What is ν?
INVISIBLES 12 and Alexei Smirnov Fest
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Leptogenesis confronting neutrino data

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The double side of Leptogenesis

Cosmology (early Universe) ← → Neutrino Physics, New Physics

- *Cosmological Puzzles* :
1. Dark matter
2. Matter - antimatter asymmetry
3. Inflation
4. Accelerating Universe

- *New stage in early Universe history* :

T		
< 10^{14} GeV	—	Inflation
	—	Leptogenesis
100 GeV	—	EWSSB
0.1- 1 MeV	—	BBN
0.1- 1 eV	—	Recombination

Leptogenesis complements low energy neutrino experiments testing the seesaw mechanism high energy parameters

Can Leptogenesis be useful to overconstrain the seesaw parameter space providing a way to understand the measured values of the neutrino parameters or to make predictions on future measurements ?

Neutrino mixing parameters

pre-T2K

(Gonzalez-Garcia, Maltoni 2008)

- best-fit point and 1σ (3σ) ranges:

$$\theta_{12} = 34.5 \pm 1.4 \left(^{+4.8}_{-4.0}\right), \quad \Delta m^2_{21} = 7.67\,^{+0.22}_{-0.21} \left(^{+0.67}_{-0.60}\right) \times 10^{-5}\ \mathrm{eV}^2,$$

$$\theta_{23} = 43.1\,^{+4.4}_{-3.5} \left(^{+10.1}_{-8.0}\right), \quad \Delta m^2_{31} = \begin{cases} -2.39 \pm 0.12 \left(^{+0.37}_{-0.40}\right) \times 10^{-3}\ \mathrm{eV}^2, \\ +2.49 \pm 0.12 \left(^{+0.39}_{-0.36}\right) \times 10^{-3}\ \mathrm{eV}^2, \end{cases}$$

$$\theta_{13} = 3.2^{+4.5} \left(^{+9.6}\right), \quad \delta_{\mathrm{CP}} \in [0, 360];$$

Nonvanishing θ_{13}

- T2K : $\sin^2 2\theta_{13}$ = 0.03 – 0.28 (90% CL NO)
- DAYA BAY: $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$
- RENO: $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$

recent global analyses

$\theta_{13} = 7.7° \div 10.2°$ (95% CL)

$\theta_{23} = 36.3° \div 40.9°$ (95% CL)

$\delta_{best\ fit} \sim \pi$

(Normal Ordering)

(Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno 2012)

Analogous results presented by T. Schwetz but $\delta_{best\ fit} \sim -\pi/3$

Neutrino masses: $m_1 < m_2 < m_3$

neutrino mixing data

2 possible schemes: normal or inverted

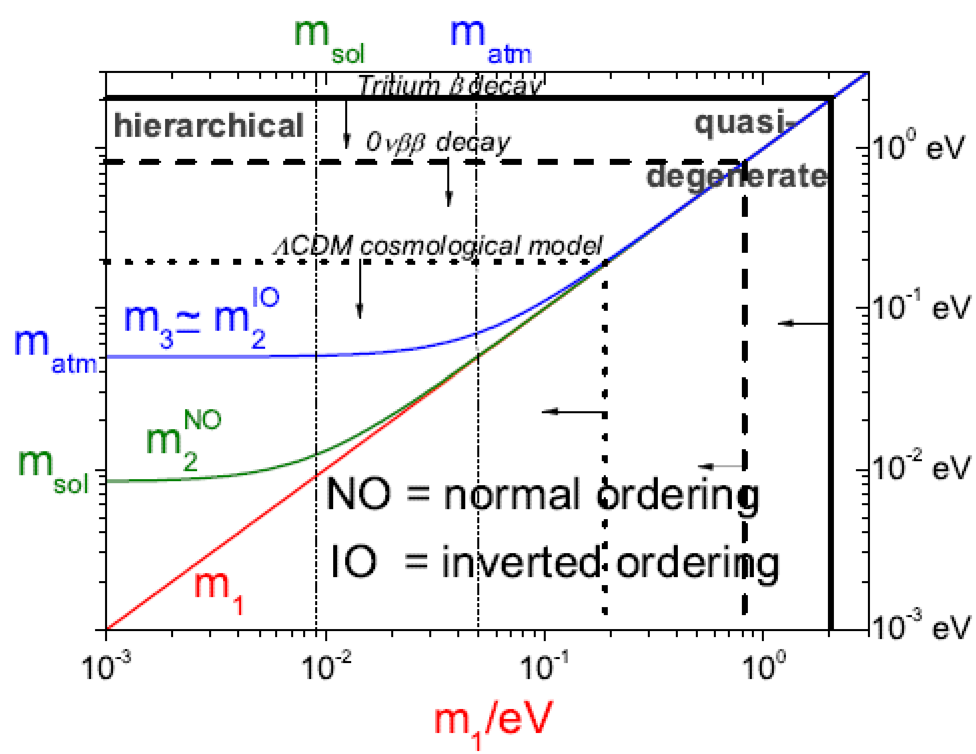
$$m_3^2 - m_2^2 = \Delta m^2_{\rm atm} \text{ or } \Delta m^2_{\rm sol} \qquad m_{\rm atm} \equiv \sqrt{\Delta m^2_{\rm atm} + \Delta m^2_{\rm sol}} \simeq 0.05\,{\rm eV}$$

$$m_2^2 - m_1^2 = \Delta m^2_{\rm sol} \text{ or } \Delta m^2_{\rm atm} \qquad m_{\rm sol} \equiv \sqrt{\Delta m^2_{\rm sol}} \simeq 0.009\,{\rm eV}$$

Tritium β decay :m_e < 2 eV
(Mainz + Troitzk 95% CL)

$\beta\beta0\nu$: $m_{\beta\beta}$< 0.34 – 0.78 eV
(CUORICINO 95% CL, similar bound from Heidelberg-Moscow)
NEW! : $m_{\beta\beta}$< 0.14 – 0.38 eV
(EXO-200 90% CL)

CMB+BAO+H0 : Σ m_i < 0.58 eV
(WMAP7+2dF+SDSS+HST, 95%CL)
CMB+LSS + Lyα :Σ m_i <0.17 eV
(Seljak et al.)



Minimal scenario of Leptogenesis

(Fukugita, Yanagida '86)

- Type I seesaw

$$\mathcal{L}^{\nu}_{\rm mass} = -\frac{1}{2}\left[(\bar{\nu}^c_L, \bar{\nu}_R)\begin{pmatrix} 0 & m_D^T \\ m_D & M \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix}\right] + h.c.$$

In the see-saw limit ($M \gg m_D$) the spectrum of mass eigenstates splits in 2 sets:

- 3 light neutrinos ν_1, ν_2, ν_3 with masses

$$\text{diag}(m_1, m_2, m_3) = -U^\dagger\, m_D\, \frac{1}{M}\, m_D^T\, U^\star$$

- 3 new heavy RH neutrinos N_1,N_2,N_3 with masses $M_3 > M_2 > M_1 \gg m_D$

On average one N_i decay produces a B-L asymmetry given by the

total CP asymmetries

$$\varepsilon_i \equiv -\frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i}$$

- Thermal production of the RH neutrinos $\Rightarrow$ $T_{RH} \gtrsim M_i / (2\div10)$

An impossible challenge?

Imposing $\eta_B = \eta_B^{CMB}$ one would like to get information on U and m_i

Problem: too many parameters

(Casas, Ibarra '01)

$$m_\nu = -m_D \frac{1}{M} m_D^T \Leftrightarrow \Omega^T \Omega = I$$

Orthogonal parameterisation

$$m_D = U \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \Omega \begin{pmatrix} \sqrt{M_1} & 0 & 0 \\ 0 & \sqrt{M_2} & 0 \\ 0 & 0 & \sqrt{M_3} \end{pmatrix} \qquad \begin{pmatrix} U^\dagger U & = & I \\ U^\dagger m_\nu U^\star & = & -D_m \end{pmatrix}$$

(in a basis where charged lepton and Majorana mass matrices are diagonal)

The **6 parameters in the orthogonal matrix Ω** encode the 3 life times and the 3 total CP asymmetries of the RH neutrinos and is an invariant

(King '07)

Possible parameter reduction from:

- Cancelation in asymmetry calculation $\eta_B = \eta_B(U, m_i;\lambda_1,\ldots,\lambda_{M<9})$
- Imposing some (model dependent) conditions on m_D one can reduce the number of parameters and arrive to a new parameterisation where

$$\Omega = \Omega\,(U,m_i;\lambda_1,\ldots,\lambda_{N<9}) \quad \text{and} \quad M_i = M_i\,(U,m_i;\lambda_1,\ldots,\lambda_{N\leq M})$$

- Both reductions

Vanilla leptogenesis

1) Flavor composition of final leptons is neglected

$$N_i \xrightarrow{\Gamma} l_i\, H^\dagger \qquad N_i \xrightarrow{\bar\Gamma} \bar{l}_i\, H$$

Total CP asymmetries $\quad \varepsilon_i \equiv -\frac{\Gamma_i - \bar\Gamma_i}{\Gamma_i + \bar\Gamma_i}$

$$N_{B-L}^{\rm fin} = \sum_i \varepsilon_i\, \kappa_i^{\rm fin} \Rightarrow \eta_B = a_{\rm sph}\, \frac{N_{B-L}^{\rm fin}}{N_\gamma^{\rm rec}}$$

baryon-to -photon number ratio

Successful leptogenesis : $\eta_B = \eta_B^{CMB} = (6.2 \pm 0.15) \times 10^{-10}$

2) Hierarchical heavy RH neutrino spectrum: $M_2 \gtrsim 3\, M_1$

3) N_3 does not interfere with N_2-decays: $(m_D^\dagger\, m_D)_{23} = 0$

From the last two assumptions

$$\Rightarrow \quad N_{B-L}^{\rm fin} = \sum_i \varepsilon_i\, \kappa_i^{\rm fin} \simeq \varepsilon_1\, \kappa_1^{\rm fin}$$

4) Barring fine-tuned mass cancellations in the seesaw

$$\Rightarrow \quad \varepsilon_1 \lesssim 10^{-6} \left(\frac{M_1}{10^{10}\,\text{GeV}}\right) \frac{m_{\text{atm}}}{m_1 + m_3}$$

(Davidson, Ibarra '02)

5) Efficiency factor from simple Boltzmann equations

$$\frac{dN_{N_1}}{dz} = -D_1 \left(N_{N_1} - N_{N_1}^{\text{eq}}\right)$$

$$\frac{dN_{B-L}}{dz} = -\varepsilon_1 \frac{dN_{N_1}}{dz} - W_1 N_{B-L}$$

$$z \equiv \frac{M_1}{T}$$

decays: $-D_1 N_{N_1}$

inverse decays: $N_{N_1}^{\text{eq}}$

wash-out: $W_1 N_{B-L}$

decay parameter

$$K_1 \equiv \frac{\Gamma_{N_1}(T=0)}{H(T=M_1)}$$

$$\kappa_1(z; K_1, z_{\text{in}}) = -\int_{z_{\text{in}}}^{z} dz' \left[\frac{dN_{N_1}}{dz'}\right] e^{-\int_{z'}^{z} dz''\, W_1(z'')}$$

Independence of the initial conditions

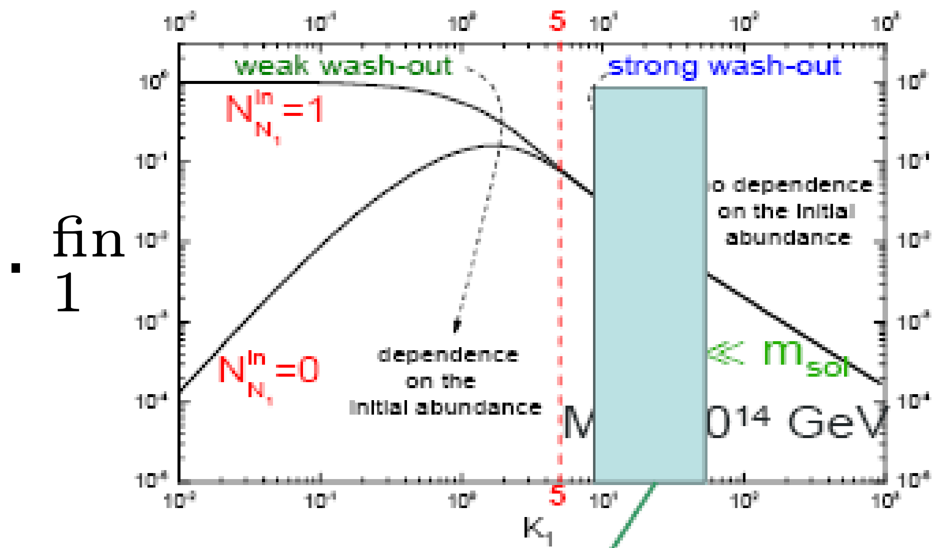
The early Universe „knows“ neutrino masses ...

(Buchmüller, PDB, Plümacher '04)

$$\eta_B \simeq 0.01\,\varepsilon_1(m_1, M_1, \Omega)\,\kappa_1^{\rm fin}(K_1)$$

decay parameter

$$K_1 \equiv \frac{\Gamma_{N_1}}{H(T=M_1)} \sim \frac{m_{\rm sol,atm}}{m_\star \sim 10^{-3}\,{\rm eV}} \sim 10 \div 50$$



$$K_{\rm sol} \simeq 9 \lesssim K_1 \lesssim 50 \simeq K_{\rm atm}$$

wash-out of a pre-existing asymmetry

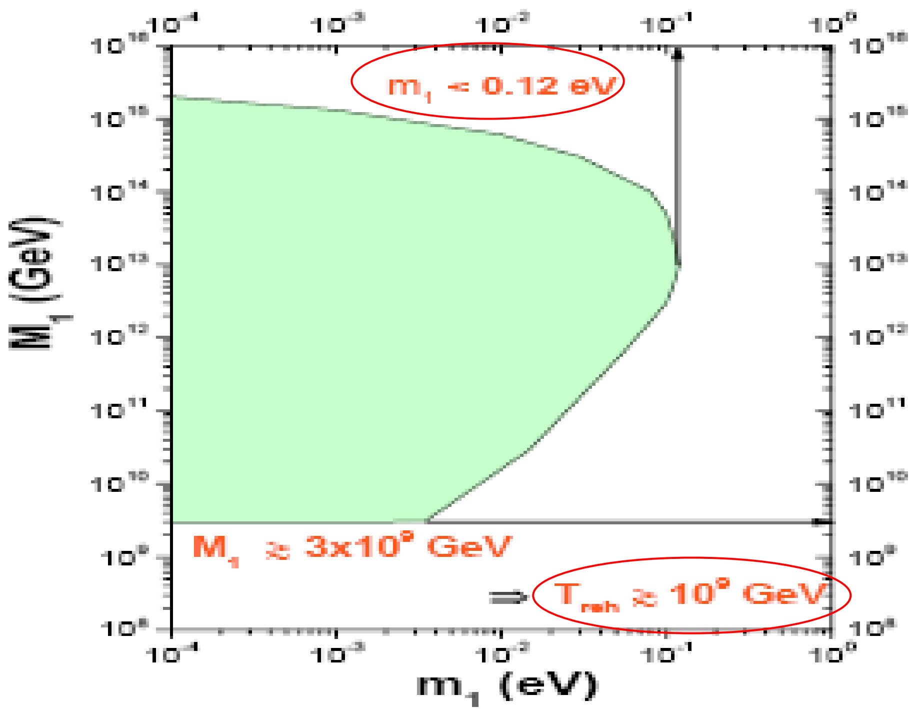
$$N_{B-L}^{\rm p,final} = N_{B-L}^{\rm p,initial}\, e^{-\frac{3\pi}{8}\,K_1} \ll N_{B-L}^{\rm f,N_1}$$

Neutrino mass bounds in vanilla leptog.

(Davidson, Ibarra '02; Buchmüller, PDB, Plümacher '02, '03, '04; Giudice et al. '04)

$$\eta_B \simeq 0.01\, \varepsilon_1(m_1, M_1, \Omega)\, \kappa_1^{\rm fin}(K_1)$$

Imposing: $\eta_B^{\rm max}(m_1, M_1) \geq \eta_B^{CMB}$



No dipendence on the leptonic mixing matrix U

SO(10)-inspired leptogenesis

(Branco et al. '02; Nezri, Orloff '02; Akhmedov, Frigerio, Smirnov '03)

Expressing the neutrino Dirac mass matrix m_D (in the basis where the Majorana mass and charged lepton mass matrices are diagonal) as:

$$m_D = V_L^\dagger \, D_{m_D} \, U_R \qquad D_{m_D} = \mathrm{diag}\{\lambda_{D1}, \lambda_{D2}, \lambda_{D3}\}$$

SO(10) inspired conditions:

$$\lambda_{D1} = \alpha_1 \, m_u \, , \; \lambda_{D2} = \alpha_2 \, m_c \, , \; \lambda_{D3} = \alpha_3 \, m_t \, , \quad (\alpha_i = \mathcal{O}(1)) \qquad V_L \simeq V_{CKM} \simeq I$$

One can express: $\Omega = D_m^{-\frac{1}{2}} \, U^\dagger \, V_L^\dagger \, D_{m_D} \, U_R \, D_M^{-\frac{1}{2}}$

and from the seesaw formula: $U_R = U_R(V_L, U)$, $M_i = M_i(V_L, U)$

one typically obtains (barring fine-tuned 'crossing level' solutions):

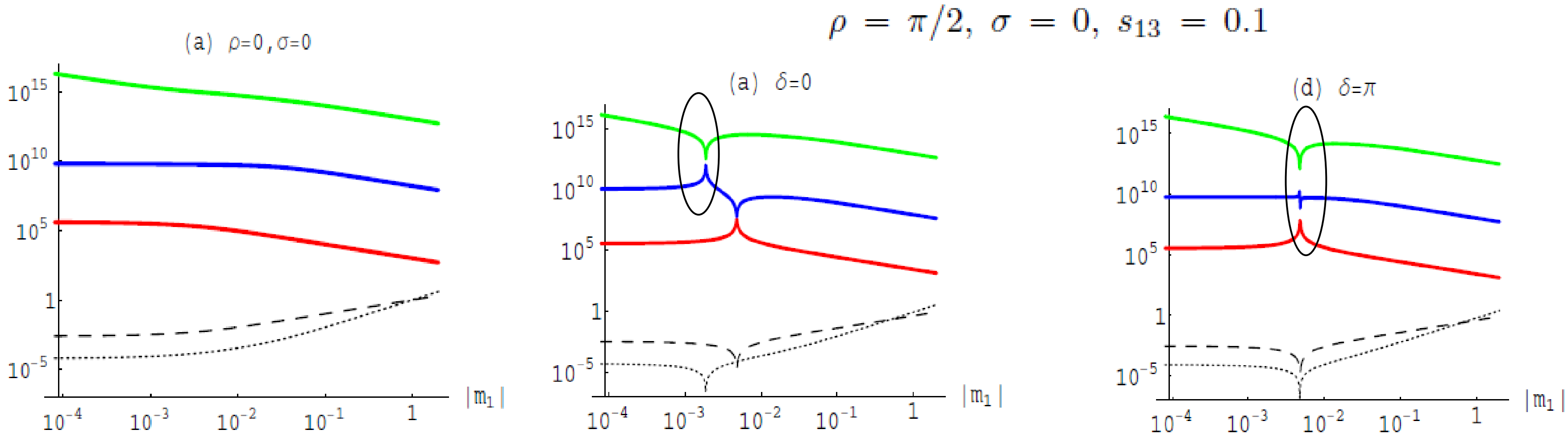
$$M_1 \sim \alpha_1^2 \, 10^5 \mathrm{GeV} \, , \;\; M_2 \sim \alpha_2^2 \, 10^{10} \, \mathrm{GeV} \, , \;\; M_3 \sim \alpha_3^2 \, 10^{15} \, \mathrm{GeV}$$

since $M_1 \ll 10^9$ GeV $\Rightarrow \eta_B(N_1) \ll \eta_B^{CMB}$!

