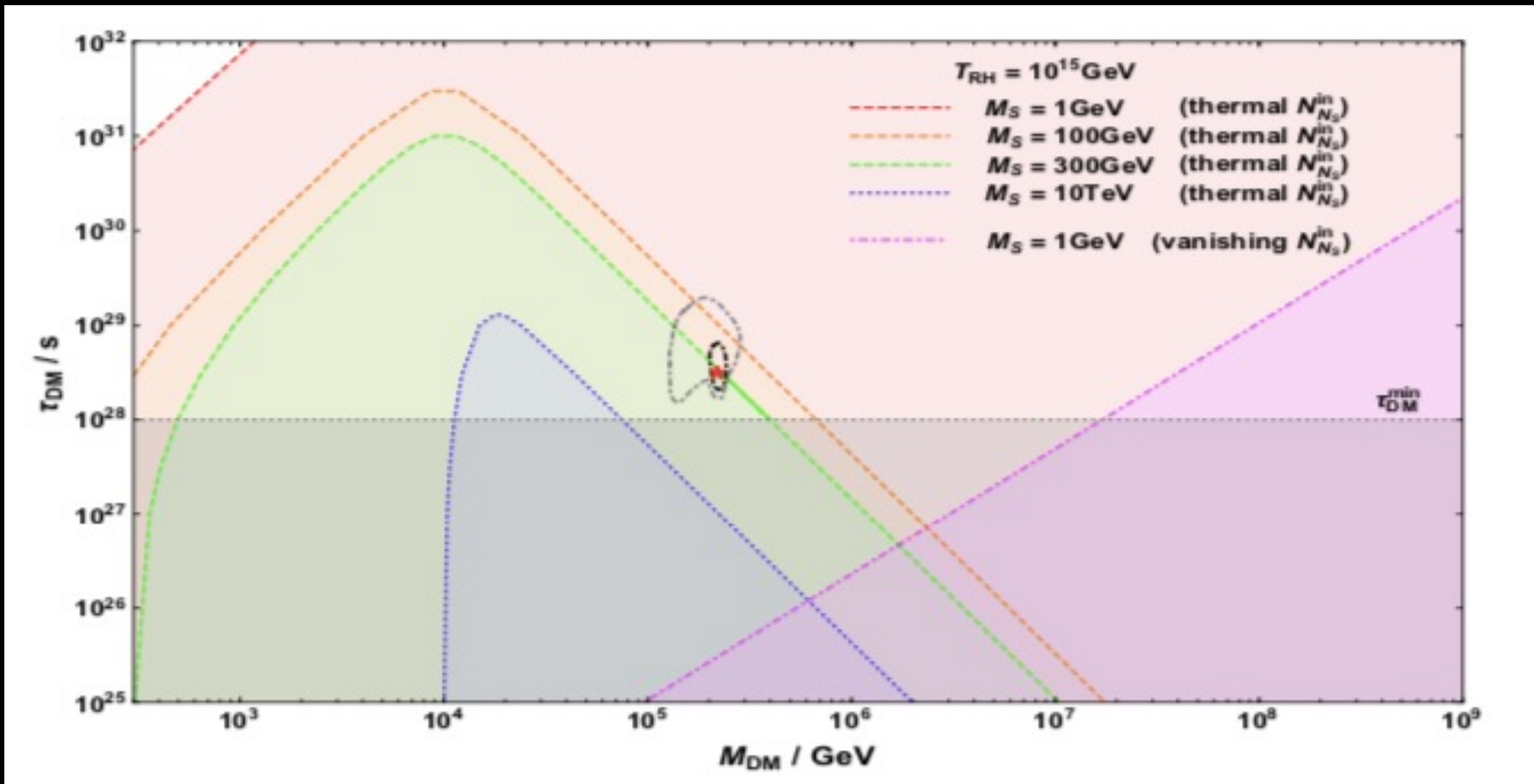
(PDB, K. Farrag, R. Samanta, Y. Zhou, 1908.00521)

A solution for initial thermal N_S abundance:



Density matrix calculation of the relic abundance

(PDB, K. Farrag, R. Samanta, Y. Zhou, 1908.00521)



Solutions only for initial thermal N_S abundance, unless M_S~ 1 GeV

Mass varying source right-handed neutrino

(PDB, D. Marfatia, YL. Zhou 2001.07637)

$$-\mathcal{L}_\lambda = \frac{1}{2} M_{\rm DM}\, \overline{N_{\rm DM}^c}\, N_{\rm DM} + \frac{1}{2} M_{\rm D}\, \overline{N_{\rm D}^c}\, N_{\rm D} + \frac{\lambda_{\rm S}}{2}\, \eta\, \overline{N_{\rm S}^c}\, N_{\rm S} + \frac{1}{\tilde{\Lambda}}\, \Phi^\dagger\, \Phi\, \overline{N_{\rm D}^c}\, N_{\rm S} + \frac{1}{\tilde{\Lambda}}\, \Phi^\dagger\, \Phi\, \overline{N_{\rm DM}^c}\, N_{\rm D} + \text{h.c.}\,. \quad (1)$$

