UNIVERSITY OF SOUTHAMPTON ELEC3028A

SEMESTER 1 EXAMINATION 2005/06

DIGITAL TRANSMISSION

Duration: 120 mins

Answer THREE questions,
at least ONE from EACH of the two sections

Calculators without text storage may be used.
An approximate marking scheme is indicated.

$$\log_2 x = \frac{\log_{10} x}{\log_{10} 2} \approx 3.322 \cdot \log_{10} x$$

$$x \log_y x = 0 \quad \text{for} \quad x = 0$$

$$e^{j\varphi} = \cos\varphi + j\sin\varphi$$

$$\cos\varphi = \frac{1}{2}(e^{j\varphi} + e^{-j\varphi}) \qquad \sin\varphi = \frac{1}{2j}(e^{j\varphi} - e^{-j\varphi})$$

$$\sin(2\varphi) = 2\sin(\varphi)\cos(\varphi)$$

$$\cos(2\varphi) = 2\cos^2(\varphi) - 1 = 1 - 2\sin^2(\varphi)$$

$$\frac{1}{1-a} = \sum_{i=0}^{\infty} a^i \qquad \text{for} \quad |a| < 1$$

Number of pages: 7

SECTION A

1. Source Encoding

a) A source emits symbols X_i, $1 \leq i \leq 8$, in binary coded decimal (BCD) format with probabilities $P(X_i)$ as given in Table 1, at a rate $R_s = 64$ kbaud (baud=symbol/s).

Table 1.

X_i	$P(X_i)$	BCD word
A	$\frac{1}{4}$	000
B	$\frac{1}{16}$	001
C	$\frac{1}{64}$	010
D	$\frac{1}{8}$	011
E	$\frac{1}{2}$	100
F	$\frac{1}{32}$	101
G	$\frac{1}{128}$	110
H	$\frac{1}{128}$	111

State (i) the information rate and (ii) the data rate of the source. (*6 marks*)

b) Apply Huffman coding to the source signal characterised in Table 1. Are there any disadvantages in the resulting code words? (*6 marks*)

c) What is the original symbol sequence of the Huffman coded signal 011000111110010? (*2 marks*)

d) What is the data rate of the signal after Huffman coding? What compression ratio has been achieved? (*5 marks*)

e) Derive the coding efficiency of both the uncoded BCD signal as well as the Huffman coded signal. (*4 marks*)

f) Apply Shannon-Fano coding to the source signal characterised in Table 1, and derive the coding efficiency of the Shannon-Fano coded signal. (*8 marks*)

g) What is the original symbol sequence of the Shannon-Fano coded signal 011000111110010? (*2 marks*)

2. Information across Channels

a) Consider the binary symmetric channel (BSC) shown in Figure 1.

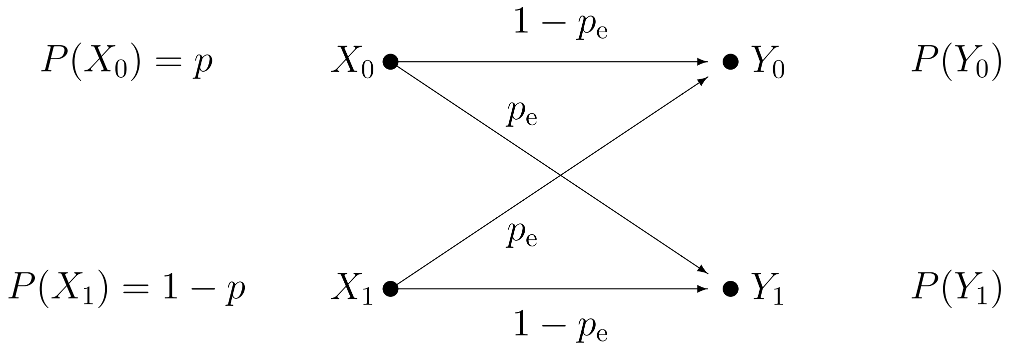

Figure 1.

From the definition of mutual information,

$$I(X,Y) = \sum_i \sum_j P(X_i, Y_j) \cdot \log_2 \frac{P(X_i|Y_j)}{P(X_i)} \quad \text{[bits/symbol]}$$

derive both

(i) a formula relating $I(X,Y)$, the source entropy $H(X)$, and the average information lost per symbol $H(X|Y)$, and (*6 marks*)

(ii) a formula relating $I(X,Y)$, the destination entropy $H(Y)$, and the error entropy $H(Y|X)$. (*6 marks*)

b) State and justify the relation ($>$, $<$, $=$, $\leq$, or $\geq$) between $H(X|Y)$ and $H(Y|X)$. (*4 marks*)

c) Considering the BSC in Figure 1, we now have $p = \frac{1}{4}$ and a channel error probability $p_e = \frac{1}{16}$. Calculate all the probabilities $P(X_i, Y_j)$ and $P(X_i|Y_j)$, and derive the numerical value for the mutual information $I(X,Y)$. (*9 marks*)

Question 2 continued on the next page

2. Information across Channels continued . . .

d) A digital communication system uses 4-ary signalling scheme. Assume that the four symbols X_i, $1 \leq i \leq 4$, are equiprobable with $P_i = P(X_i) = \frac{1}{4}$, and this symbol source can be modelled as a first-order Markov process with the transition probabilities $p_{i,j} = P(X_j|X_i) = \frac{1}{4}$ for $1 \leq i, j \leq 4$.