$\Rightarrow$ failure of the N_1-dominated scenario !

Crossing level solutions

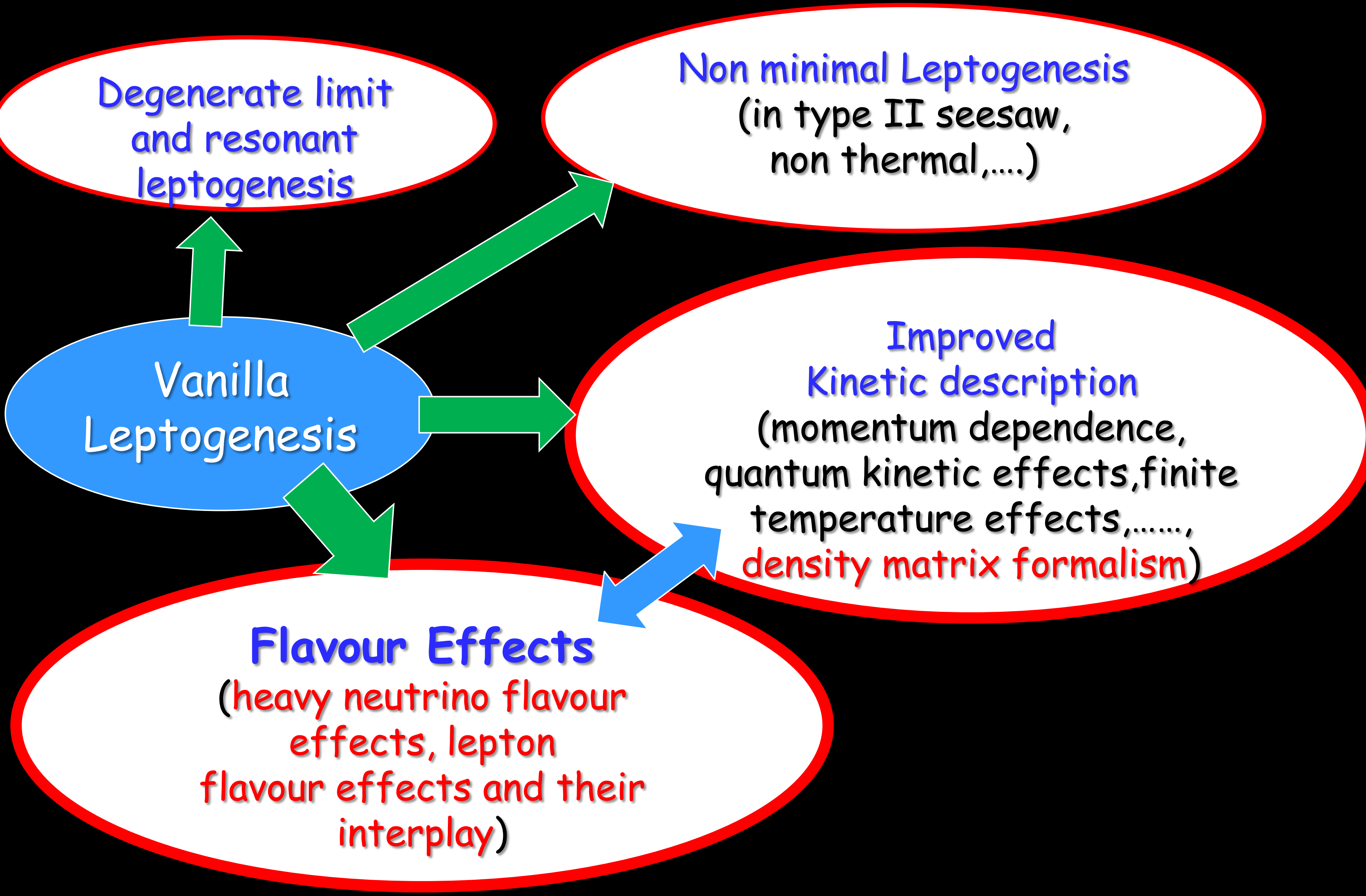
(Akhmedov, Frigerio, Smirnov '03)



At the crossing the CP asymmetries undergo a resonant enhancement (Covi,Roulet,Vissani '96; Pilaftsis '98; Pilaftsis,Underwood '04; ...) and fine tuning parameters the correct baryon asymmetry can be attained

Recently one of this kind of solutions has been studied including flavour effects as well (Buccella, Falcone, Nardi et al '12)

Beyond vanilla Leptogenesis



Lepton flavour effects

(Abada, Davidson, Losada, Josse-Michaux, Riotto'06; Nardi, Nir, Roulet, Racker '06; Blanchet, PDB, Raffelt '06; Riotto, De Simone '06)

Flavor composition of lepton quantum states:

$$|l_1\rangle = \sum_\alpha \langle l_\alpha | l_1 \rangle \, |l_\alpha\rangle \quad (\alpha = e, \mu, \tau) \qquad P_{1\alpha} \equiv |\langle l_\alpha | l_1 \rangle|^2$$

$$|\bar{l}_1'\rangle = \sum_\alpha \langle l_\alpha | \bar{l}_1' \rangle \, |\bar{l}_\alpha\rangle \qquad \bar{P}_{1\alpha} \equiv |\langle \bar{l}_\alpha | \bar{l}_1' \rangle|^2$$

But for $T \lesssim 10^{12}$ GeV $\Longrightarrow$ τ-Yukawa interactions $(\bar{l}_{L\tau}\, \phi\, f_{\tau\tau}\, e_{R\tau})$ are fast enough to break the coherent evolution of $|l_1\rangle$ and $|\bar{l}_1'\rangle$

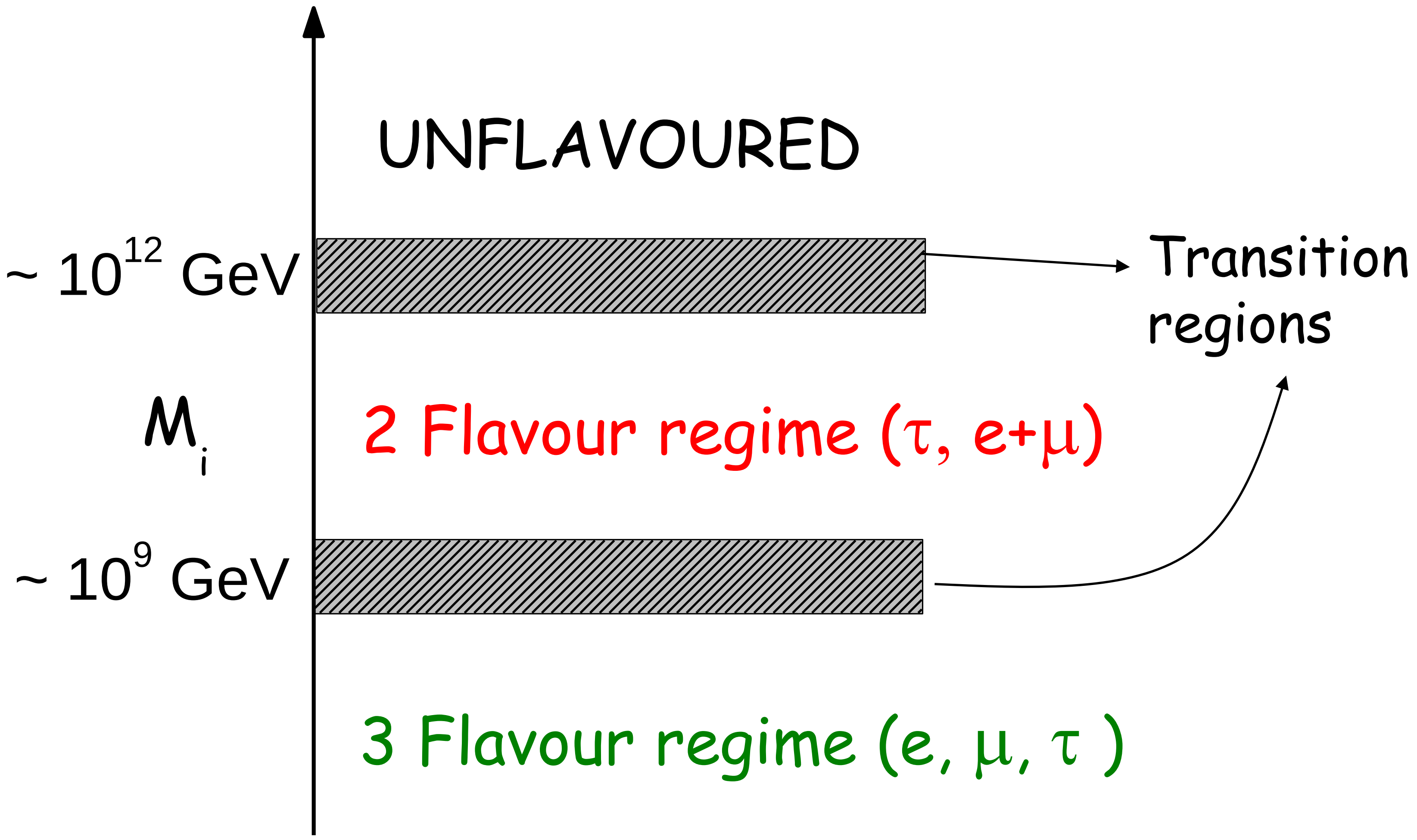
$\Longrightarrow$ they become an incoherent mixture of a τ and of μ+e

At $T \lesssim 10^9$ GeV then also μ- Yukawas in equilibrium $\Longrightarrow$ 3-flavor regime

$$\Rightarrow \boxed{N_{B-L}^{\rm fin} = \sum_{i,\alpha} \varepsilon_{i\alpha}\, \kappa_{i\alpha}^{\rm fin} \quad (\alpha = e, \mu, \tau)}$$

heavy neutrino flavor index (i) — lepton flavor index (α)

Since leptogenesis occurs at $T \sim M_i$, temperatures regimes translate into different mass ranges regimes for the calculation of the asymmetry:



Fully two-flavored regime

- $(\alpha = \tau, e+\mu)$

$$P_{1\alpha} \equiv |\langle l_\alpha | l_1 \rangle|^2 = P^0_{1\alpha} + \Delta P_{1\alpha}/2 \qquad \left(\sum_\alpha P^0_{1\alpha} = 1 \right)$$

$$\bar{P}_{1\alpha} \equiv |\langle \bar{l}_\alpha | \bar{l}'_1 \rangle|^2 = P^0_{1\alpha} - \Delta P_{1\alpha}/2 \qquad \left(\sum_\alpha \Delta P_{1\alpha} = 0 \right)$$

These 2 terms correspond respectively to 2 different flavor effects:

1) wash-out is in general reduced: $K_1 \rightarrow K_{1\alpha} \equiv K_1 \, P^0_{1\alpha}$

2) additional CP violating contribution $\left(|\bar{l}'_1\rangle \neq CP|l_1\rangle \right)$

$$\Rightarrow \quad \varepsilon_{1\alpha} \equiv -\frac{P_{1\alpha}\Gamma_1 - \bar{P}_{1\alpha}\bar{\Gamma}_1}{\Gamma_1 + \bar{\Gamma}_1} = P^0_{1\alpha}\,\varepsilon_1 + \Delta P_{1\alpha}(\Omega, U)/2$$

- Classic Kinetic Equations (in their simplest form)

$$\frac{dN_{N_1}}{dz} = -D_1 \left(N_{N_1} - N^{\rm eq}_{N_1} \right)$$

$$\frac{dN_{\Delta_\alpha}}{dz} = -\varepsilon_{1\alpha} \frac{dN_{N_1}}{dz} - P^0_{1\alpha} W_1 \, N_{\Delta_\alpha}$$

$$\Rightarrow \quad N_{B-L} = \sum_\alpha N_{\Delta_\alpha} \qquad (\Delta_\alpha \equiv B/3 - L_\alpha)$$

$$\Rightarrow N^{\rm fin}_{B-L} = \sum_\alpha \varepsilon_{1\alpha}\,\kappa^{\rm fin}_{1\alpha} \simeq N_{\rm fl}\,\varepsilon_1\,\kappa^{\rm fin}_1 + \frac{\Delta P_{1\alpha}}{2}\left[\kappa^{\rm fin}_{1\alpha} - \kappa^{\rm fin}_{1\beta}\right]$$

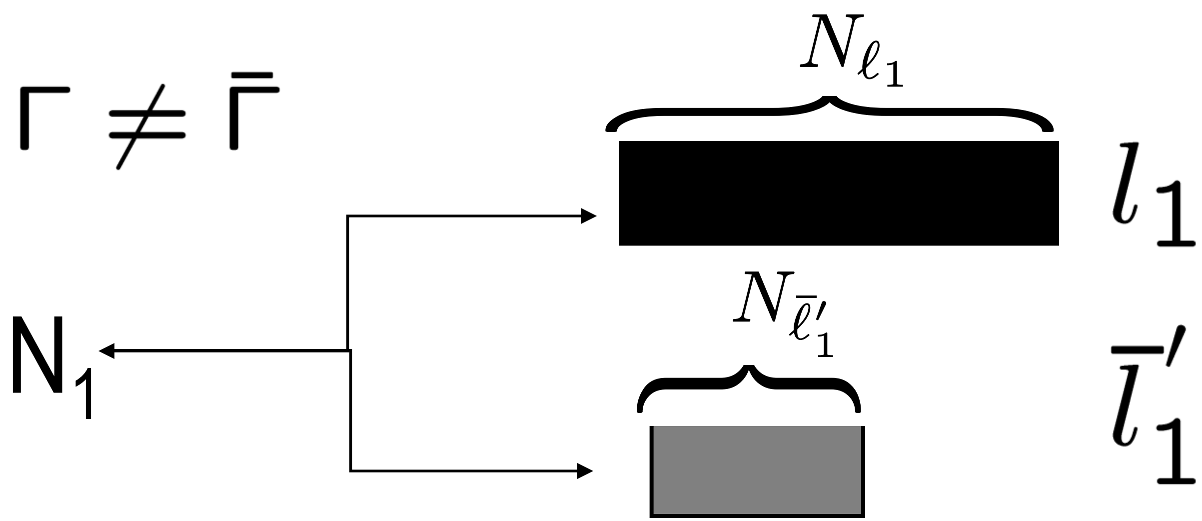
Additional contribution to CP violation:

(α = τ, e+μ)

$$\varepsilon_{1\alpha} = P^0_{1\alpha}\,\varepsilon_1 + \frac{\Delta P_{1\alpha}}{2}$$

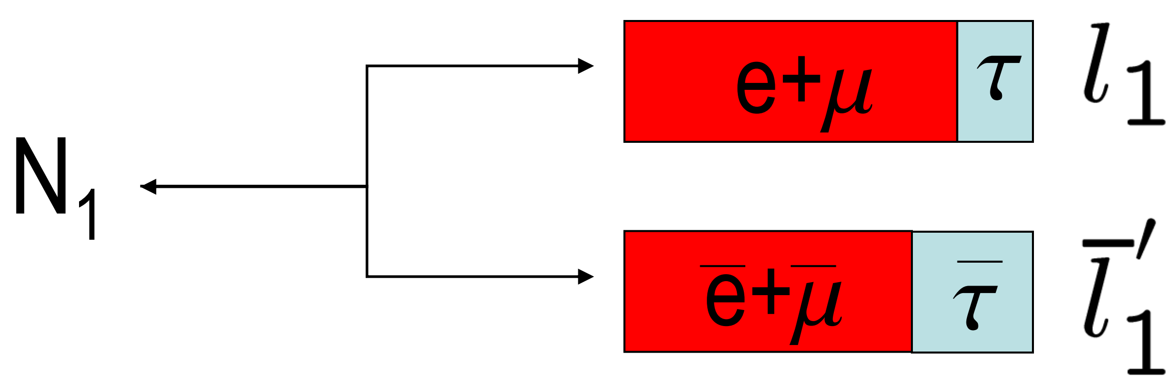
depends on U !