The scalar field η acquires a vev v_η during a first order phase transition and accordingly N_S acquires a space-time dependent mass:

$$M_s(x,t) = \lambda_s\, v_\eta(x,t)$$

The bubble wall profile is well described by a kink solution found in the thin wall approximation:

$$v_\eta(r,t) = \frac{1}{2}\,\bar{v}_\eta \left[1 - \tanh\left(\frac{r - v_{\rm w}\,(t - t_\star)}{\Delta_{\rm w}}\right)\right]\,,$$

Thermal effects and density matrix equation

(PDB, D. Marfatia, YL. Zhou 2001.07637)

Also this time we need to account for thermal masses from both interactions:

$$\widetilde{M}^2_{\rm S}(x,t) = M^2_{\rm S}(x,t) + \frac{T^2}{4}\,h^2_{\rm S} + \frac{T^2}{8}\,\lambda^2_{\rm S}\,N_{N_{\rm S}}\,N_\eta$$

We get then the following effective Hamiltonian:

$$\Delta\mathcal{H}_{IJ} \simeq \begin{pmatrix} -\frac{\Delta\widetilde{M}^2}{4\,p} & \Delta H_{\rm mix} \\ \Delta H_{\rm mix} & \frac{\Delta\widetilde{M}^2}{4\,p} \end{pmatrix}$$

$$\Delta H_{\rm mix} \equiv T^2/(12\,\widetilde{\Lambda})$$

$$\widetilde{\Delta M^2}(x,t) = M_S^2(x,t) - M_D^2$$

And again we can write a density matrix equation (I,J = D,S):

$$\frac{dN_{IJ}}{dt} = -i\,[\Delta\mathcal{H}, N]_{IJ} - \begin{pmatrix} 0 & \Gamma_{\rm dec} \\ \Gamma_{\rm dec} & \Gamma_{\rm prod} \end{pmatrix}$$

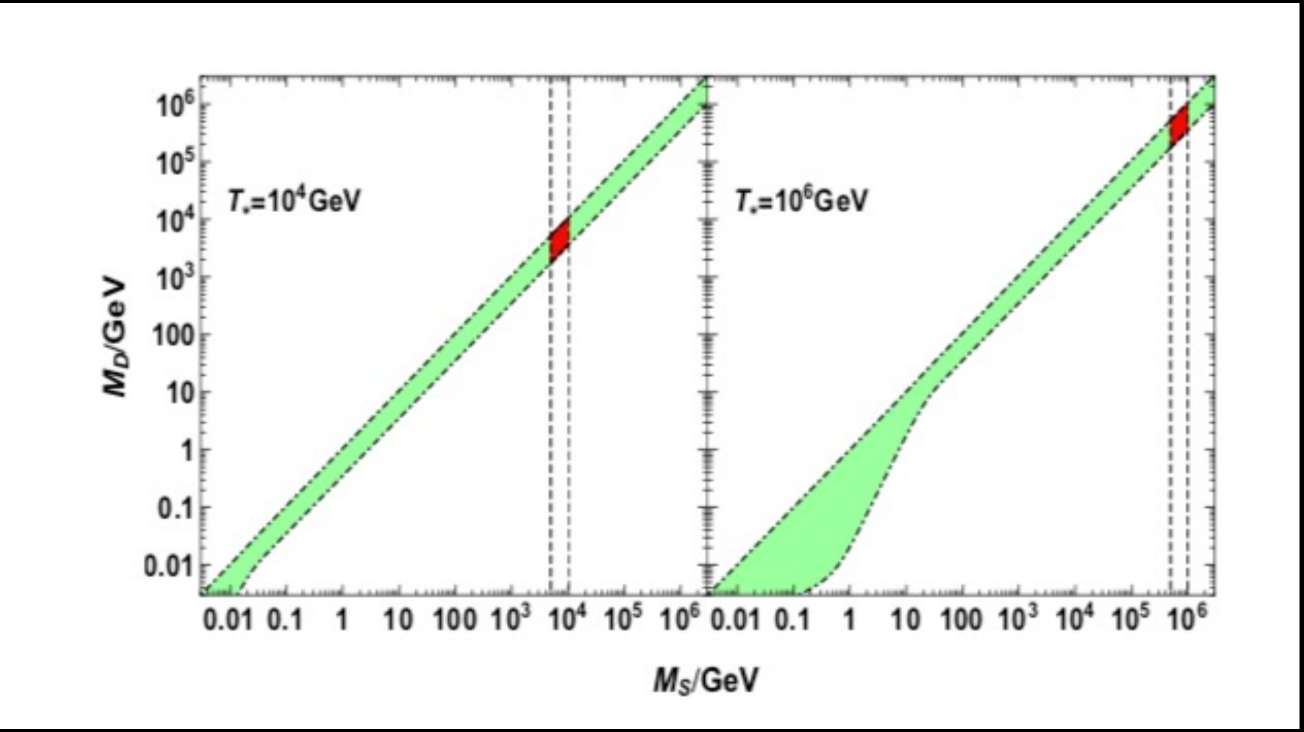
$$\frac{dN_{N_S}}{dt} = -\Gamma_{prod}(N_{N_S} - N_{N_S}^{eq}) \quad with \;\; \Gamma_{prod} \simeq 2\,\Gamma_{\eta\to N_S N_S} \Rightarrow N_{N_S} \simeq N_{N_S}^{eq}$$

