SEMESTER 1 (2016-17): FIRST REVIEW PROBLEM SHEET

(POSTED ON FRIDAY 11 NOVEMBER 2016)

Cosmology and the early Universe

Duration: 1 WEEK

This paper contains 5 questions.

Answer **all** questions in **Section A**

To be handed in Faculty Reception (Building 59) indicating your ID number

DEADLINE : Friday 18 November at 2 pm

This test will contribute 5% to the overall mark

The default system of units is SI. You may give expressions in natural units, but you should state when you start to use them and note when you change unit systems. Throughout the paper the scale factor is normalized in such a way that at the present time $a_0 = 1$.

Section A

A1. Consider the value of the Hubble constant $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

(i) Convert this value in hertz (Hz);

(ii) Calculate the present value of the Hubble radius, $R_{H,0} \equiv c\,H_0^{-1}$, in Gpc (2 significant figures).

[1]

A2. Consider Hubble's law,

$$z = \frac{H_0}{c}\,d_L\,,$$

relating the cosmological redshift z of an astronomical object to its luminosity distance d_L and valid for redshift $z \lesssim 0.1$. How is the redshift defined in terms of the wavelength of the radiation at emission, $\lambda_{\rm em}$, and the observed wavelength, $\lambda_{\rm obs}$? How is the cosmological redshift interpreted within the Milne-McCrea model based on a Newtonian description of the cosmological expansion? How is the cosmological redshift interpreted within the Friedmann models based on General Relativity and how is it expressed in terms of the scale factor at the time of the emission $a_{\rm em}$? [1]

A3. The physical size of the visible component of our Galaxy at the present time is about $25\,\mathrm{kpc}$. Consider a future time $t_\star$ such that $a(t_\star) = 3$. What will the approximate physical size of the of the visible component of our Galaxy be at $t_\star$ neglecting any astrophysical evolution? What will the comoving size of the of the visible component of our Galaxy be at $t_\star$? [1]

A4. Consider a 4-dim Euclidean space with metric

$$d\ell^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$$

and an embedded 3-dim hyperbolic space (also called 3-dim pseudo-sphere) described by the parametric equation

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = -R^2 \,,$$

with R positive. Show that the metric for this space can be written as

$$d\ell^2 = R^2 \left(\frac{dr^2}{1+r^2} + r^2 \, d\Omega^2 \right) \,,$$

with r and Ω properly defined. [1]

A5. Consider a Universe described by an equation of state $p = w\,\varepsilon$ with $w = -2/3$.

(i) Find $\varepsilon(a)$, i.e. how the energy density ε depends on the scale factor a;

(ii) Calculate the age of the Universe in Gigayears using $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

[1]

END OF PAPER