1) $\Gamma \neq \bar{\Gamma}$



$\Rightarrow P^0_{1\alpha}\,\varepsilon_1$

2) $|\bar{l}'_1\rangle \neq CP|l_1\rangle$ **+**



$$\Rightarrow \frac{\Delta P_{1\alpha}}{2}$$

Low energy phases can be the only source of CP violation

(Blanchet,PDB, '06; Pascoli, Petcov Riotto '06; Anisimov, Blanchet, PDB '08)

- Assume real $\Omega \Longrightarrow$ $\varepsilon_1 = 0 \Longrightarrow \varepsilon_{1\alpha} = P^0_{1\alpha}\,\varepsilon_1 + \dfrac{\Delta P_{1\alpha}}{2}$ (Nardi et al.´06)

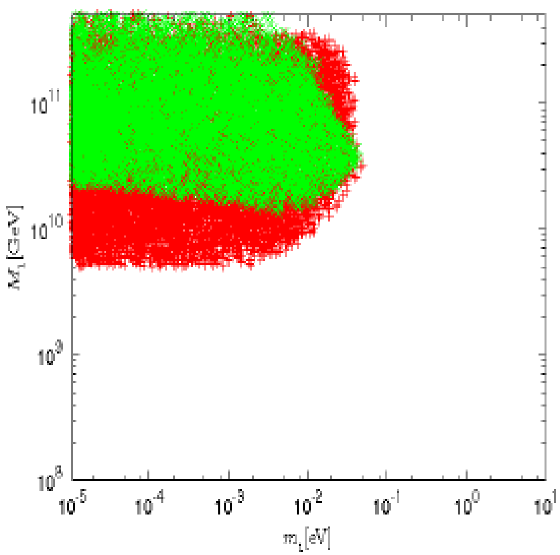
$$\Longrightarrow N_{B-L} \simeq 2\varepsilon_1 \kappa_1^{fin} + \Delta P_{1\alpha}(\kappa_{1\alpha}^{fin} - \kappa_{1\beta}^{fin})$$

($\alpha = \tau$, e+μ)

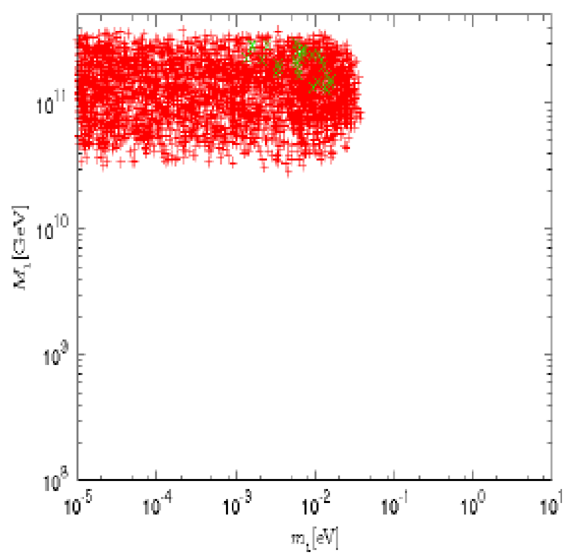
- Assume even vanishing Majorana phases

$\Longrightarrow$ a Dirac phase with non-vanishing θ_{13} ($J_{CP} \neq 0$) would be the only source of CP violation (testable)

initial thermal N_1 abundance



independent of initial N_1 abundance



Green points: only Dirac phase with $\sin\theta_{13} = 0.2$ $|\sin\delta| = 1$

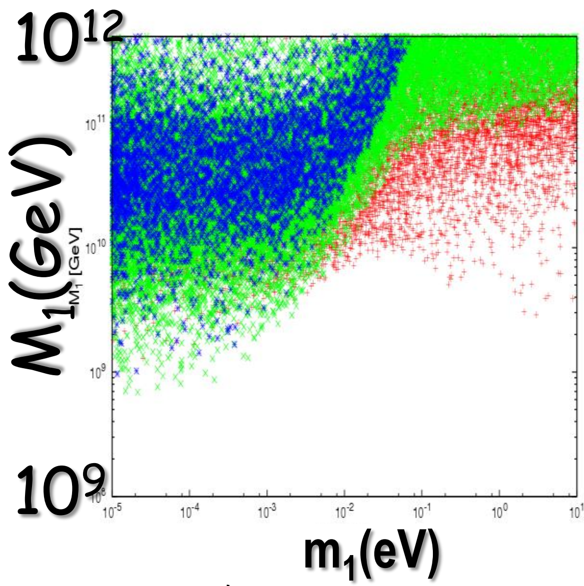
Red points: only Majorana phases

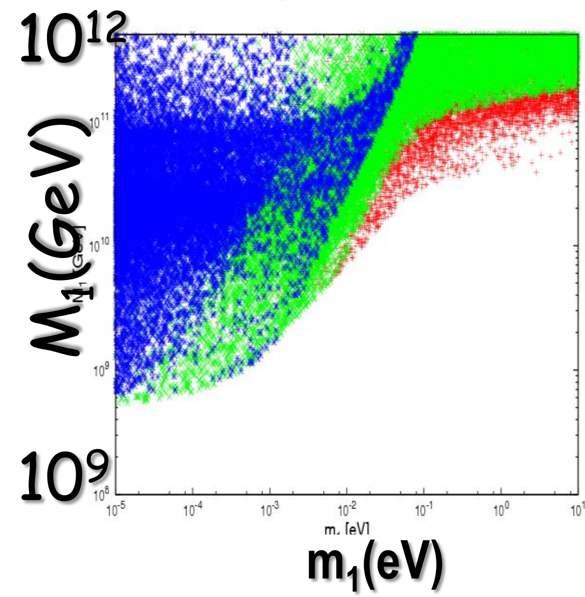
No known reasons for these assumptions to be rigorously satisfied but they are approximately satisfied within specific scenarios in some region of the parameter space: in any case it is by itself interesting that CP violation in neutrino mixing could be sufficient to reproduce the BAU

Upper bound on m_1

(Abada et al.' 07 Blanchet,PDB '08)

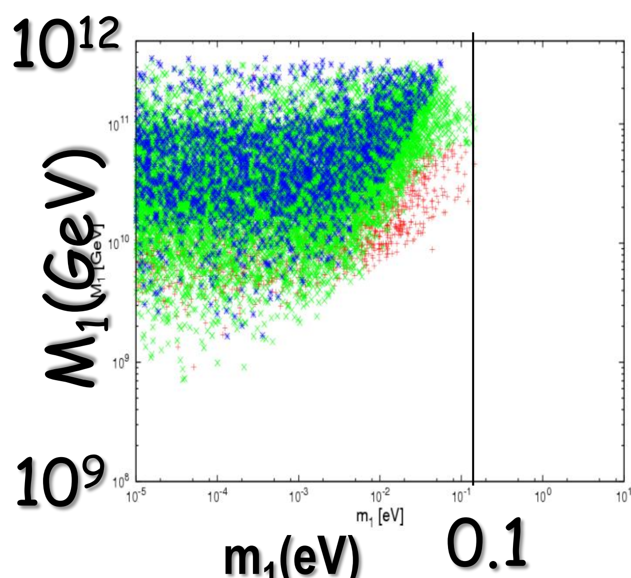
PMNS phases off





$M_1 \lesssim 10^{12}\,\mathrm{GeV}/W_1(T_B)$

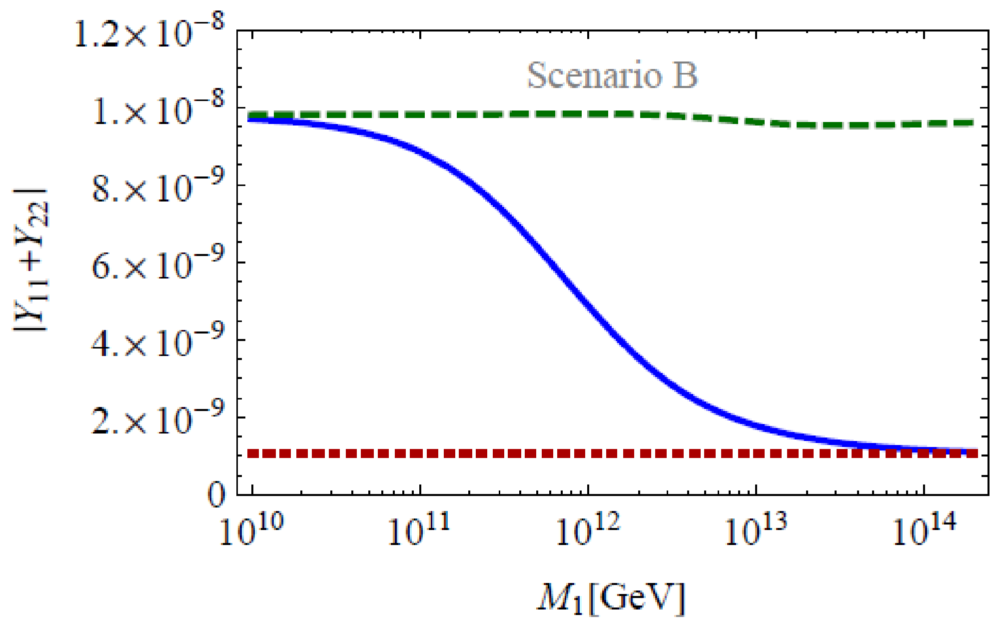
imposing a condition of validity of Boltzmann equations



Density matrix and CTP formalism to describe the transition regimes

(De Simone, Riotto '06; Beneke, Gabrecht, Fidler, Herranen, Schwaller '10)

$$\frac{dY_{\alpha\beta}}{dz} = \frac{1}{szH(z)}\left[(\gamma_D + \gamma_{\Delta L=1})\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\epsilon_{\alpha\beta} - \frac{1}{2Y_\ell^{eq}}\left\{\gamma_D + \gamma_{\Delta L=1}, Y\right\}_{\alpha\beta}\right] - \left[\sigma_2 \mathrm{Re}(\Lambda) + \sigma_1|\mathrm{Im}(\Lambda)|\right] Y_{\alpha\beta}$$



Fully two-flavoured regime limit

Unflavoured regime limit

Heavy neutrino flavour effects: N_2-dominated scenario

(PDB '05)

If light flavour effects are neglected the asymmetry from the next-to-lightest (N_2) RH neutrinos is typically negligible:

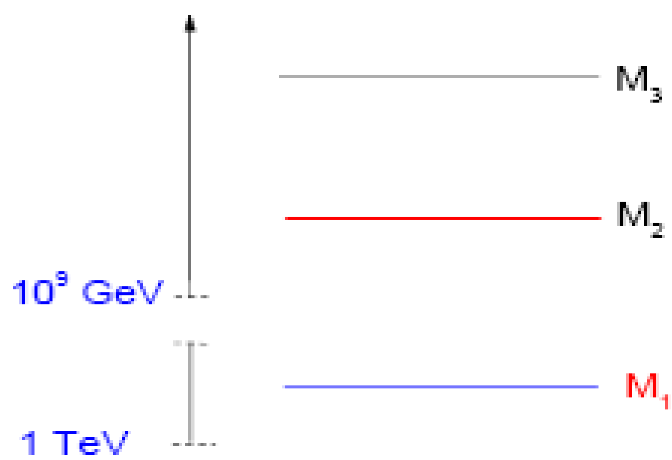
$$N_{B-L}^{f,N_2} = \varepsilon_2 \cdot (K_2)\, e^{-\frac{3\pi}{8} K_1} \ll N_{B-L}^{f,N_1} = \varepsilon_1 \cdot (K_1)$$

...except for a special choice of $\Omega = R_{23}$ when $K_1 = m_1/m_* \ll 1$ and $\varepsilon_1 = 0$:

$$\Rightarrow \quad N_{B-L}^{\rm fin} = \sum_i \varepsilon_i\, \kappa_i^{\rm fin} \simeq \varepsilon_2\, \kappa_2^{\rm fin} \qquad \varepsilon_2 \lesssim 10^{-6} \left(\frac{M_2}{10^{10}\,\mathrm{GeV}}\right)$$

The lower bound on M_1 disappears and is replaced by a lower bound on M_2 ...

that however still implies a lower bound on T_{reh} !



Interplay between lepton and heavy neutrino flavour effects:

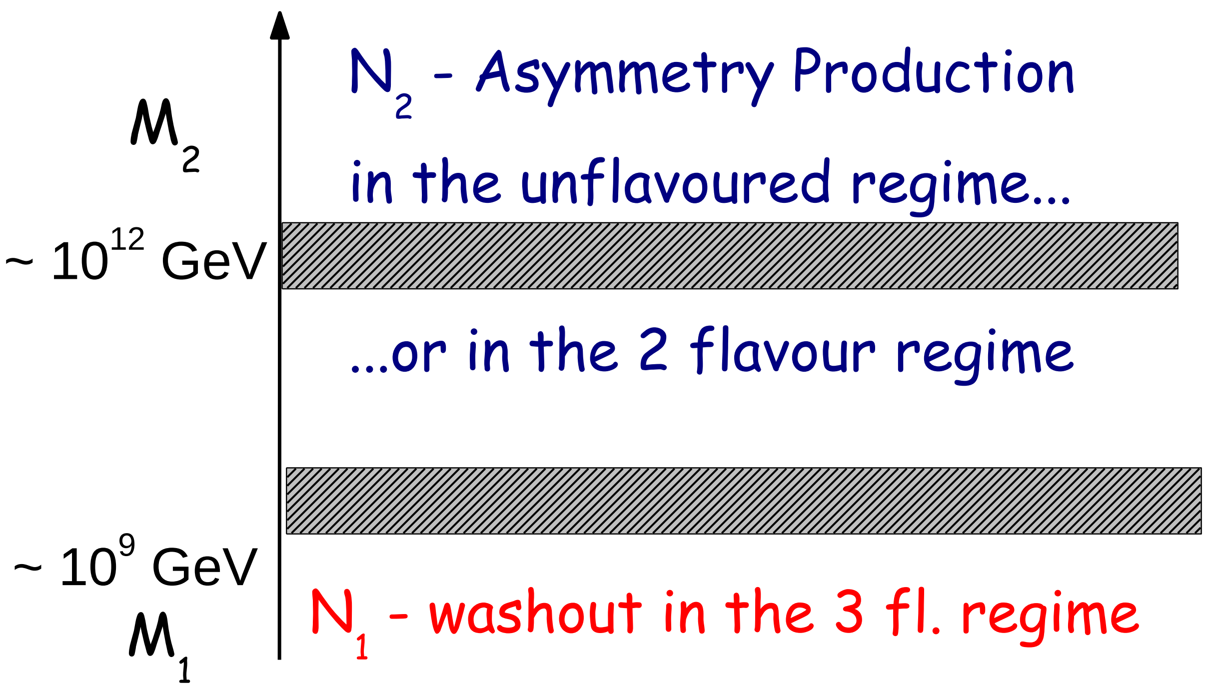
1. N_2 flavoured leptogenesis
2. Flavour projection
3. Phantom leptogenesis

N_2-flavored leptogenesis

(Vives '05; Blanchet, PDB '06; Blanchet, PDB '08)

Combining together lepton and heavy neutrino flavour effects one has

A two stage process:



Notice that the presence of the heaviest RH neutrino N_3 is necessary for the CP asymmetries of N_2 not to be negligible !

Without lepton flavour effects we had:

$$N_{B-L}^{\mathrm{f},\mathrm{N}_2} = \varepsilon_2 \cdot (K_2)\, e^{-\frac{3\pi}{8} K_1} \ll N_{B-L}^{\mathrm{f},\mathrm{N}_1} = \varepsilon_1 \cdot (K_1)$$

Now if, for example, the N_2 production is assumed in the 3 fl. regime $\Rightarrow$

$$N_{B-L}^{\mathrm{f}}(N_2) = P_{2e}^{0}\, \varepsilon_2 \cdot (K_2)\, e^{-\frac{3\pi}{8} K_{1e}} + P_{2\mu}^{0}\, \varepsilon_2 \cdot (K_2)\, e^{-\frac{3\pi}{8} K_{1\mu}} + P_{2\tau}^{0}\, \varepsilon_2 \cdot (K_2)\, e^{-\frac{3\pi}{8} K_{1\tau}}$$

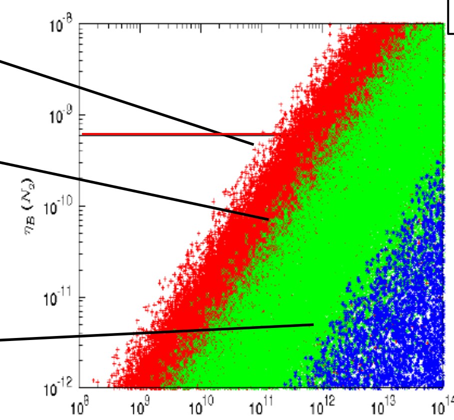
Notice that $K_1 = K_{1e} + K_{1\mu} + K_{1\tau}$

$$\Omega = R_{12}(\omega_{12})\, R_{13}(\omega_{13})$$

N_1 wash-out is neglected

Wash-out and flavor effects are both taken into account

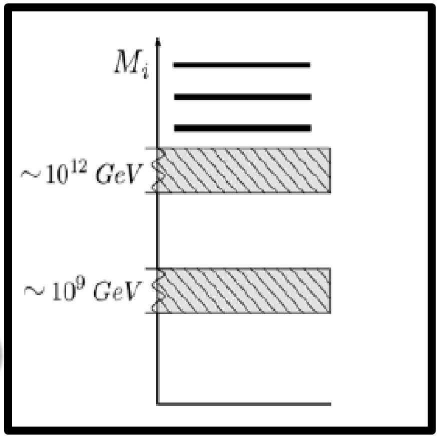
Unflavored case



Thanks to flavor effects the domain of applicability extends much beyond the particular choice $\Omega = R_{23}$

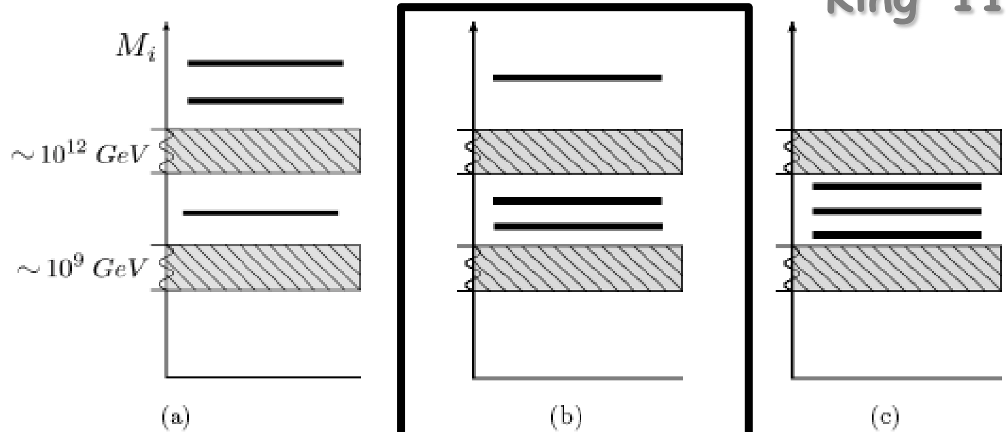
Heavy neutrino flavored scenario

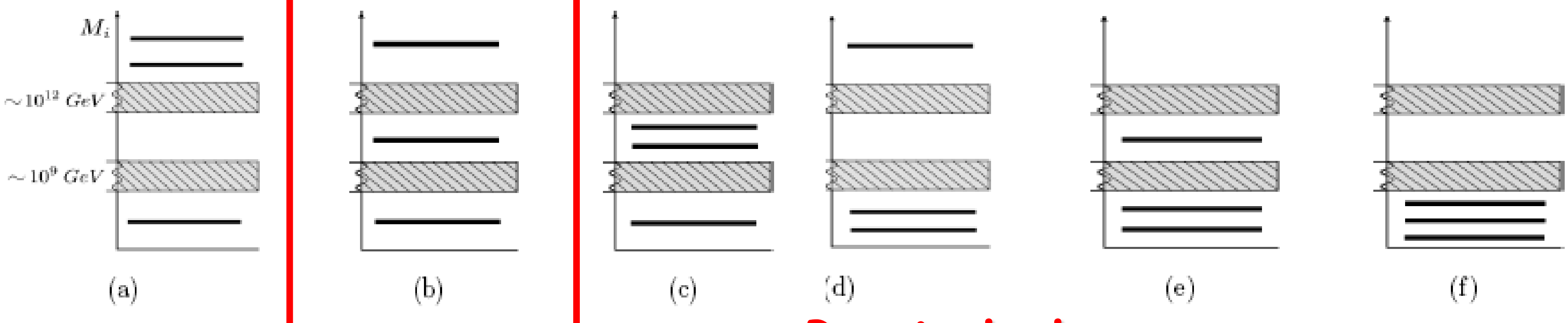
(Bertuzzo, PDB, Marzola '09)



2 RH neutrino scenario

(Antusch, PDB, Jones, King '11)





N_2-dominated scenario → **Particularly attractive for two reasons**

1) It is just that one realised in SO(10) inspired models!

Can they be reconciled with leptogenesis?

The N_2-dominated scenario rescues SO(10) inspired models

(PDB, Riotto '08)

$$N_{B-L}^{\rm f} \simeq \varepsilon_{2e}\,\kappa(K_{2e+\mu})\,e^{-\frac{3\pi}{8}\,K_{1e}} + \varepsilon_{2\mu}\,\kappa(K_{2e+\mu})\,e^{-\frac{3\pi}{8}\,K_{1\mu}} + \varepsilon_{2\tau}\,\kappa(K_{2\tau})\,e^{-\frac{3\pi}{8}\,K_{1\tau}}\,.$$

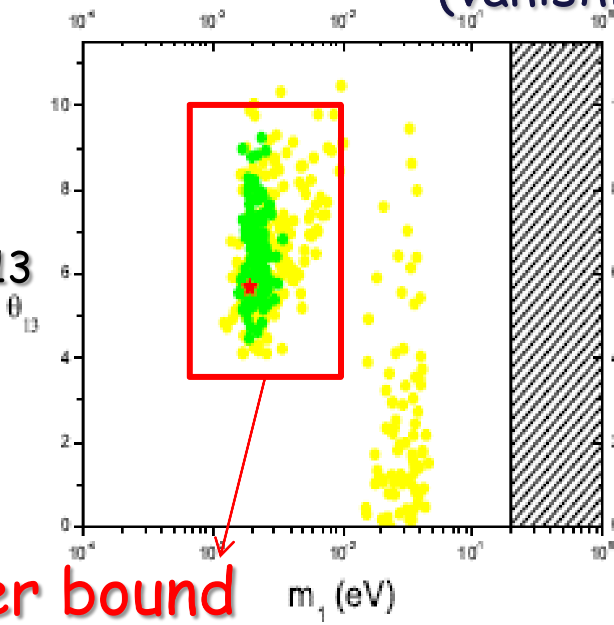
Independent of α_1 and α_3 !