Resonance condition

(PDB, D. Marfatia, YL. Zhou 2001.07637)

$$\Delta\widetilde{M}^2(r, t_{\rm res}) = 0, \iff \widetilde{M}_{\rm S}^2(r, t_{\rm res}) = M_{\rm D}^2$$

$$\iff \frac{M_{\rm D}^2}{M_{\rm S}^2} \simeq \left[\frac{1}{2} - \frac{1}{2}\tanh\left(\frac{r - v_{\rm w}\,(t_{\rm res} - t_\star)}{\Delta_{\rm w}}\right)\right]^2 + \frac{T_\star^2}{6\bar{v}_\eta^2},$$



Dark matter genesis

(PDB, D. Marfatia, YL. Zhou 2001.07637)

This time a Landau-Zener formula as far as the bubble wall moves fast enough and the resonance width is larger than the oscillation length:

$$N_{N_D}^{\rm res} \simeq 12\pi\, N_{N_S}(T_\star)\, T_\star \left.\frac{\Delta H^2_{\rm mix}}{d\Delta\widetilde{M}^2/dt}\right|_{t_{\rm res}}$$

Calculating explicitly the time derivative:

$$N_{N_D}^{\rm res} \simeq \frac{N_{N_S}(T_\star)\,(\pi/48)\,(\Delta_{\rm w}/v_{\rm w})\,T_\star^5}{\widetilde{\Lambda}^2\, M_S^2(r,t_{\rm res})\,[1 - M_S(r,t_{\rm res})/M_S]},$$

All N_D's decay to N_{DM} via $N_D \to N_{DM}+2A$ and the contribution to the dark matter energy density parameter is then given by:

$$\Omega_{N_{\rm DM}}\, h^2 = 1.0875 \times 10^6\, N_{N_D}^{\rm res}\, \frac{M_{\rm DM}}{\rm GeV}$$

To be compared with the measureded value

$$\Omega_{\rm DM}\, h^2 = 0.11933 \pm 0.00091$$

Constraints from dark matter decays

(PDB, D. Marfatia, YL. Zhou 2001.07637)

Since the DM mass is below the Higgs and gauge boson mass the dominant decaying mode is DM → νe^+e^-

$$\Gamma_{N_{\rm DM}\to\nu\,\ell_\alpha^+\ell_\alpha^-} = \frac{(\theta_{\Lambda 0}^{\rm D-S}\,\theta_{\Lambda 0}^{\rm DM-D})^2}{96\,\pi^3}\,\frac{\overline{m}_\alpha}{M_{\rm S}}\,G_{\rm F}^2\,M_{\rm DM}^5\,,$$

$\overline{m}_\alpha \simeq 0.1\, m_{sol} \sim 1\,\mathrm{meV}$

$$\theta_{\Lambda 0}^{\rm D-S(DM-D)} = 2v^2/(\widetilde{\Lambda}\,(M_{\rm S(D)} - M_{\rm D(DM)}))$$

