UNIVERSITY OF SOUTHAMPTON PHYS6005W1

SEMESTER 1 (2016-17): SECOND REVIEW PROBLEM SHEET

(POSTED ON FRIDAY 9 DECEMBER 2016)

COSMOLOGY AND THE EARLY UNIVERSE

Duration: 1 WEEK

This paper contains 5 questions.

Answer **all** questions in **Section A**

To be handed in Faculty Reception (Building 59) indicating your ID number

DEADLINE : Friday 16 December at 2 pm

This test will contribute 5% to the overall mark

The default system of units is SI. You may give expressions in natural units, but you should state when you start to use them and note when you change unit systems. Throughout the paper the scale factor is normalized in such a way that at the present time $a_0 = 1$.

Section A

A1. Given the present value of the Λ-like fluid contribution to the total energy density parameter, $\Omega_{\Lambda,0} = 0.7$, and using for the Hubble constant $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, calculate (in Gpc) what would be the radius R_E of an Einstein static Universe with the same amount of energy density ε_Λ. [1]

A2. Calculate the lookback time (in years) for a quasar of redshift $z = 10$ in a Einstein-de Sitter model with $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. [1]

A3. The Draco galaxy is a dwarf galaxy within the Local Group. Its luminosity is $L = 1.8 \times 10^5\,L_\odot$ and half of its total mass is contained within a sphere of radius $r_h = 120\,\mathrm{pc}$. The mean square velocity of the red giant stars in the Draco galaxy is $\langle v^2 \rangle = 10^4\,\mathrm{km^2\,s^{-2}}$. Assuming that the Draco galaxy obeys the virial theorem and that its gravitational potential energy is given by $W = -\alpha\,G\,M^2/r_h$ with $\alpha = 0.45$, what is its mass M (in solar mass units)? What is its mass-to-light ratio (in solar units)? [1]

A4. Consider the deceleration parameter $q(t) \equiv -\ddot{a}(t)/[H^2(t)\,a(t)]$ at an arbitrary time t. Assuming that the Universe expansion is described by a flat Lemaitre model with $\Omega_{M,0} = 0.3$ and $\Omega_{\Lambda,0} = 0.7$, what is the asymptotic value of $q(t)$ for $t \to \infty$? [1]

A5. Using for the current age of the Universe $t_0 = 14\,\mathrm{Gyr}$, for the CMB radiation temperature $k_B\,T_0 = 2 \times 10^{-4}\,\mathrm{eV}$, estimate (2 significant figures) the age of the Universe at the recombination time in a flat Lemaitre model with $\Omega_{M,0} = 0.3$ and $\Omega_{\Lambda,0} = 0.7$. [1]

END OF PAPER