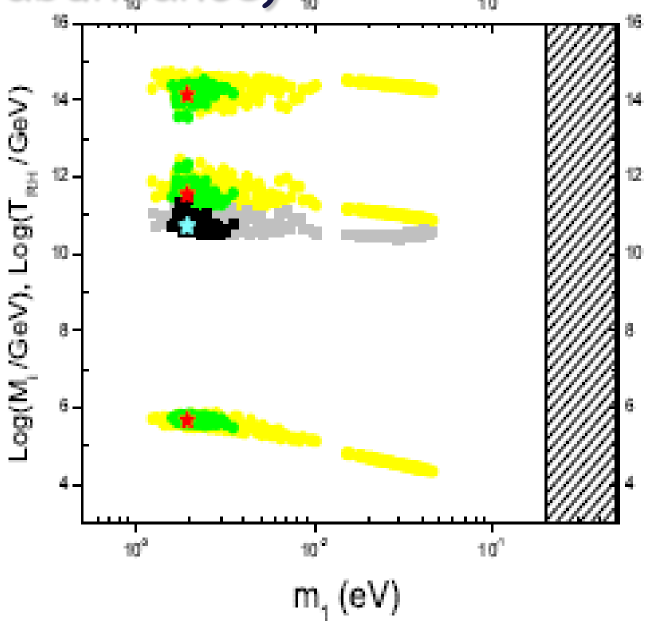
α_2=5 α_2=4 α_2=3

$V_L = I$ Normal ordering

(vanishing initial N_2-abundance)

Θ_{13}





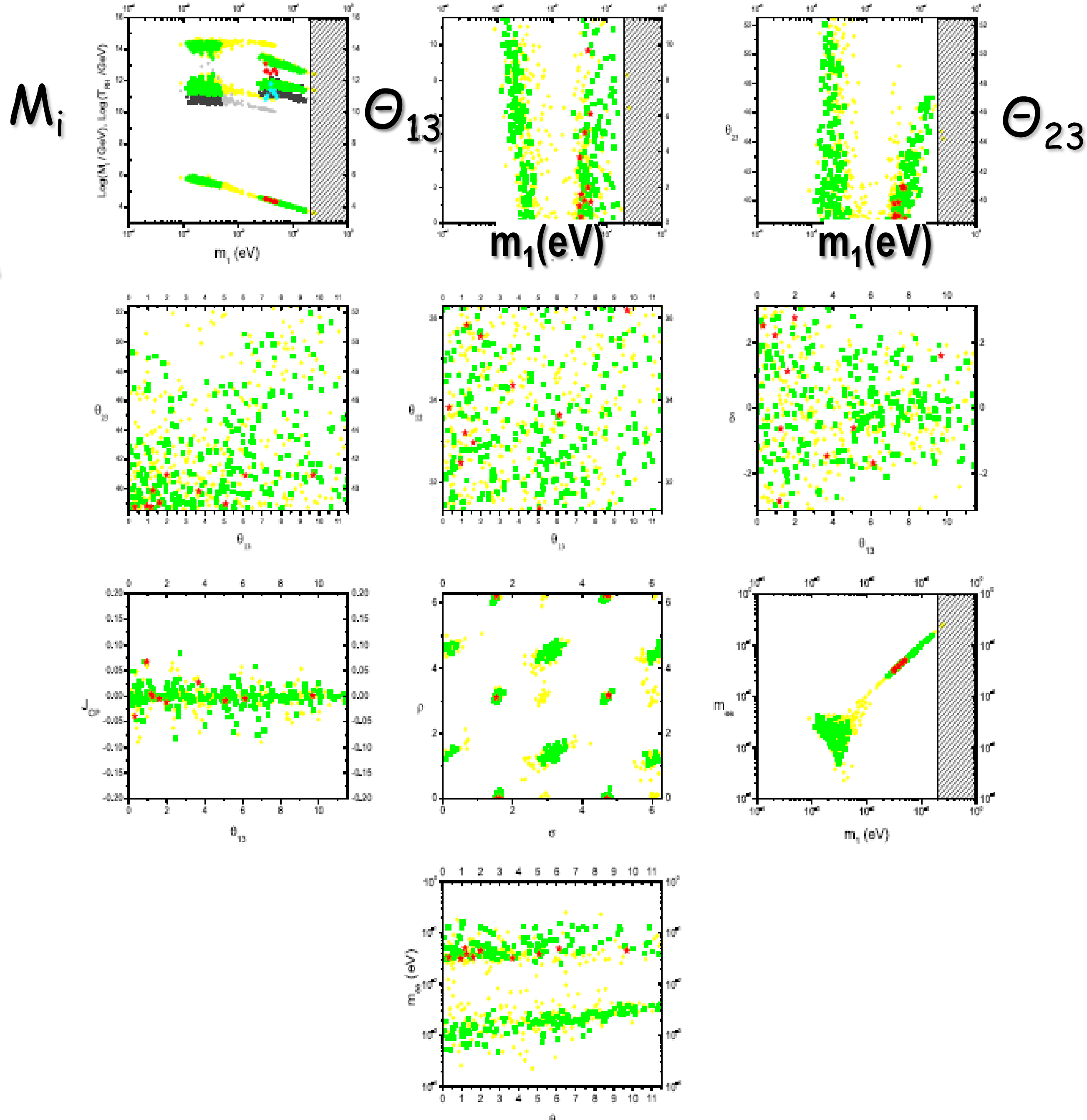
lower bound on Θ_{13} ?

Another way to rescue SO(10) inspired models is by considering a left-right symmetric seesaw (Abada, Hosteins, Josse-Michaux, Lavignac'08)

The model yields constraints on all low energy neutrino observables !

$I < V_L < V_{CKM}$

NORMAL ORDERING

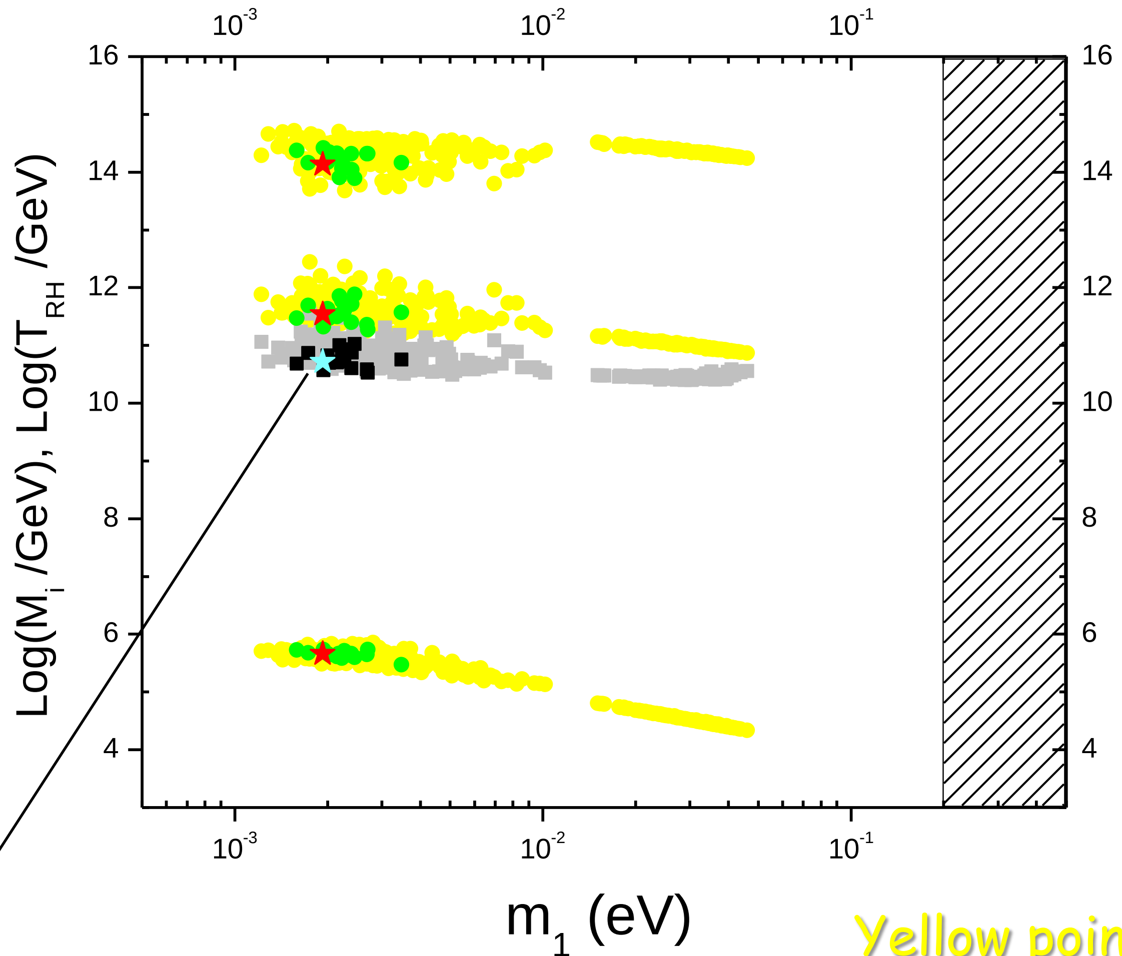


$\alpha_2=5$

$\alpha_2=4$

$\alpha_2=1$

(PDB, Riotto '10)



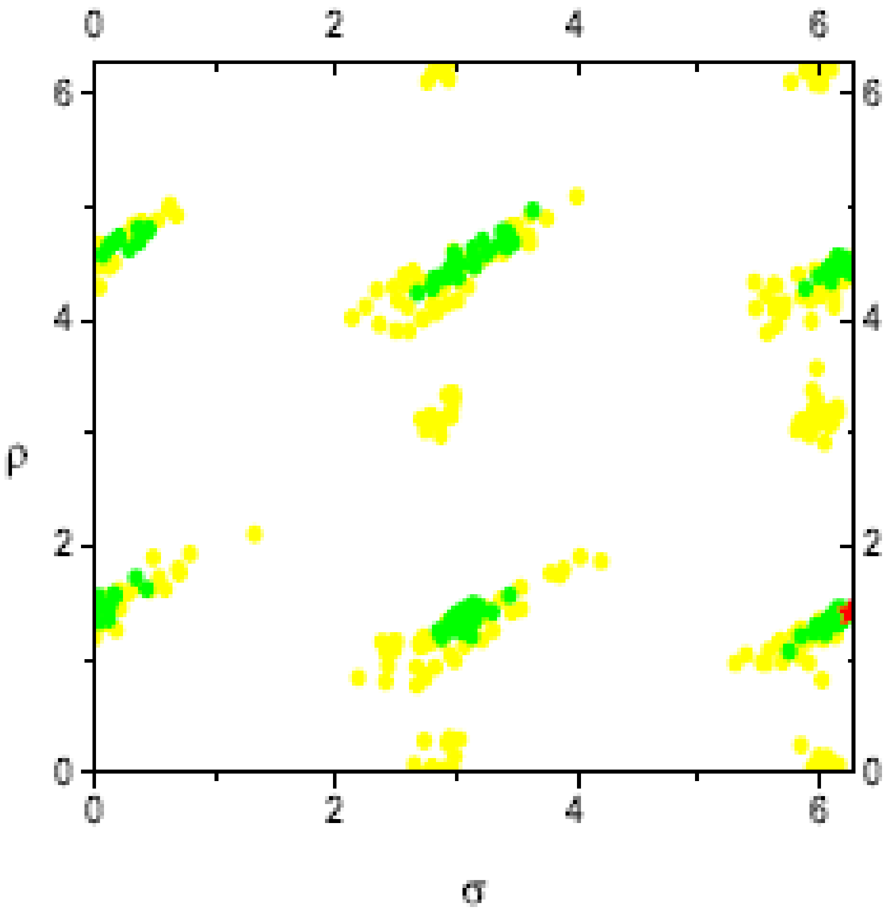
The reheat temperature lower bound is $\sim 4\times10^{10}$ GeV

Yellow points: α_2=5

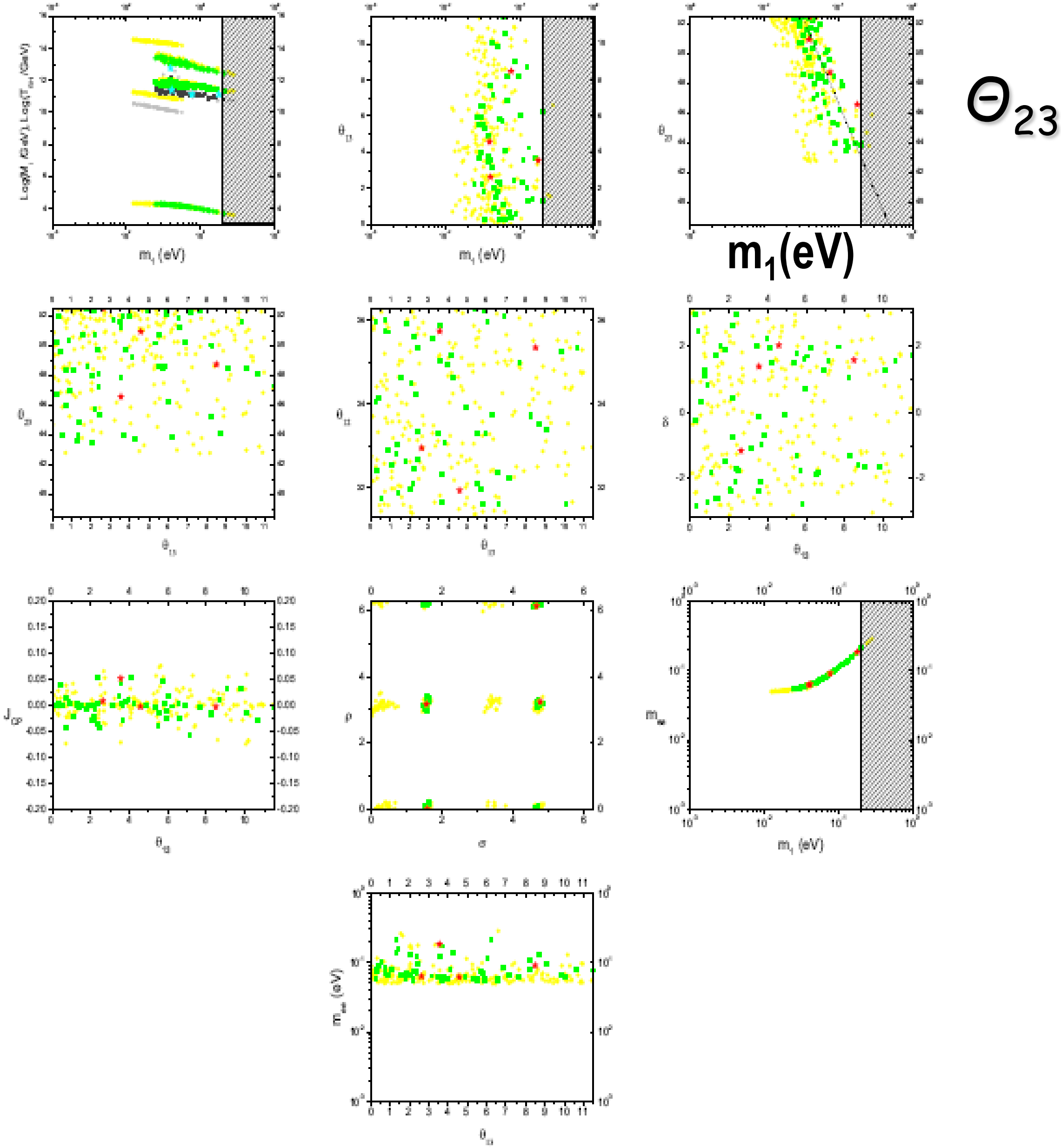
Green points: α_2=4

Red star : α_2=3

The Majorana phases need to be around very specific values



$I < V_L < V_{CKM}$



INVERTED
ORDERING

α_2=5

α_2=4

α_2=1.5

An improved analysis

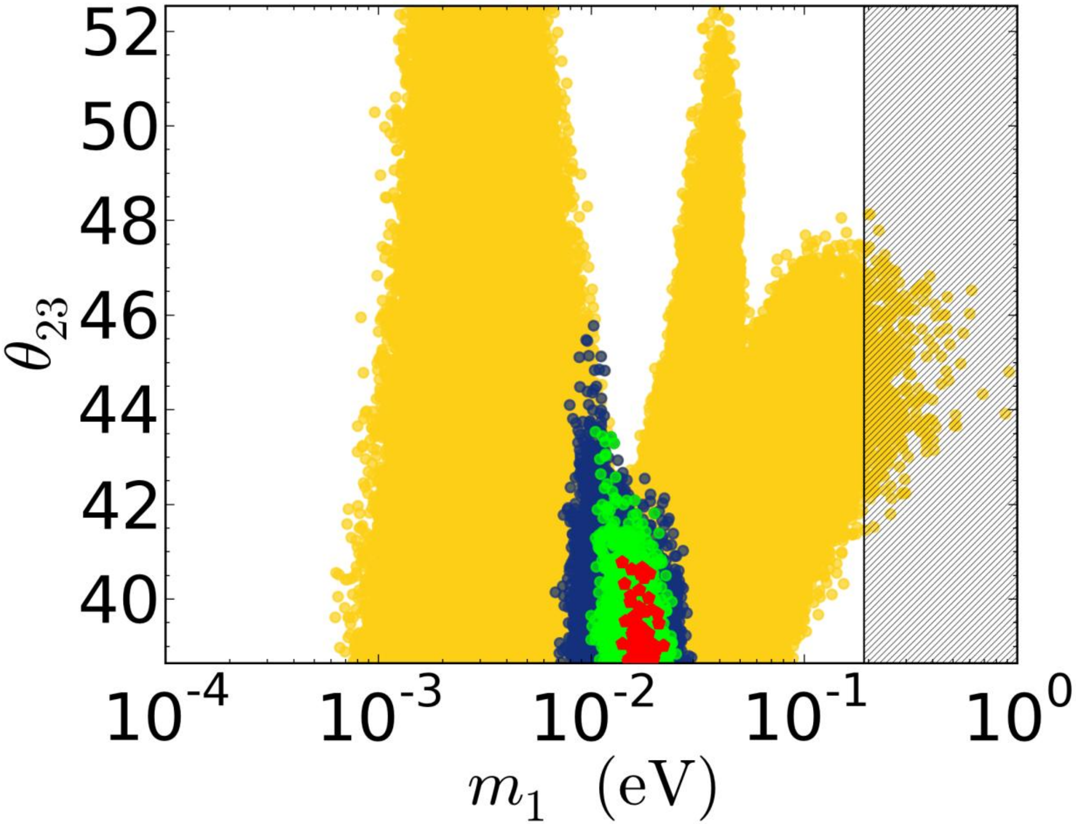
(PDB, Marzola '11-'12)

We optimised the procedure increasing of two orders of magnitudes the number of solutions (focus on yellow points for the time being):

α_2=5

NORMAL ORDERING

$I < V_L < V_{CKM}$



Why? Just to have sharper borders ? NO…. i) statistical analysis
ii) ….