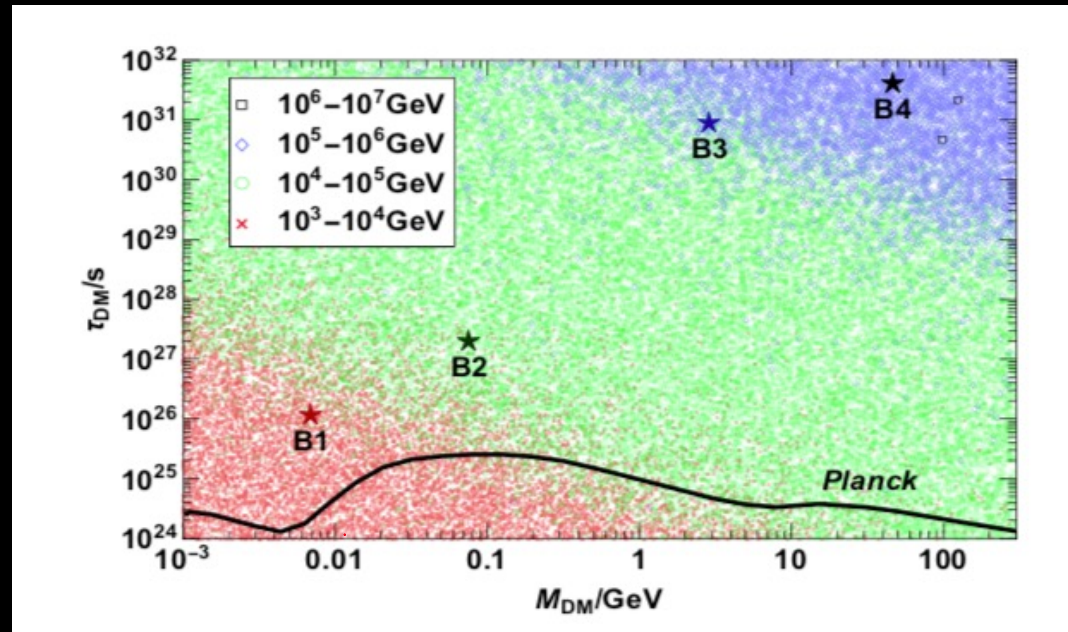
One has constraints both from CMB anisotropies (decays affect reionization history) and from X and γ-ray diffuse backgrounds (EGRET, FERMI, INTEGRAL) placing a lower bound $\tau_{\rm DM} \gtrsim 10^{25}\,\mathrm{s}$

Searching for solutions

(PDB, D. Marfatia, YL. Zhou 2001.07637)

The scan is done over $M_{DM}, M_S, T^*, \tau_{DM}$

All points reproduce $\Omega_{DM}h^2 \sim 0.12$



	$\frac{T_\star}{\text{PeV}}$	$\frac{\tau_{DM}}{10^{26}\text{s}}$	$\frac{M_S}{\text{TeV}}$	$\frac{M_D}{\text{TeV}}$	$\frac{M_{DM}}{\text{GeV}}$	v_w	α	$\frac{\beta}{H_\star}$
B1	$3\cdot10^{-3}$	1.219	1.57	0.567	$7\cdot10^{-3}$	0.90	0.10	10
B2	0.016	21.26	12.9	7.72	0.077	0.90	0.10	10
B3	0.106	$9.25\cdot10^4$	93.3	72.6	2.92	0.90	0.10	10
B4	1.052	$4.24\cdot10^5$	666	666	46.69	0.95	0.15	5
B5	10.75	$4.69\cdot10^{17}$	$8.7\cdot10^3$	$5.3\cdot10^3$	175.8	0.95	0.15	5

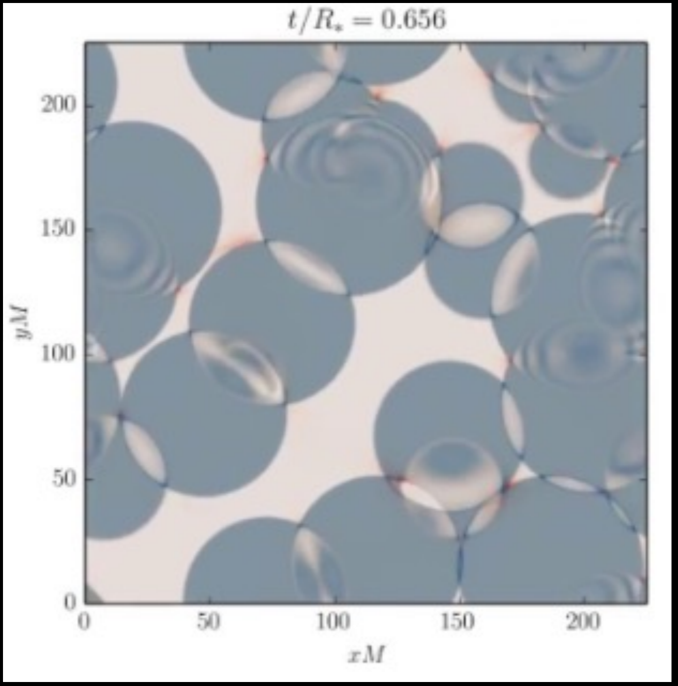
TABLE I: Benchmark points obtained for $v_w/\Delta_w = T_\star/50$.