No link between the sign of the asymmetry and J_{CP}

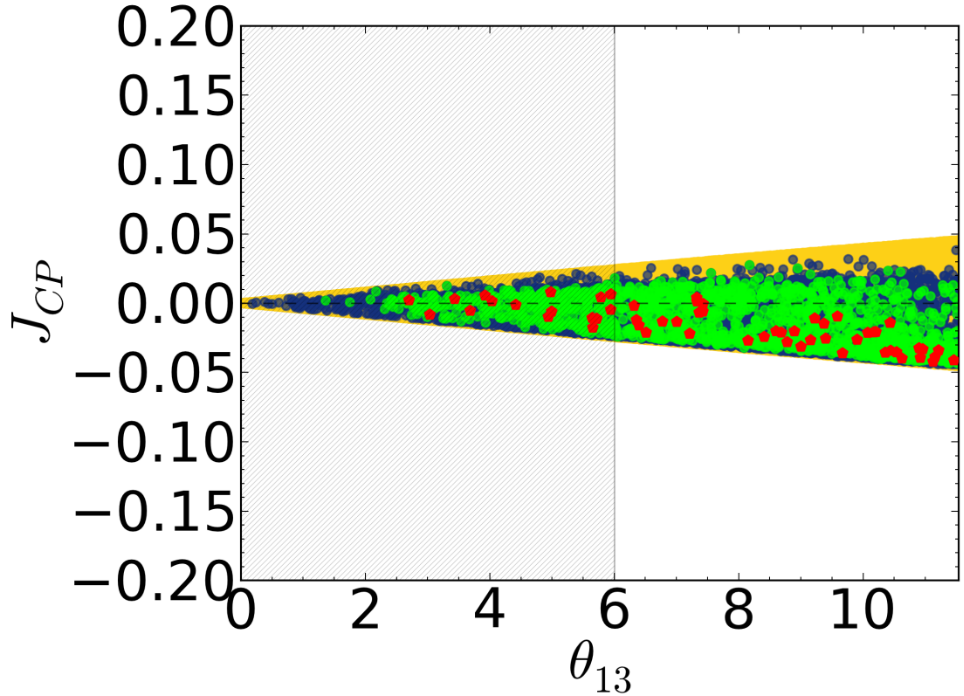
(PDB, Marzola '11-'12)

We optimised the procedure increasing of two orders of magnitudes the number of solutions:

α_2=5

NORMAL ORDERING

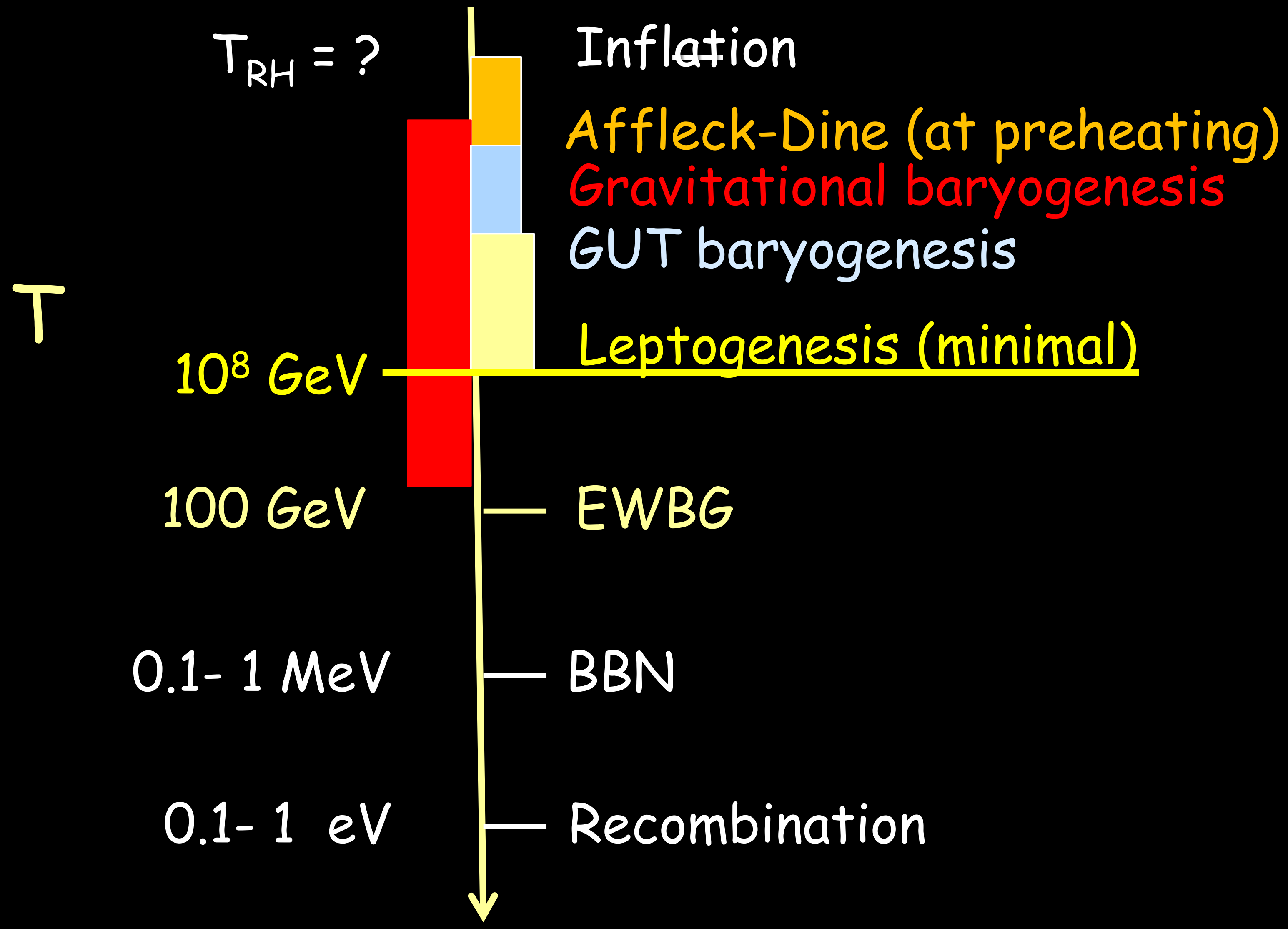
$I < V_L < V_{CKM}$



It is confirmed that there is no link between the matter-antimatter asymmetry and CP violation in neutrino mixing…….for the yellow points

WHAT ARE THE NON-YELLOW POINTS?

Baryogenesis and the early Universe history

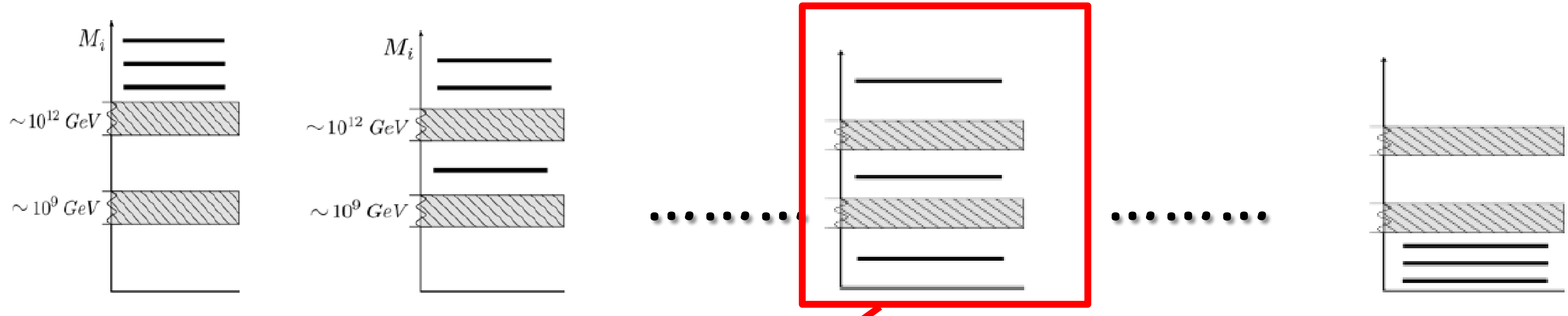


(Bertuzzo,PDB,Marzola '10)

Residual "pre-existing" asymmetry possibly generated by some external mechanism

$$N_{B-L}^{\rm f} = N_{B-L}^{\rm p,f} + N_{B-L}^{\rm lep,f}$$

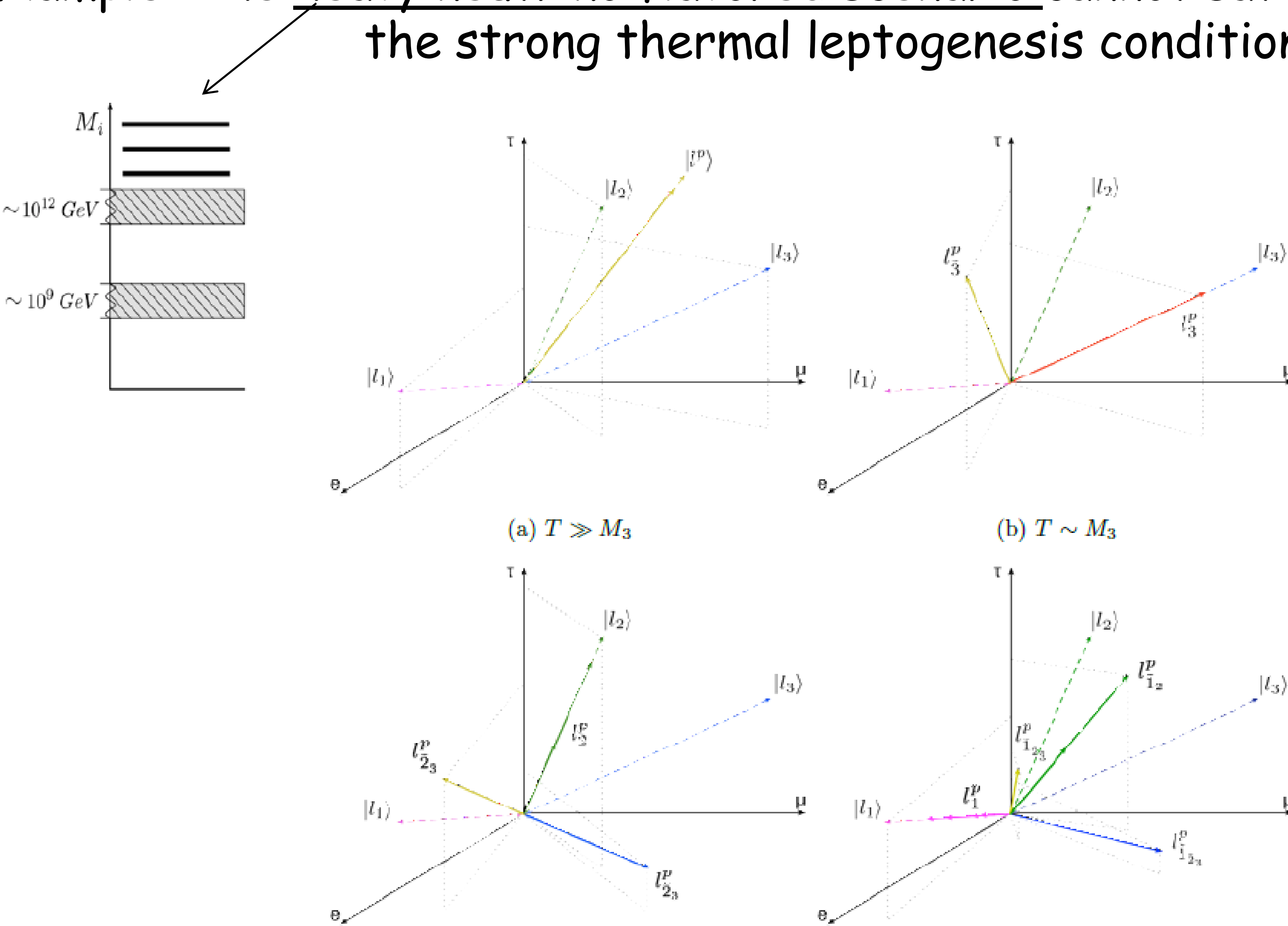
Asymmetry generated from leptogenesis



The conditions for the wash-out of a pre-existing asymmetry (= '**strong thermal leptogenesis**') can be realised only within a N_2-dominated scenario where the final asymmetry is dominantly produced in the tauon flavour

This mass pattern is just that one realized in the SO(10) inspired models: can they realise strong thermal leptogenesis?

Example: The heavy neutrino flavored scenario cannot satisfy the strong thermal leptogenesis condition



(a) $T \gg M_3$

(b) $T \sim M_3$

(c) $T \sim M_2$

(d) $T \sim M_1$

SO(10)-inspired+strong thermal leptogenesis

(PDB, Marzola '11-'12)

$$N_{B-L}^{\rm f} = N_{B-L}^{\rm p,f} + N_{B-L}^{\rm lep,f}\,,$$

Imposing both successful SO(10)-inspired leptogenesis

$\eta_B = \eta_B^{CMB} = (6.2 \pm 0.15) \times 10^{-10}$ and $N_{B-L}^{P,f} \ll N_{B-L}^{leP,f}$

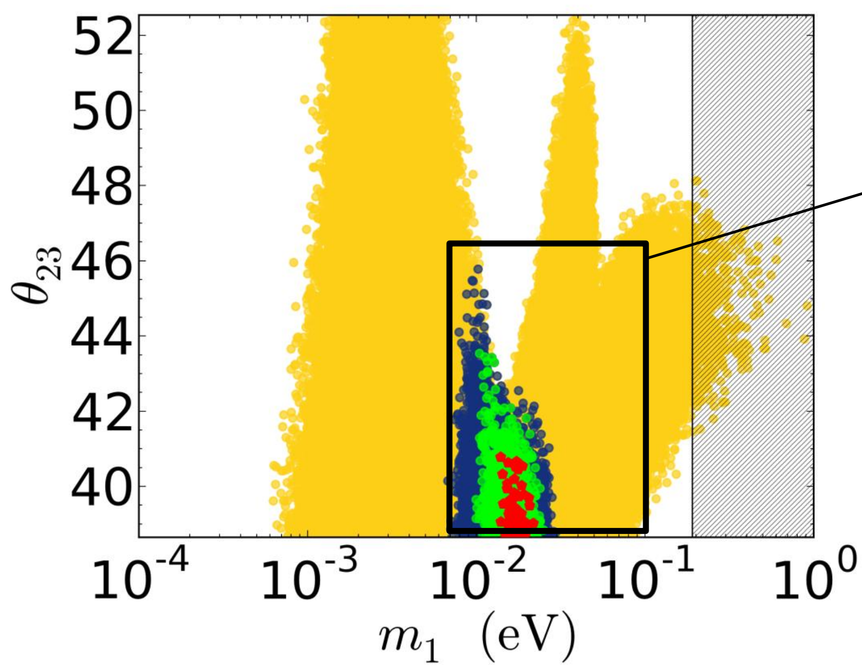
NO Solutions for Inverted Ordering ! But…

..for Normal Ordering there is a subset with interesting predictions

UPPER BOUND ON THE ATMOSPHERIC MIXING ANGLE

N_{B-L}= 0
0.001
0.01
0.1

α_2=5



Small atmospheric mixing angle

SO(10)-inspired+strong thermal leptogenesis

(PDB, Marzola '11-'12)

$$N^{\rm f}_{B-L} = N^{\rm p,f}_{B-L} + N^{\rm lep,f}_{B-L}\,,$$

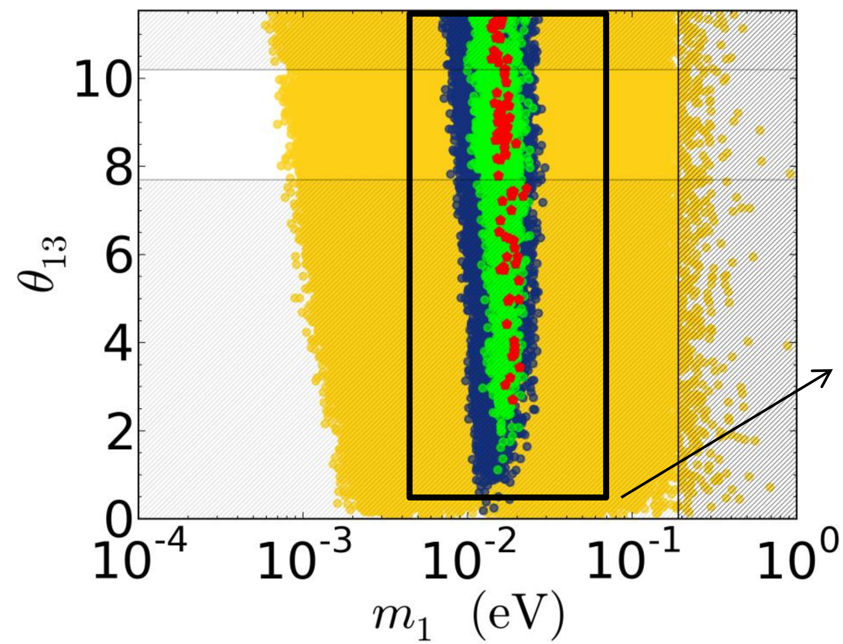
Imposing both successful SO(10)-inspired leptogenesis

$\eta_B = \eta_B^{CMB} = (6.2 \pm 0.15) \times 10^{-10}$ and $N^{P,f}_{B-L} \ll N^{leP,f}_{B-L}$

NON-VANISHING REACTOR MIXING ANGLE

$N_{B-L} =$ 0
0.001
0.01
0.1

$\alpha_2 = 5$



non-vanishing Θ_{13} (green and red points)

SO(10)-inspired+strong thermal leptogenesis

(PDB, Marzola '11-'12)

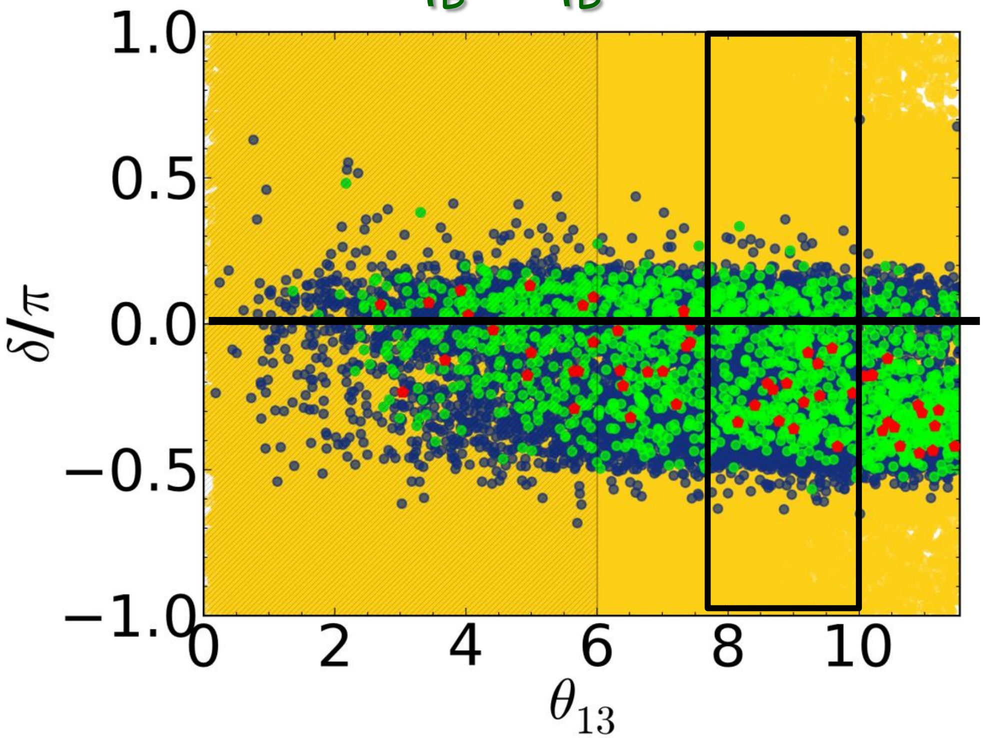
$$N_{B-L}^{\rm f} = N_{B-L}^{\rm p,f} + N_{B-L}^{\rm lep,f}\,,$$

Imposing both successful SO(10)-inspired leptogenesis

$\eta_B = \eta_B^{CMB} = (6.2 \pm 0.15) \times 10^{-10}$ and $N_{B-L}^{P,f} \ll N_{B-L}^{leP,f}$

Link between the sign of J_{CP} and the sign of the asymmetry

$\eta_B = \eta_B^{CMB}$ $\eta_B = -\eta_B^{CMB}$



A Dirac phase $\delta \sim -60^\circ$ is favoured for large θ_{13}

SO(10)-inspired+strong thermal leptogenesis

(PDB, Marzola '11-'12)

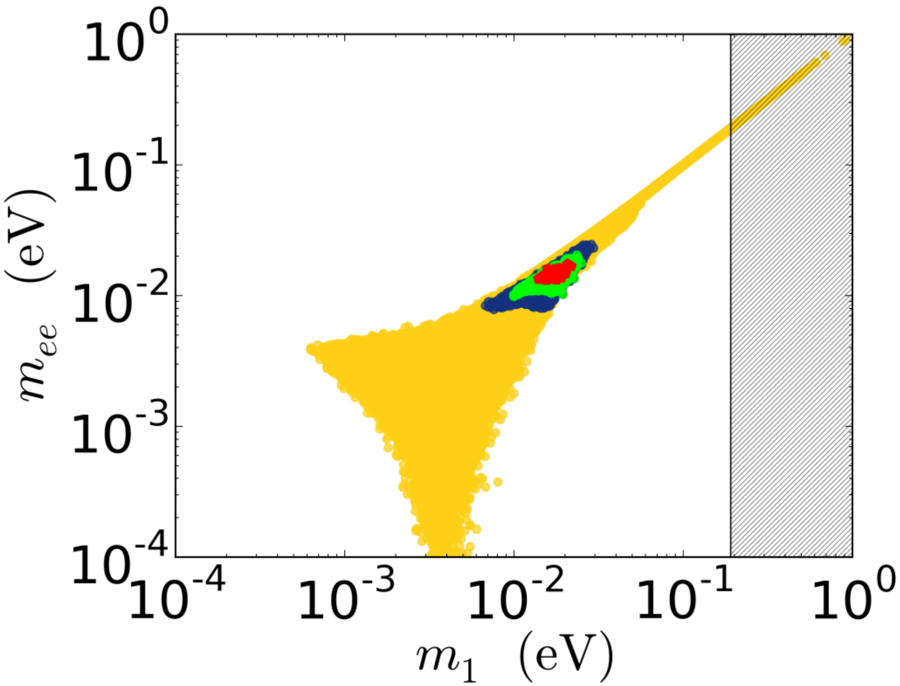
$$N^{\rm f}_{B-L} = N^{\rm p,f}_{B-L} + N^{\rm lep,f}_{B-L} \, ,$$

Imposing both successful SO(10)-inspired leptogenesis $\eta_B = \eta_B^{CMB} = (6.2 \pm 0.15) \times 10^{-10}$ and $N_{B-L}^{P,f} \ll N_{B-L}^{leP,f}$

NO Solutions for Inverted Ordering, while for Normal Ordering there is a subset with interesting predictions:

N_{B-L} = 0
0.001
0.01
0.1

α_2=5



A sharp prediction on the absolute neutrino mass scales

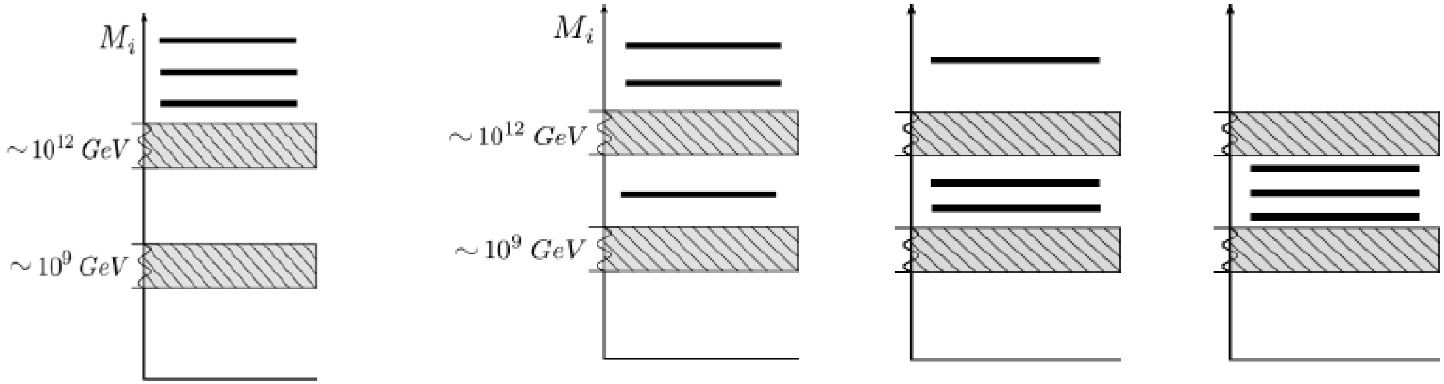
Concluding remarks

- SO(10)-inspired leptogenesis is not only alive but it produces a set of solutions able to satisfy a very difficult condition when flavour effects are taken into account: *independence of the initial conditions (strong thermal leptogenesis)*

- It is not necessary to believe it or not…it is sufficient just to wait expected improvements in low energy neutrino data with already data taking (or soon starting) experiments: any step can rule them out (for example IO)

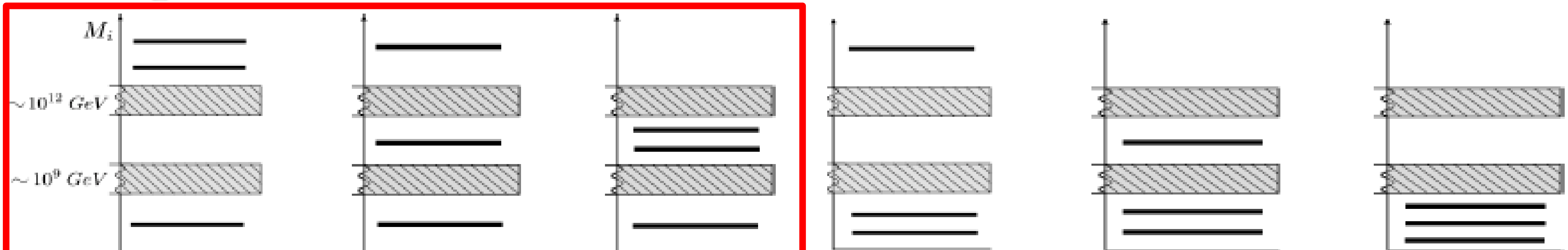
- At the moment the predictions are in quite a nice agreement with the current data but the absolute neutrino mass scale experiments would be the ultimate test, in case, since the predictions are quite sharp and, therefore, if satisfied all together with the others, they would form quite a strong case

More generally one has to distinguish 10 different RH neutrino mass patterns (Bertuzzo,PDB,Marzola '10)



(a) (b) (c)

N_2 dominated scenario

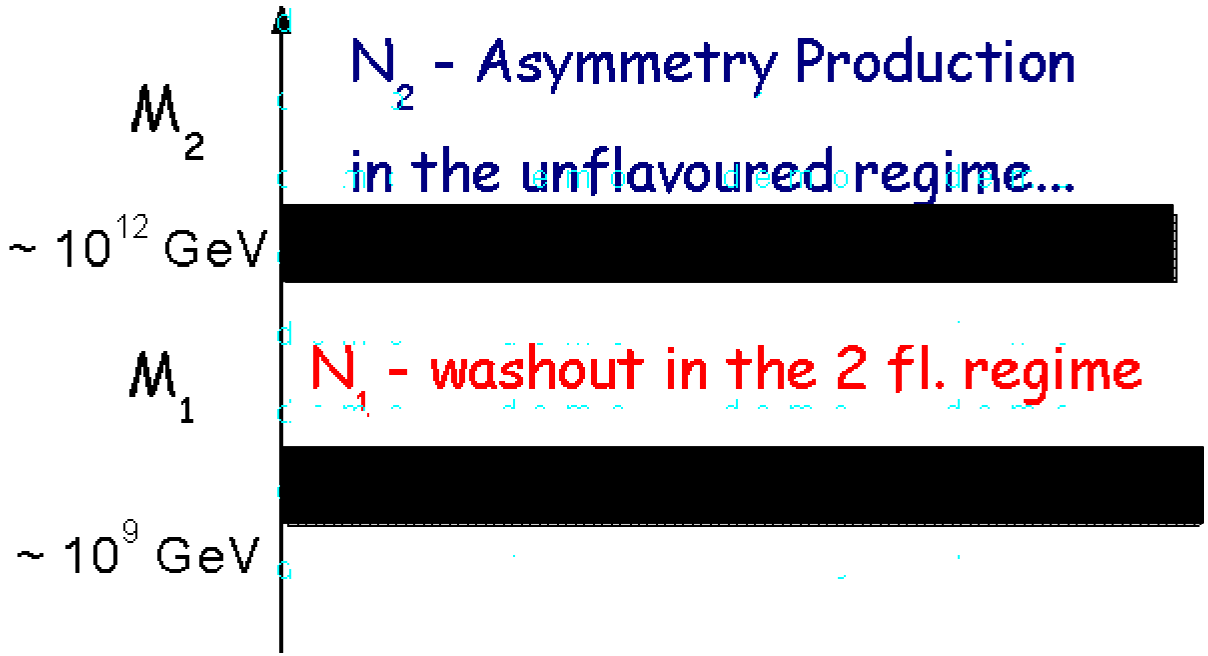


(a) (b) (c) (d) (e) (f)

For each pattern a specific set of Boltzmann equations has to be considered !

Phantom Leptogenesis

Consider this situation



What happens to N_{B-L} at $T \sim 10^{12}$ GeV?

How does it split into a $N_{\Delta\tau}$ component and into a $N_{\Delta e+\mu}$ component?

One could think:

$$N_{\Delta\tau} = p_{2\tau}\, N_{B-L},$$

$$N_{\Delta e+\mu} = p_{2\, e+\mu}\, N_{B-L}$$

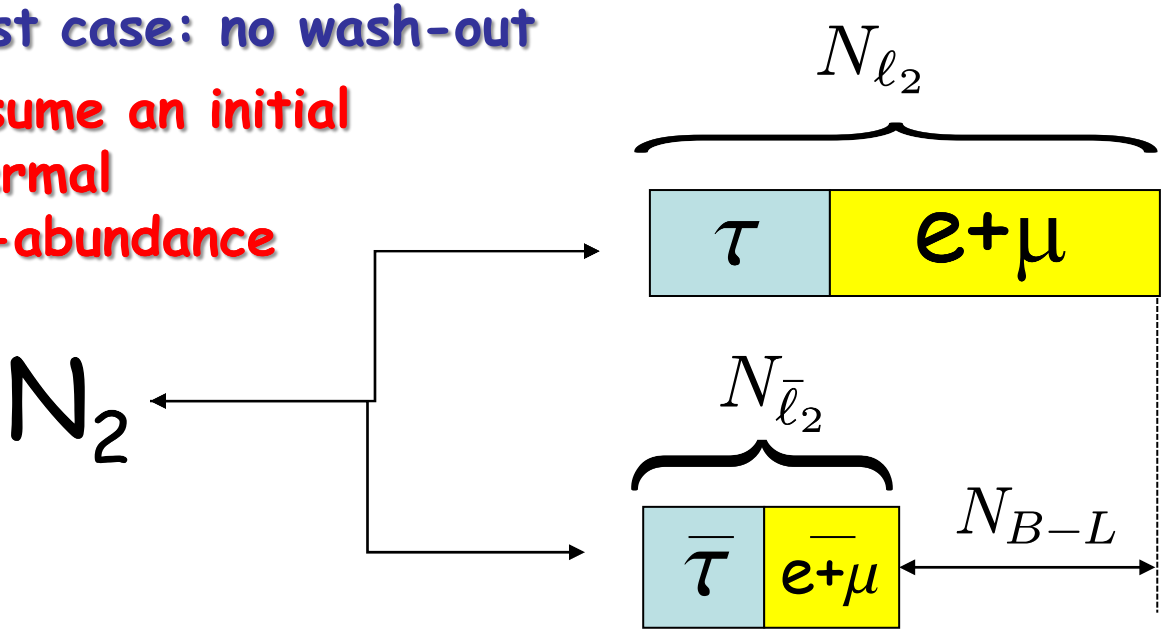
Phantom terms

However one has to consider that in the unflavoured case there are contributions to $N_{\Delta\tau}$ and $N_{\Delta e+\mu}$ that are not just proportional to N_{B-L}

Remember that:
$$\varepsilon_{1\alpha} = P^0_{1\alpha}\,\varepsilon_1 + \frac{\Delta P_{1\alpha}}{2}$$

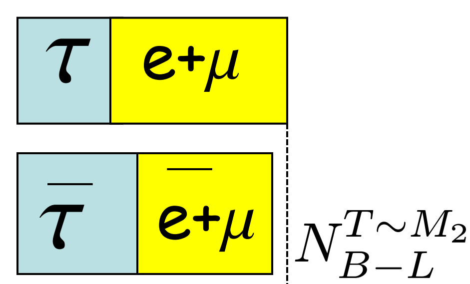
First case: no wash-out

Assume an initial thermal N_2-abundance



Second case: strong washout

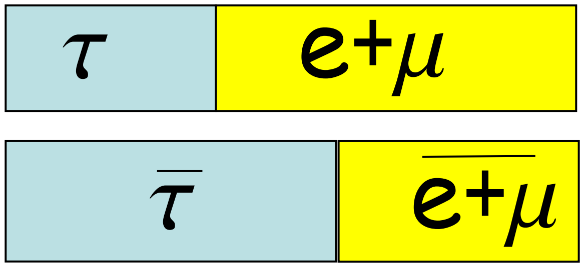
The N_2 wash-out can only suppress the B-L-asymmetry but it cannot change the flavour compositions of ℓ_2 and $\bar{\ell}'_2$



Phantom Leptogenesis

We can have then a situation where $K_2 >> 1$ so that at the end of the N_2 washout the total asymmetry is negligible:

$T \sim M_2$



$$N_{B-L}^{T\sim M_2} = N_{\Delta_\tau}^{T\sim M_2} + N_{\Delta_{e+\mu}}^{T\sim M_2} \simeq 0\ !$$

10^{12} GeV $\gtrsim T >> M_1$

$$N_{B-L}^{T\sim M_2} = N_{\Delta\tau}^{T\sim M_2} + N_{\Delta_{e+\mu}}^{T\sim M_2} \simeq 0\ !$$

$T \simeq M_1$

Assume $K_{1\tau} \lesssim 1$ and $K_{1e+\mu} >> 1$

$$N_{B-L}^{\rm f} \simeq N_{\Delta_\tau}^{T\sim M_2}\ !$$

The N_1 wash-out un-reveal the phantom term and effectively it creates a N_{B-L} asymmetry ! There is nothing esoteric but there is a...

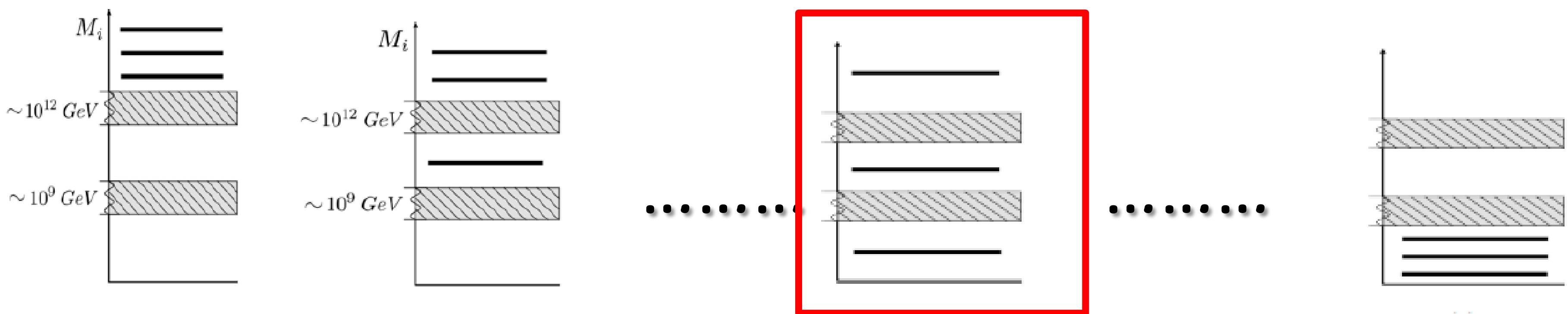
The problem of the initial conditions in flavoured leptogenesis

(Bertuzzo,PDB,Marzola '10)

$$N_{B-L}^{\rm f} = N_{B-L}^{\rm p,f} + N_{B-L}^{\rm lep,f}$$

Residual "pre-existing" asymmetry possibly generated by some external mechanism

Asymmetry generated from leptogenesis



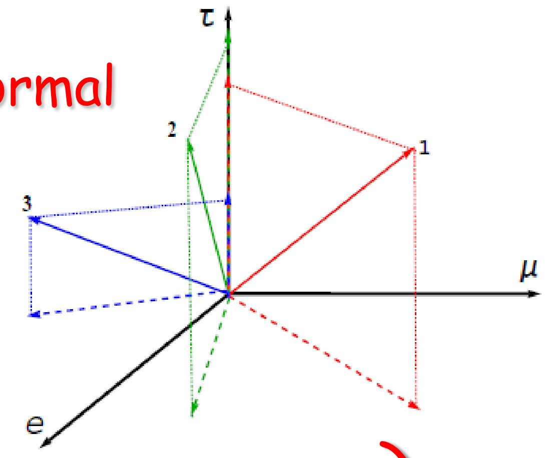
The wash-out of a pre-existing asymmetry is guaranteed only in a N_2-dominated scenario where the final asymmetry is dominantly in the tauon flavour

Flavour projection

(Engelhard, Nir, Nardi '08 , Bertuzzo,PDB,Marzola '10)

Assume $M_{i+1} \gtrsim 3M_i$ (i=1,2)

The heavy neutrino flavour basis cannot be orthonormal otherwise the CP asymmetries would vanish: this complicates the calculation of the final asymmetry



$$p_{ij} = |\langle \ell_i | \ell_j \rangle|^2 \qquad p_{ij} = \frac{\left|(m_D^\dagger\, m_D)_{ij}\right|^2}{(m_D^\dagger\, m_D)_{ii}\,(m_D^\dagger\, m_D)_{jj}}\,.$$

$$N_{B-L}^{(N_2)}(T \ll M_1) = N_{\Delta_1}^{(N_2)}(T \ll M_1) + N_{\Delta_{1\perp}}^{(N_2)}(T \ll M_1)$$

$\propto p_{12}$ $\qquad$ $\propto (1-p_{12})$

Component from heavier RH neutrinos **parallel** to l_1 and washed-out by N_1 inverse decays

Contribution from heavier RH neutrinos **orthogonal** to l_1 and escaping N_1 wash-out

$$N_{\Delta_1}^{(N_2)}(T \ll M_1) = p_{12}\, e^{-\frac{3\pi}{8}K_1}\, N_{B-L}^{(N_2)}(T \sim M_2)$$

?