Gravitational waves from first order phase transitions

(See Caprini et al. 1512.06239 for details)

3 contributions: bubble wall collision, sound waves and turbolence

$$h^2\Omega_{\rm GW} \simeq h^2\Omega_\phi + h^2\Omega_{\rm sw} + h^2\Omega_{\rm turb}\,.$$



In our case the sound wave contribution is realistically the dominant one:

$$h^2\Omega_{\rm sw}(f) = 2.65\times10^{-6}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}} v_w\, S_{\rm sw}(f)$$

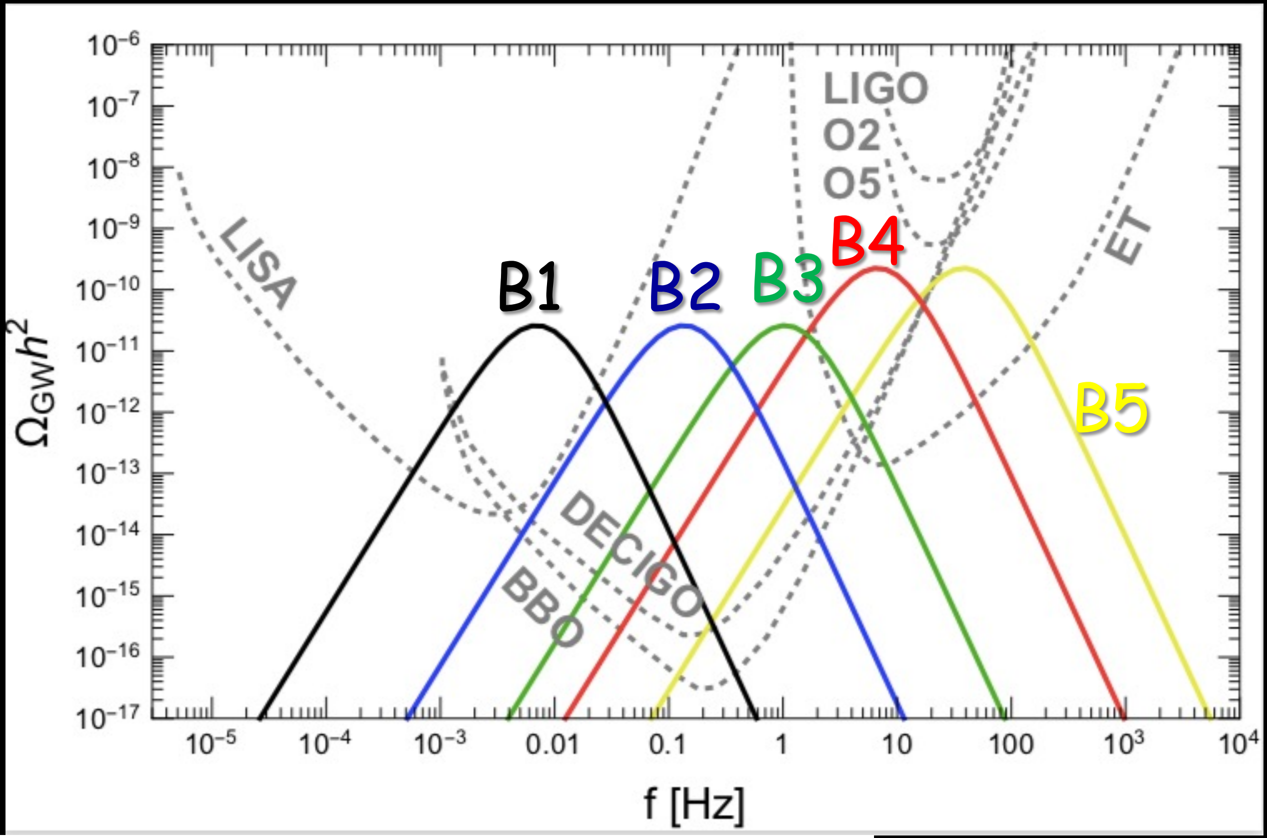
$$S_{\rm sw}(f) = (f/f_{\rm sw})^3\left(\frac{7}{4+3\,(f/f_{\rm sw})^2}\right)^{7/2}$$

Peak frequency

$$f_{\rm sw} = 1.9\times10^{-2}\,{\rm mHz}\,\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

Testing dark matter genesis at GW interferometers

(PDB, D. Marfatia, YL. Zhou 2001.07637)



	$\frac{T_*}{\text{PeV}}$	$\frac{\tau_{DM}}{10^{26}\text{s}}$	$\frac{M_S}{\text{MeV}}$	$\frac{M_{DM}}{\text{MeV}}$	v_w	α	β/H_*
B1	0.0033	0.37	5.35	1.80	0.90	0.10	10
B2	0.0635	1.78	133	19.73	0.90	0.10	10
B3	0.486	1.08	805	37.36	0.90	0.10	10
B4	6.64	3.80	6.4×10^3	176.1	0.95	0.15	5
B5	37.7	0.74	411×10^3	15.4×10^3	0.95	0.15	5

Notice that if T_* is sufficiently large then $M_S \gtrsim 300\text{GeV}$: one can have also successful leptogenesis

Can this scenario be realised within a model ?