Drawback of phantom Leptogenesis

We assumed an initial N_2 thermal abundance but if we were assuming An initial vanishing N_2 abundance the phantom terms were just zero !

$$N_{\Delta_\tau}^{\rm phantom} = \frac{\Delta p_{2\tau}}{2} N_{N_2}^{\rm in}$$

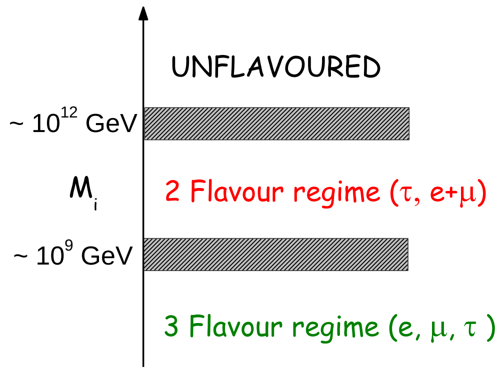
The reason is that if one starts from a vanishing abundance during the N_2 production one creates a contribution to the phantom term by inverse decays with opposite sign and exactly cancelling with what is created in the decays

In conclusion ....phantom leptogenesis introduces a strong dependence on the initial conditions

Limitations of Boltzmann equations

All results have been obtained within Boltzmann kinetic formalism assuming that leptons are either pure states or a full incoherent admixture of lepton flavour eigenstates (mixed states)

Limitations:



- Asymmetry cannot be calculated when masses fall in transition regions

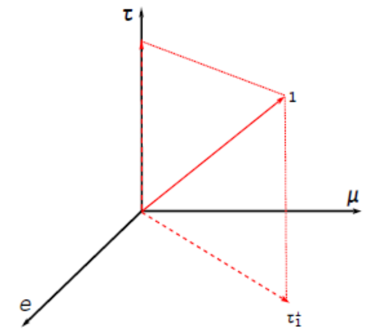
- Even in the fully flavoured regimes, the simultaneous occurrence of many effects makes the calculation quite contrived and one should worry whether everything is consistently taken into account

- More insight is certainly needed!

Density matrix formalism

Within a density matrix formalism it is possible to describe consistently a system that is a statistical ensemble of several elementary quantum states that are either pure states or mixed states.

Consider our leptons ℓ_1 produced by the decays of N_1



$$|1\rangle = \mathcal{C}_{1\tau}\,|\tau\rangle + \mathcal{C}_{1\tau_1^\perp}\,|\tau_1^\perp\rangle\,, \quad \mathcal{C}_{1\alpha} \equiv \langle\alpha|1\rangle$$

$(\alpha = \tau, \tau_1^\perp)$

Density operator $\hat{\rho}^{\ell_1} \equiv |1\rangle\langle 1| = \sum_{\alpha,\beta} \rho_{\alpha\beta}|\alpha\rangle\langle\beta|$

For a pure state $\hat{\rho}^2 = \hat{\rho}$ Moreover since $\rho = \rho^\dagger$ there is always a basis where is diagonal, in this case the basis is simply $|1\rangle,\ |1^\perp\rangle$

$$\rho_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad (i,j = 1, 1^\perp)$$

Density matrix formalism

When the ℓ_1 start to interact with the thermal bath, there will be

the early Universe starts to be populated both with pure states $|1\rangle$ and with mixed states $|\tau\rangle, |\tau_1^\perp\rangle$

I can still find a basis $|A\rangle, |B\rangle$ where the density matrix is diagonal:

$$\rho_{AB} = \mathrm{diag}(p_A, p_B), \quad \text{where } p_A + p_B = 1 \quad \text{but now } \rho \neq \rho^2$$

- When all states are pure simply $|A\rangle = |1\rangle, |B\rangle = |1^\perp\rangle$

- When all states are mixed $|A\rangle = |\tau\rangle, |B\rangle = |\tau_1^\perp\rangle$ but this time

$$\rho_{\tau\tau_1^\perp} = \mathrm{diag}(p_{1\tau}, 1 - p_{1\tau})$$

We can also introduce the lepton number density matrix simply as

$$N_{ij}^\ell = N_{\ell_1}\, \rho_{ij}^\ell$$

Density matrix formalism

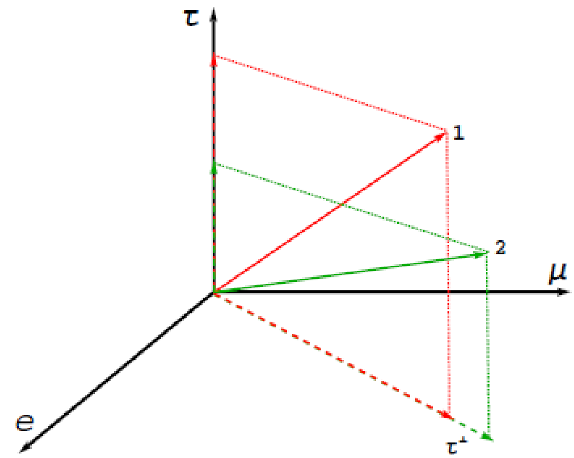
In the charged lepton flavour basis $|\tau\rangle, |\tau_1^{\perp}\rangle$ one has a transition from a matrix with off-diaagonal elements to a diagonal matrix. This evolution can be described with kinetic equations introducing decoherence due to the scatterings with the thermal bath

The result (subtracting density matrix for leptons and anti-leptons) for the B-L asymmetry matrix is

$$\begin{aligned}\frac{dN^{B-L}_{\alpha\beta}}{dz} &= \varepsilon^{(1)}_{\alpha\beta}\, D_1\,(N_{N_1} - N_{N_1}^{\rm eq}) - \frac{1}{2}\, W_1\,\left\{\mathcal{P}^{0(1)}, N^{B-L}\right\}_{\alpha\beta} \\ &+ \ {\rm i}\,\frac{{\rm Re}(\Lambda_\tau)}{H\,z}\left[\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, N^{\ell+\bar{\ell}}\right]_{\alpha\beta} - \frac{{\rm Im}(\Lambda_\tau)}{H\,z}\left[\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, \left[\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, N^{B-L}\right]\right]_{\alpha\beta},\end{aligned} \tag{45}$$

Density matrix formalism

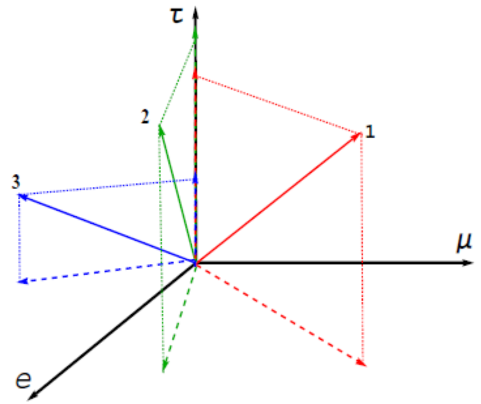
When more than 1 heavy neutrino flavour is included but still one has only 2 lepton flavours $|\tau\rangle, |\tau_1^{\perp}\rangle$
The equation includes 2 source terms for the asymmetry



$$\frac{dN^{B-L}_{\alpha\beta}}{dz} = \varepsilon^{(1)}_{\alpha\beta} D_1 (N_{N_1} - N^{\rm eq}_{N_1}) - \frac{1}{2} W_1 \{\mathcal{P}^{0(1)}, N^{B-L}\}_{\alpha\beta} \quad (50)$$
$$+ \varepsilon^{(2)}_{\alpha\beta} D_2 (N_{N_2} - N^{\rm eq}_{N_2}) - \frac{1}{2} W_2 \{\mathcal{P}^{0(2)}, N^{B-L}\}_{\alpha\beta}$$
$$+ \,{\rm i}\,{\rm Re}(\Lambda_\tau) \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, N^{\ell+\bar\ell} \right]_{\alpha\beta} - {\rm Im}(\Lambda_\tau) \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, N^{B-L} \right] \right]_{\alpha\beta} .$$

Density matrix formalism

When the whole 3 flavour structure is taken into account

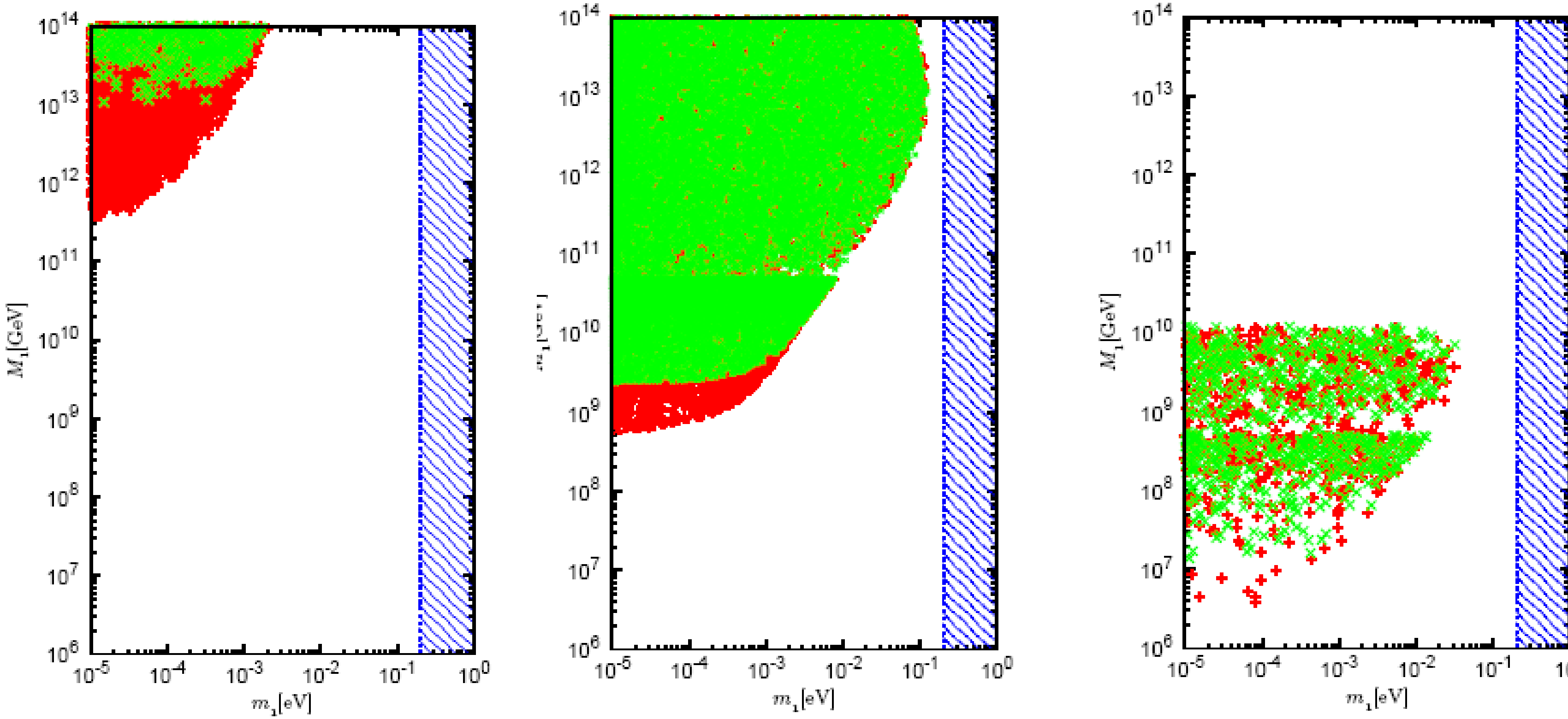


The result is a monster equation:

$$
\begin{aligned}
\frac{dN^{B-L}_{\alpha\beta}}{dz} &= \varepsilon^{(1)}_{\alpha\beta}\, D_1\,(N_{N_1} - N^{\rm eq}_{N_1}) - \frac{1}{2}\, W_1\, \{\mathcal{P}^{0(1)}, N^{B-L}\}_{\alpha\beta} \qquad (80)\\
&+ \varepsilon^{(2)}_{\alpha\beta}\, D_2\,(N_{N_2} - N^{\rm eq}_{N_2}) - \frac{1}{2}\, W_2\, \{\mathcal{P}^{0(2)}, N^{B-L}\}_{\alpha\beta}\\
&+ \varepsilon^{(3)}_{\alpha\beta}\, D_3\,(N_{N_3} - N^{\rm eq}_{N_3}) - \frac{1}{2}\, W_3\, \{\mathcal{P}^{0(3)}, N^{B-L}\}_{\alpha\beta}\\
&+ {\rm i\,Re}(\Lambda_\tau) \left[\begin{pmatrix} 1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, N^{\ell+\bar\ell}\right]_{\alpha\beta} - {\rm Im}(\Lambda_\tau) \left[\begin{pmatrix} 1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, \left[\begin{pmatrix} 1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, N^{B-L}\right]\right]_{\alpha\beta}\\
&+ {\rm i\,Re}(\Lambda_\mu) \left[\begin{pmatrix} 0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}, N^{\ell+\bar\ell}\right]_{\alpha\beta} - {\rm Im}(\Lambda_\mu) \left[\begin{pmatrix} 0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}, \left[\begin{pmatrix} 0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}, N^{B-L}\right]\right]_{\alpha\beta} .
\end{aligned}
$$

A first encouraging coincidence

$m_{atm} = 10^{-5}\, eV$ $\quad$ $m_{atm} = 0.05\, eV$ $\quad$ $m_{atm} = 10\, eV$



Green points: Unflavored

Red points: Flavored

SO(10)-inspired leptogenesis

(Branco et al. '02; Nezri, Orloff '02; Akhmedov, Frigerio, Smirnov '03)

Expressing the neutrino Dirac mass matrix m_D (in the basis where the Majorana mass and charged lepton mass matrices are diagonal):

$$m_D = V_L^\dagger \, D_{m_D} \, U_R$$ (bi-unitary parametrization) *

where $D_{m_D} = \mathrm{diag}\{\lambda_{D1}, \lambda_{D2}, \lambda_{D3}\}$

and assuming:

1) $\lambda_{D1} = \alpha_1 \, m_u \, , \, \lambda_{D2} = \alpha_2 \, m_c \, , \, \lambda_{D3} = \alpha_3 \, m_t \, , \quad (\alpha_i = \mathcal{O}(1))$

2) $V_L \simeq V_{CKM} \simeq I$

one typically obtains (barring fine-tuned exceptions):

$$M_1 \sim \alpha_1^2 \, 10^5 \mathrm{GeV} \, , \;\; M_2 \sim \alpha_2^2 \, 10^{10} \, \mathrm{GeV} \, , \;\; M_3 \sim \alpha_3^2 \, 10^{15} \, \mathrm{GeV}$$

since $M_1 \ll 10^9$ GeV $\Rightarrow \eta_B(N_1) \ll \eta_B^{CMB}$!

$\Rightarrow$ failure of the N_1-dominated scenario !

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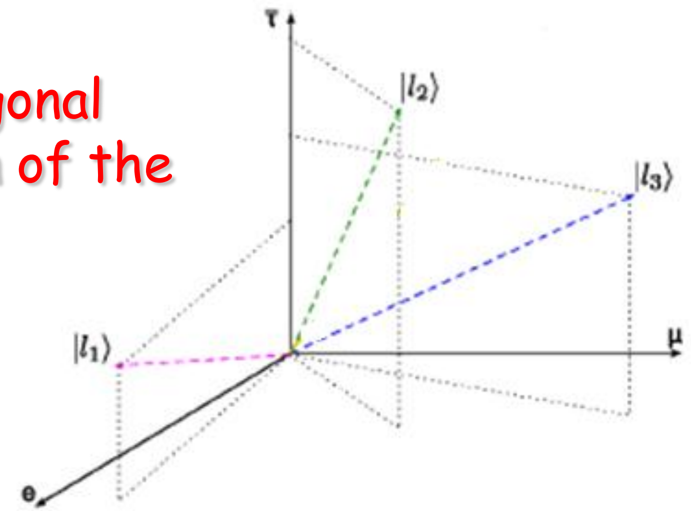
* Note that: $\Omega = D_m^{-\frac{1}{2}} \, U^\dagger \, V_L^\dagger \, D_{m_D} \, U_R \, D_M^{-\frac{1}{2}}$

Heavy neutrino flavored scenario

(Engelhard, Nir, Nardi '08 , Bertuzzo,PDB,Marzola '10)

Assume $M_{i+1} \gtrsim 3M_i$ (i=1,2)



The heavy neutrino flavour basis is not orthogonal in general and this complicates the calculation of the final asymmetry

$$p_{ij} = |\langle \ell_i | \ell_j \rangle|^2 \qquad p_{ij} = \frac{\left|(m_D^\dagger\, m_D)_{ij}\right|^2}{(m_D^\dagger\, m_D)_{ii}\,(m_D^\dagger\, m_D)_{jj}}\,.$$

$$N_{B-L}^{\rm lep}(T_{B1}) = N_{\Delta_1}^{\rm lep}(T_{B1}) + N_{\Delta_{\bar 1}}^{\rm lep}(T_{B1})\,,$$

$$\begin{aligned} N_{\Delta_1}^{\rm lep}(T_{B1}) &= p_{21}\,p_{32}\,\varepsilon_3\,\kappa(K_3)\,e^{-\frac{3\pi}{8}\,(K_1+K_2)} \\ &+ p_{21}\,\varepsilon_2\,\kappa(K_2)\,e^{-\frac{3\pi}{8}\,K_1} \\ &+ p_{\tilde{3}_3 1}\,(1-p_{32})\,\varepsilon_3\,\kappa(K_3)\,e^{-\frac{3\pi}{8}\,K_1} \\ &+ \varepsilon_1\,\kappa(K_1) \end{aligned}$$

$$\begin{aligned} N_{\Delta_{\bar 1}}^{\rm lep}(T_{B1}) &= (1-p_{21})\,[p_{32}\,\varepsilon_3\,\kappa(K_3)\,e^{-\frac{3\pi}{8}\,K_2} + \varepsilon_2\,\kappa(K_2)] \\ &+ (1-p_{\tilde{2}_3 1})\,(1-p_{32})\,\varepsilon_3\,\kappa(K_3)\,. \end{aligned}$$

Contribution from heavier RH neutrinos orthogonal to l_1 and escaping N_1 wash-out

Notice that some deviation from orthogonality is typically necessary since otherwise (e.g. with tri-bimaximal mixing) one would have vanishing CP asymmetries and therefore no asymmetry produced from leptogenesis (Antusch, King, Riotto '08; Aristizabal,Bazzocchi,Merlo,Morisi '09)

A recent global analysis

Global analysis of neutrino masses, mixings and phases: entering the era of leptonic CP violation searches

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TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. We remind that Δm^2 is defined herein as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH.

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.34 – 2.50	2.26 – 2.58	2.15 – 2.66
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.42	2.32 – 2.49	2.25 – 2.56	2.14 – 2.65
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.45	2.14 – 2.79	1.81 – 3.11	1.49 – 3.44
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.46	2.15 – 2.80	1.83 – 3.13	1.50 – 3.47
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.98	3.72 – 4.28	3.50 – 4.75	3.30 – 6.38
$\sin^2\theta_{23}/10^{-1}$ (IH)	4.08	3.78 – 4.43	3.55 – 6.27	3.35 – 6.58
δ/π (NH)	0.89	0.45 – 1.18	—	—
δ/π (IH)	0.90	0.47 – 1.22	—	—

2) The lower bounds on M_1 and on T_{reh} get relaxed:

(Blanchet, PDB '08)

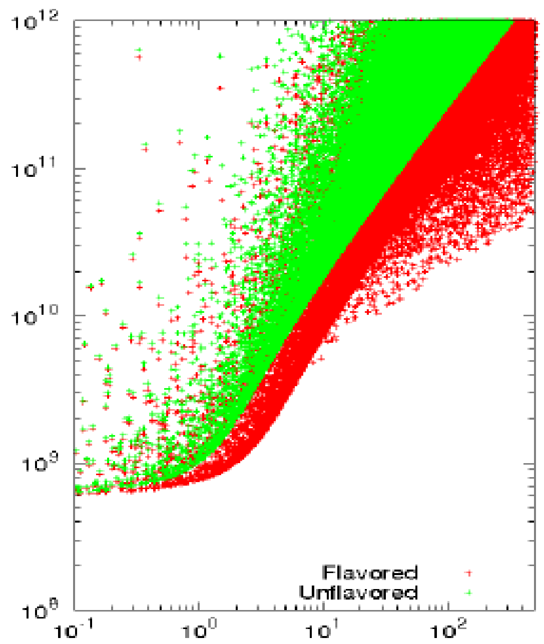
$$\frac{\Delta P_{i\alpha}}{2} \simeq \frac{1}{8\,\pi\,(h^\dagger h)_{ii}} \sum_{j\neq i} \left\{ \mathrm{Im}\left[h^\star_{\alpha i}\, h_{\alpha j} \left(\frac{3}{2\sqrt{x_j}} (h^\dagger h)_{ij} + \left(\qquad \right) \right] \right\}$$

$$x_j = \frac{M_j^2}{M_1^2}$$

It dominates for $|\Omega_{ij}| \lesssim 1$ but is upper bounded because of Ω orthogonality:

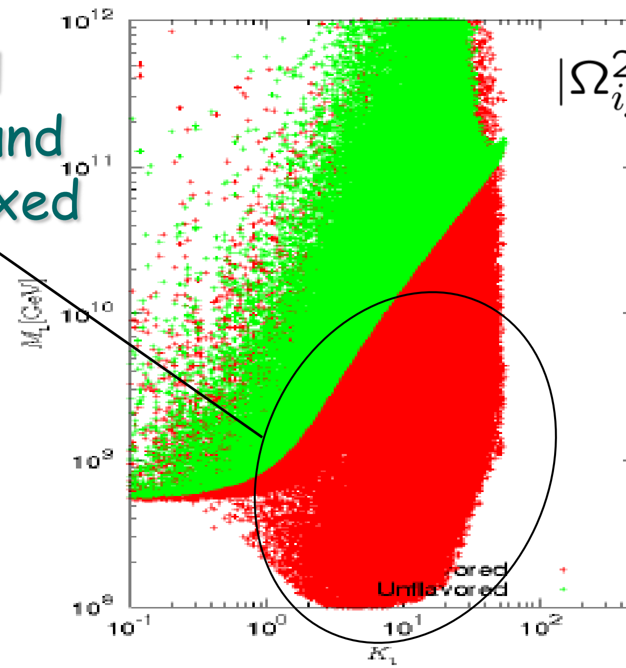
$$\left| \frac{\Delta P_{1\alpha}}{2} \right| < \overline{\varepsilon}(M_1)\, \sqrt{P^0_{1\alpha}}$$

It is usually neglected but since it is not upper bounded by orthogonality, for $|\Omega_{ij}| \gtrsim 1$ it can be important

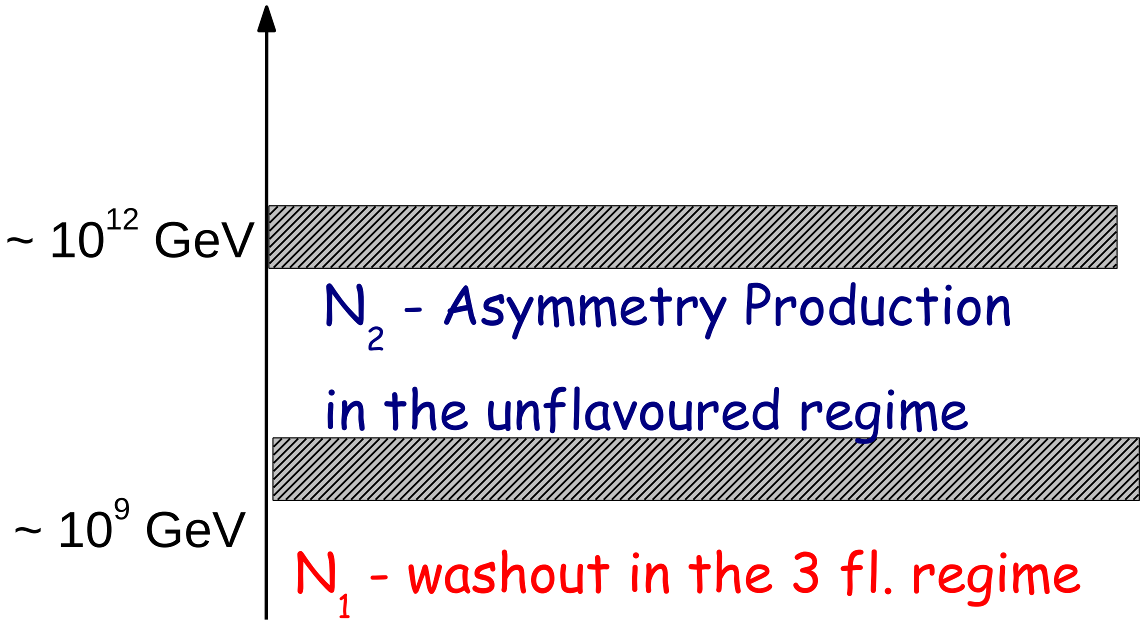


The usual lower bound gets relaxed

$|\Omega^2_{ij}| \lesssim 10$



Analogous results hold in the case when the production occurs in the 2 flavour regime for 10^{12} GeV $\gtrsim M_2 \gtrsim 10^9$ GeV:

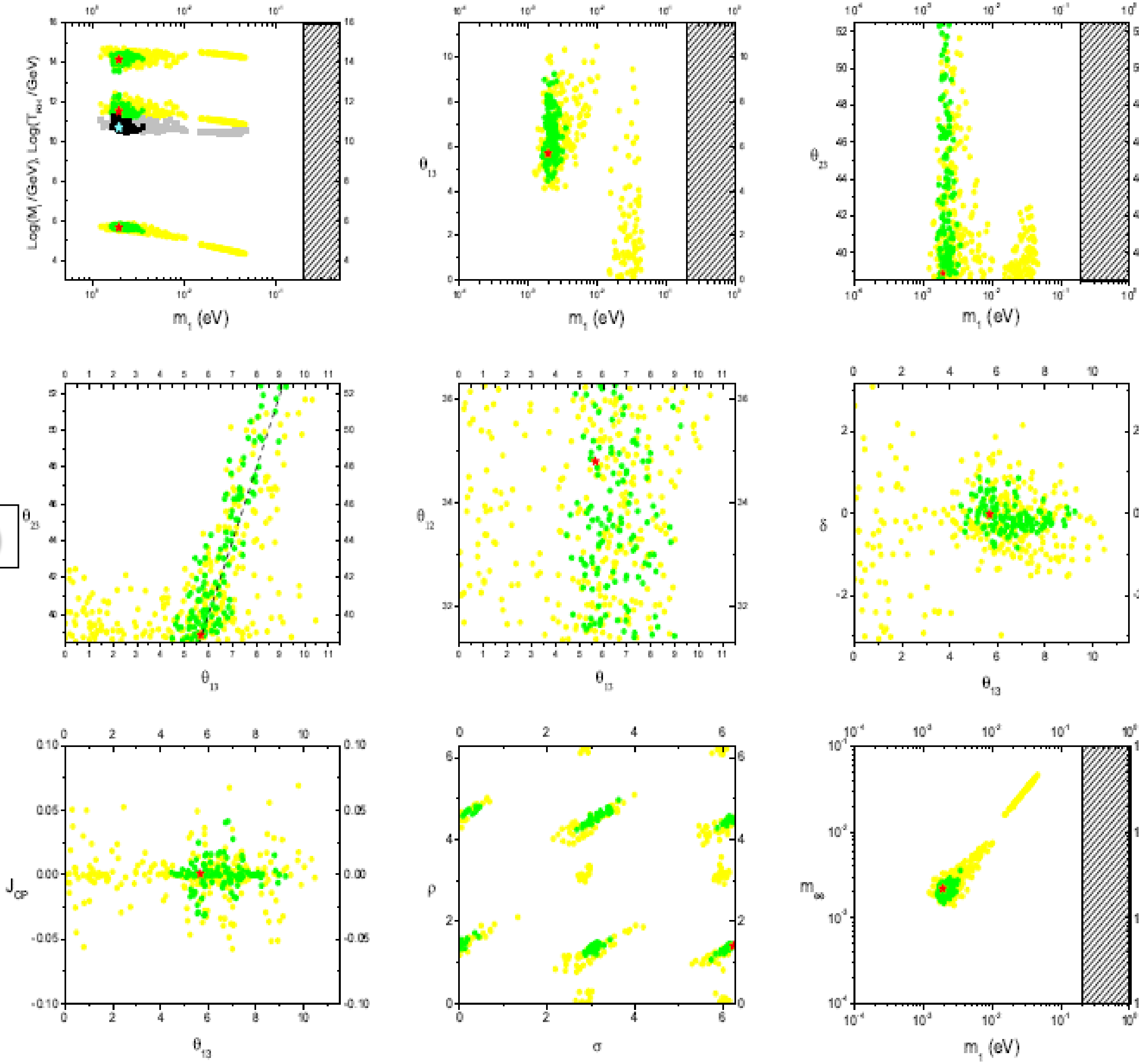


$$N_{B-L}^{\rm f} \simeq \varepsilon_{2e}\,\kappa(K_{2e+\mu})\,e^{-\frac{3\pi}{8}\,K_{1e}} + \varepsilon_{2\mu}\,\kappa(K_{2e+\mu})\,e^{-\frac{3\pi}{8}\,K_{1\mu}} + \varepsilon_{2\tau}\,\kappa(K_{2\tau})\,e^{-\frac{3\pi}{8}\,K_{1\tau}}\,.$$

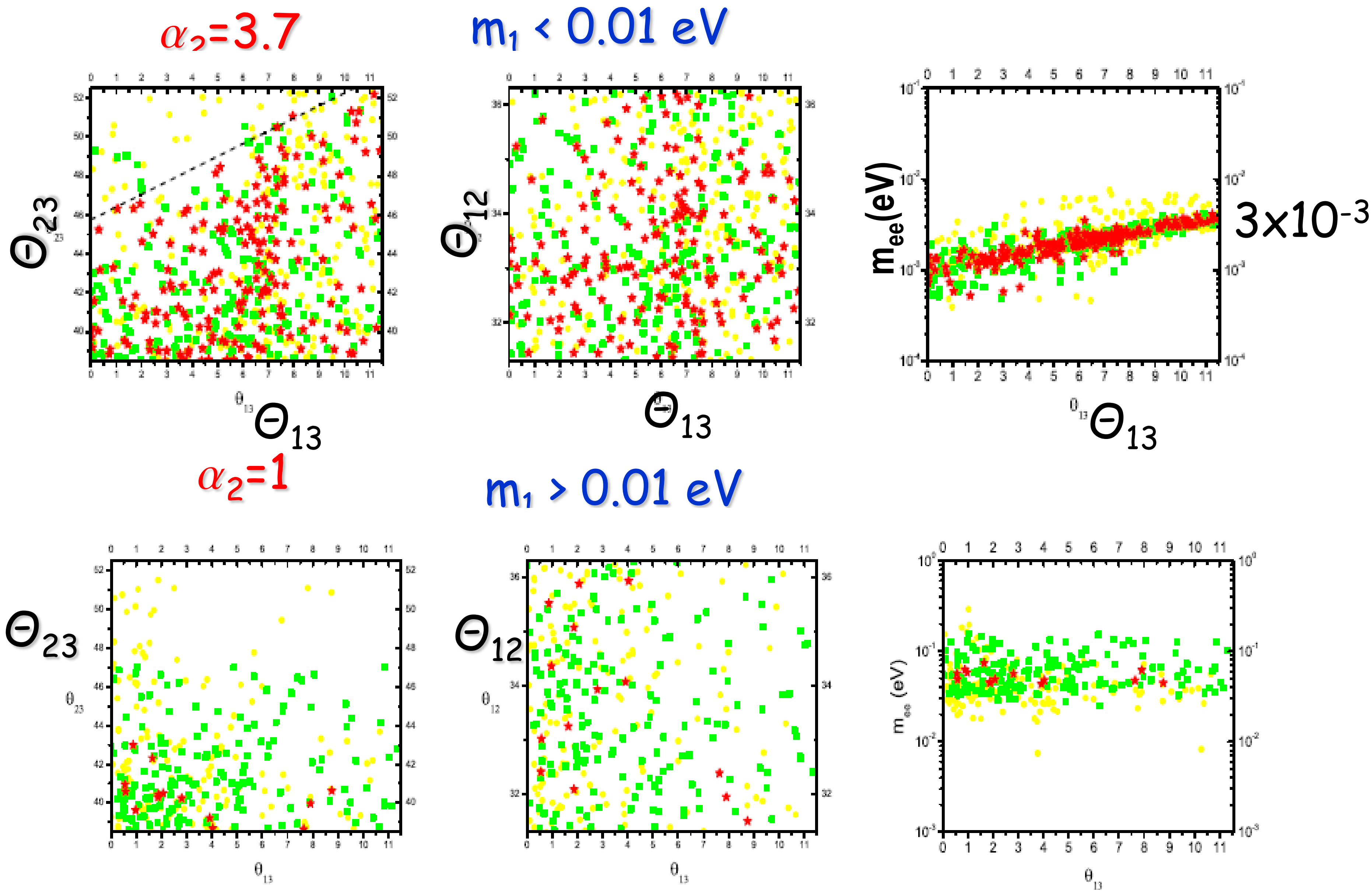
$V_L = I$

NORMAL ORDERING

(PDB, Riotto '10)



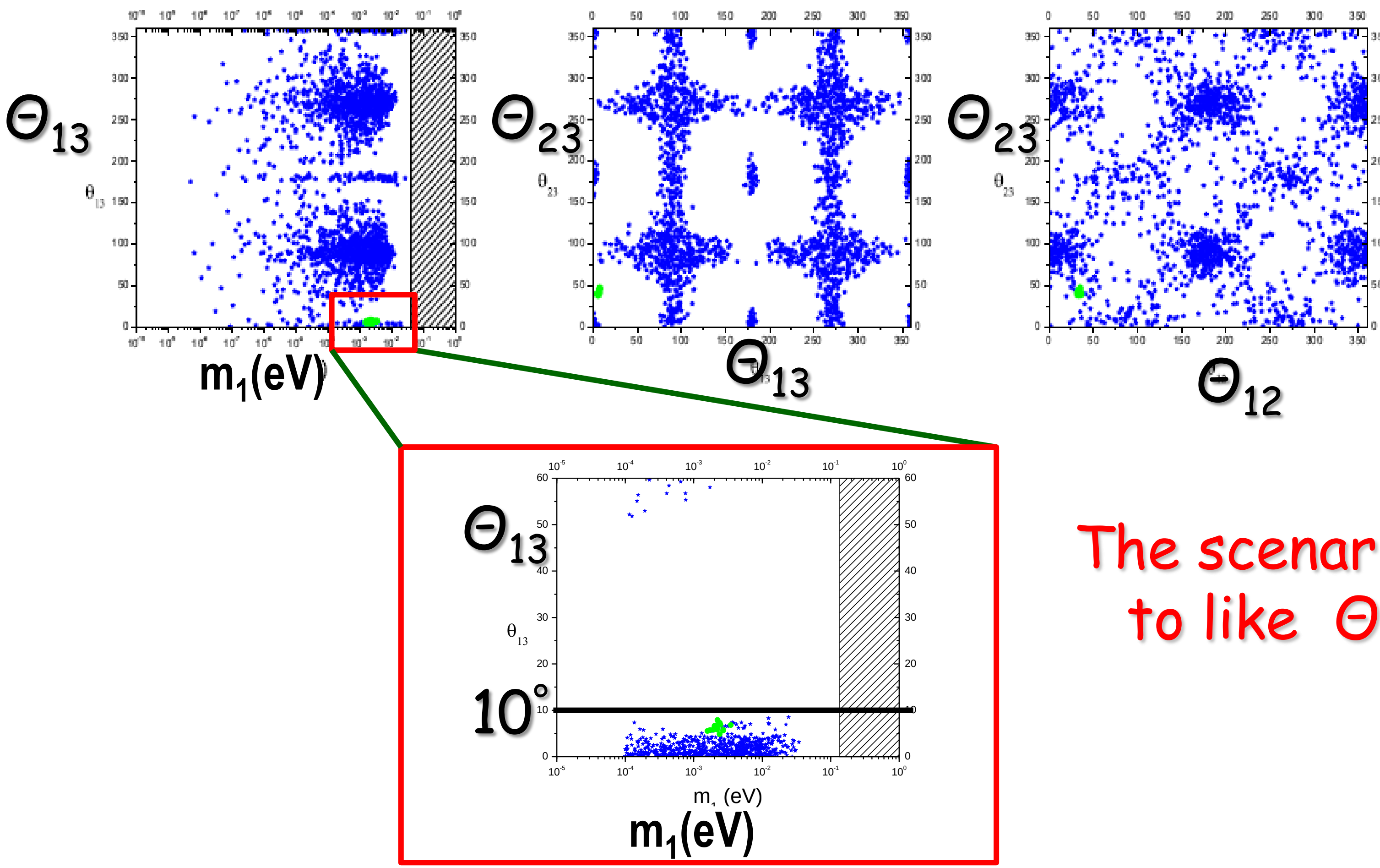
$I < V_L < V_{CKM}$ NORMAL ORDERING $\alpha_2=5$ $\alpha_2=4$



Are the data pointing in the right direction?

(PDB, Riotto '10)

Blue points: $\alpha_2=4$ and mixing angles let free in (0,180°)
Green points: $\alpha_2=4$ and current experimental constraints imposed on mixing angles



The scenario seems to like $\Theta_{13} \lesssim 10°$