(PDB, D. Marfatia, YL. Zhou, in preparation)

- Yes, we do find a model with a polinomial potential but more realistically $\beta/H_* \gtrsim 100$ for α~0.1 (in agreement with Ellis, Lewicki, No 2020) but still the signal might be detected by future interferometers

- At low scales ($T_* < 1$ MeV) one could interestingly try to reproduce NANOGrav results claiming discovery of a stochastic background at much lower frequencies (10^{-8}Hz) and also get some extra radiation able to relax partially Hubble tension
(PDB,King, Merle 2013; Nakai,Suzuki, Takahashi, Yamada 2020)

First order phase transition in the early universe

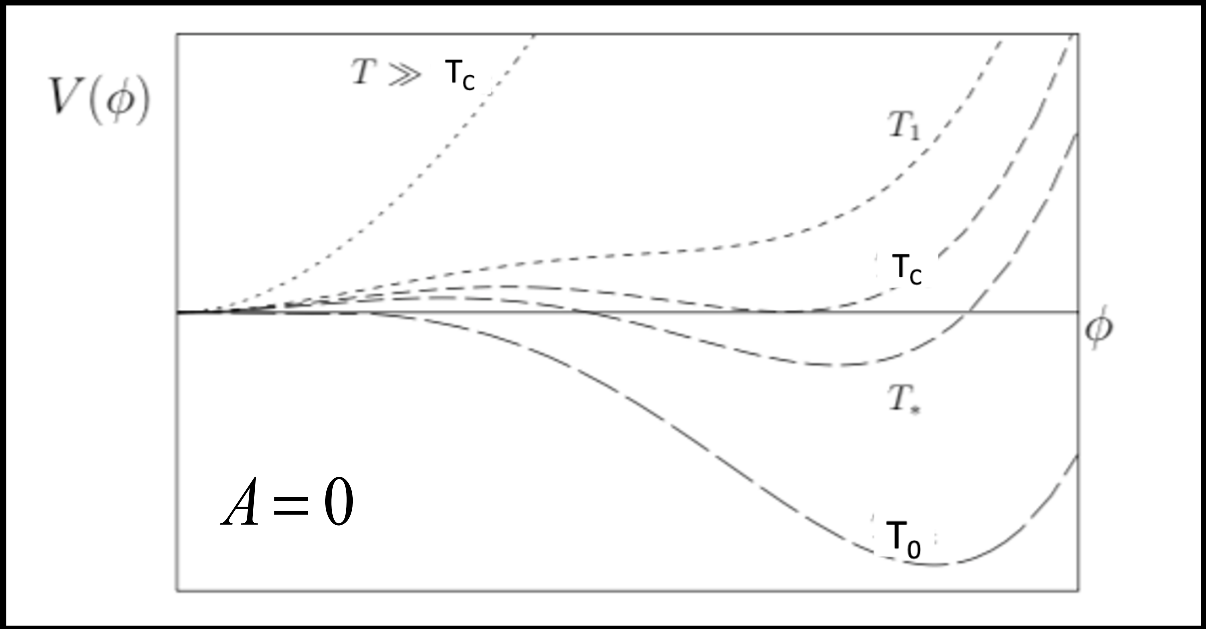
(Kirzhnits,Linde '72; Dolan, Jackiw '74; Anderson, Hall '92; Dine et al. '92; Quiros '98, Curtin et al. 2016)

effective potential

$$V(\phi,T) = V_{\text{tree}}(\phi) + \sum_i V^i_{\text{CW}}(\phi) + \sum_i V^i_{\text{T}}(\phi,T)$$

1-loop zero T → $\sum_i V^i_{\text{CW}}$; 1 loop thermal potential → $\sum_i V^i_{\text{T}}$

$$\simeq D(T-T_0)^2\phi^2 - (ET+A)\phi^3 + \frac{\lambda(T)}{4}\phi^4$$



This picture relies on the validity of perturbative expansion and in the SM this would imply $M_H < M_W$. With the large M_h measured value, there is not even a PT in the SM, just a smooth crossover.

From the effective potential to the Euclidean action

(Coleman '77; Linde '82;)

Probability of bubble nucleation per unit volume per unit time

$$\Gamma(T) = \Gamma_0(T)\, e^{-S_E(T)}$$

$$\begin{cases} \Gamma_0 = \mathcal{O}(1)\, T^4 \\ S_E(T \geq T_c) \to \infty \\ S_E(T \to T_0) \to 0 \end{cases}$$

Euclidean action

$$S_E(\phi) = \int d\tau\, d^3x \left[\frac{1}{2}\left(\frac{d\phi}{d\tau}\right)^2 + \frac{1}{2}\left(\vec{\nabla}\phi\right)^2 + V(\phi) \right]$$

At finite temperatures

$$S_E(\phi,T) = \int_0^{1/T} d\tau\, d^3x \left[\frac{1}{2}\left(\frac{d\phi}{d\tau}\right)^2 + \frac{1}{2}\left(\vec{\nabla}\phi\right)^2 + V(\phi) \right] \xrightarrow{T \gg R^{-1}(0)} \frac{S_3(\phi,T)}{T}$$

Spatial Euclidean action

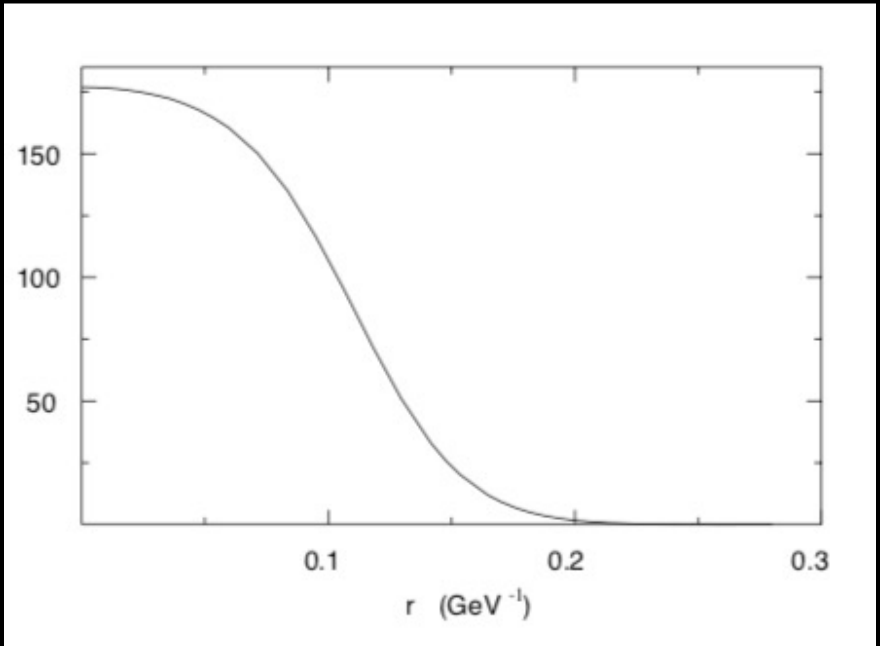
$$S_3(\phi,T) = \int d^3x \left[\frac{1}{2}\left(\vec{\nabla}\phi\right)^2 + V(\phi,T) \right] = 4\pi \int dr\, r^2 \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi,T) \right]$$

Euler-Lagrange Equation for the bubble solution

$$\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + \frac{2}{r}\frac{d\phi}{dr} - \frac{\partial V(\phi,T)}{\partial \phi} = 0\,, \qquad \phi(r=\infty) = 0\,, \qquad \left.\frac{d\phi}{dr}\right|_{r=0} = 0$$

In general a bounce solution is found numerically by (overshooting-undershooting) trials and errors procedure (in the `thin-wall' approximation the kink solution is found analytically.

Typical solution (T_*~100 GeV)

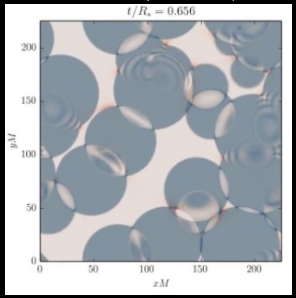


From the Euclidean action to the GW spectrum

(Kamionkowski,Kosowsky,Turner '93;Apreda et al 2001; Grogejan,Servant 2006; Ellis,Lewicki,No 2020)

time and temperature of nucleation

$$\int_o^{t_*} \frac{dt\,\Gamma}{H^3} \sim 1 \Rightarrow \int_{T_*}^{\infty} \frac{dT}{T}\left(\frac{90}{8\pi^3 g_*}\right)^2 \left(\frac{T}{M_P}\right)^4 e^{-S_3/T} = 1 \Rightarrow \frac{S_3(T_*)}{T_*} \approx -4\ln\left(\frac{T_*}{M_P}\right) \Rightarrow T_*$$



More precisely T* has to be identified with the *percolation temperature,*
Slightly more involved definition than the nucleation temperature

$$\beta = \frac{\dot{\Gamma}}{\Gamma}, \quad \Gamma = \Gamma_0\, e^{-S(t)} \simeq \Gamma_0 e^{-S(t_*)}\, e^{-\left.\frac{dS}{dt}\right|_{t_*}(t-t_*)} \Rightarrow \beta \simeq -\left.\frac{dS}{dt}\right|_{t_*} \Rightarrow \frac{\beta}{H_*} = T_* \left.\frac{d(S_3/T)}{dT}\right|_{T_*}$$

Notice that $\beta/2\pi$ gives the characteristic frequency f* of the FOPT while β the time scale of its duration

Latent heat freed in the PT

$$\varepsilon = -\Delta V(\phi) - T\Delta s = V(\phi_{\text{false}}) - V(\phi_{true}) + T\frac{\partial V}{\partial T} \Rightarrow \alpha = \frac{\varepsilon(T_*)}{\rho_R(T_*)}$$

Strength of the PT

From α and β one can calculate the GW spectrum: $\Omega_{\text{GW}}^{\text{PT}} h^2(f) \simeq \omega(f;\alpha,\beta/H_*)$

Example: sound wave contribution

(See Caprini et al. 1512.06239 for details)

3 contributions: bubble wall collision, sound waves and turbolence

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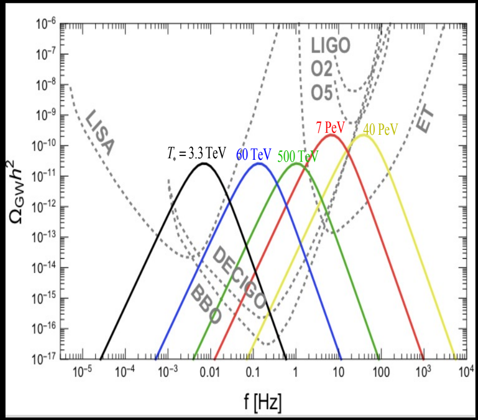
Bubble wall velocity (detonation)

$$v_w = \frac{\sqrt{1/3}+\sqrt{\alpha^2+2\alpha/3}}{1+\alpha} \geq c_s$$

Efficiency factor

$$\kappa = \frac{\sqrt{\alpha}}{0.135+\sqrt{0.98+\alpha}}$$



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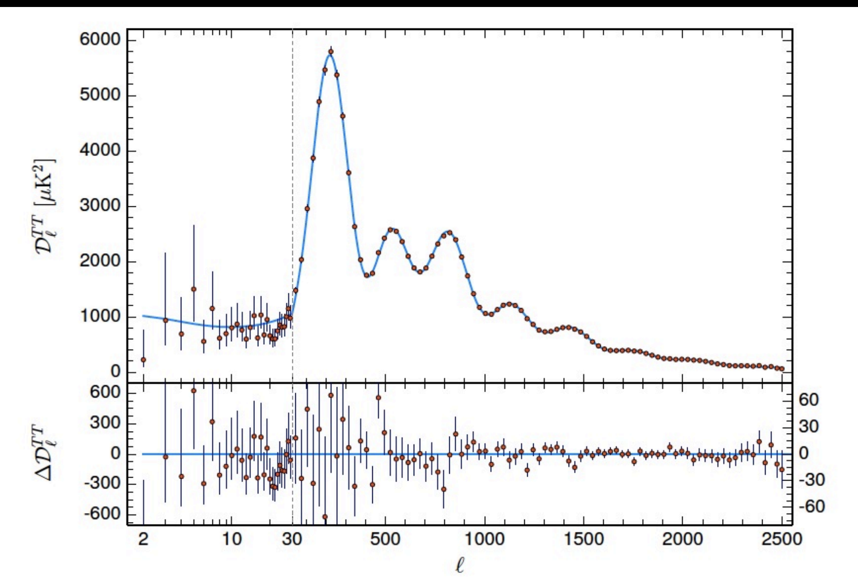
SUMMARY

- Origin of (Majorana) neutrino mass can not only reproduce neutrino masses with type I seesaw models and address cosmological origin of matter but it might also give rise to a GW stochastic background.
- RH-RH neutrino mixing can thermalize a dark particle from some hidden sector via Higgs portal interactions
- This can reproduce the DM abundance but also give rise to specific signatures
- The production can be also associated to the same first order phase transition also responsible for neutrino mass genesis
- The phase transition has to be 1st order and this generates GWs
- The spectrum can be within the sensitivity of future GW interferometers
- Also decays can give signatures (cosmological or in X and γ-ray diffuse backgrounds

ΛCDM model

It is a minimal flat cosmological model with only 6 parameters : baryon and cold dark matter abundances, angular size of sound horizon at recombination, reionization optical depth, amplitude and spectral index of primordial perturbations.

ΛCDM best fit to the *Planck* 2018 data (TT+TE+EE+low E+lensing)
(Planck Collaboration, *arXiv 1807.06209*)



Parameter	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10}A_s)$	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9649 ± 0.0042	0.9665 ± 0.0038

(Planck 2018 results, 1807.06209)

Planck results are in good agreement with BAO, SNe and galaxy lensing observations. The only significant (~4σ) tension is with local measurement of the Hubble constant

In the ΛCDM model, expansion is described by a flat Friedmann-Lemaitre cosmological model