The channel is an ideal channel with additive white Gaussian noise (AWGN), the transmission rate is 5 Mbaud (5×10^6 symbols/s), and the channel signal to noise ratio is known to be 31.

Determine the minimum channel bandwidth required to achieve error-free transmission. (*8 marks*)

3. Modem

a) Consider the case where the transmission rate in a digital communication system is $f_s = 10$ kHz, the combined transfer function of the transmit and receive filters has a raised-cosine characteristic with a roll-off factor of 0.2, and the transmit and receive filters are identical.

(i) Determine the required baseband transmission bandwidth. (*2 marks*)

(ii) The channel noise is known to have a flat power spectral density (PSD) of 0.5×10^{-6} Watts/Hz. Determine the noise power at the receiver output. (*4 marks*)

b) Based on a QAM signal $x(t) = x_i(t) + jx_q(t)$, the transmitted signal in Figure 2 is given by $s(t) = x_i(t) \cos \omega_c t + x_q(t) \sin \omega_c t$.

Question 3 continued on the next page

3. Modem continued . . .

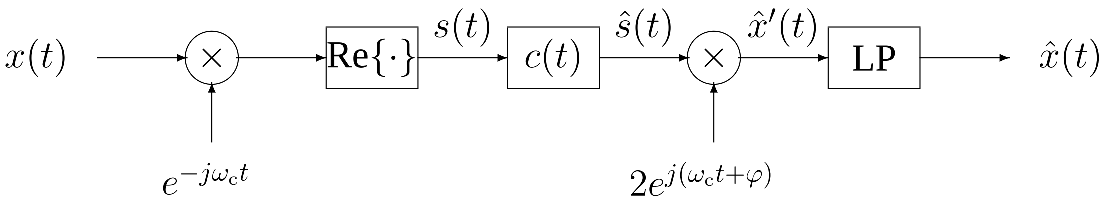

Figure 2.

(i) With a channel impulse response $c(t) = \delta(t - 0.2T_s)$ where T_s is the symbol period, and a suitably selected lowpass filter LP, show that the receiver output is given by

$$\hat{x}(t) = x(t - 0.2T_s) \cdot e^{j(\varphi + \omega_c 0.2T_s)}$$

Sketch the magnitude response of the lowpass filter LP. (*9 marks*)

(ii) What is the best value of φ for the demodulator? Name the component in the receiver that is used to lock into this optimal carrier phase offset. (*3 marks*)

(iii) In the receiver, $\hat{x}(t)$ is sampled at $t = kT_s + \tau$ to produce samples $\hat{x}[k]$. Determine the best value of τ for the sampler. Name the component in the receiver that is used to find this optimal sampling offset. (*3 marks*)

(c) If the channel $c(t)$ exhibits severe amplitude and phase distortions, an equaliser is required at the receiver.

(i) Draw the schematic of adaptive channel equalisation. Explain the two operational modes of the adaptive equaliser. (*8 marks*)

(ii) Assume that a linear equaliser of M taps with a decision delay d is used. Write down the tap weight adaptation equation of the least mean square algorithm during training. (*4 marks*)

SECTION B

4. Source Coding

a) Characterise the behaviour of both voiced and unvoiced speech signals with the aid of their statistical description using both their stylised Autocorrelation Function (ACF) as well as their Power Spectral Density (PSD). With reference to these functions explain, how redundancy manifests itself in both the time-domain and frequency-domain. (*6 marks*)

b) Characterize the speech codecs known to you by assigning them to the appropriate point on the bit-rate versus speech quality plane and comment on their computational complexity, delay and robustness against transmission errors.

(*4 marks*)

c) Under the assumption that a PCM codec's quantisation error $e(n)$ is uniformly distributed across the quantization interval, derive a formula for the variance, ie. the power of $e(n)$. (*4 marks*)

d) Under the assumption that the input signal is also uniformly distributed across the linear quantiser characteristic's entire dynamic range and no quantiser overload is encountered, derive a similar formula for the input signal's variance. Based on these two formulae express the PCM codec's Signal to Noise Ratio (SNR) in terms of dB. (*4 marks*)

e) Sketch the block diagram of a Differential Pulse Code Modulated (DPCM) speech codec and explain the operation of each constituent block with the aid of time- and/or frequency-domain sketches, as deemed appropriate.

(*6 marks*)

f) Provide an equation for the predictor coefficient value of the optimum one-tap predictor. (*6 marks*)

g) Assuming that the one-step correlation of the input speech signal is $r = 0.9$, express the prediction gain achieved by the one-tap predictor. (*3 marks*)

5. Channel Coding

a) Given the generator polynomials of $g_1(z) = 1 + z^2$ and $g_2(z) = 1 + z + z^2$, draw the corresponding convolutional encoder's schematic.

(*6 marks*)

b) Draw the encoder's state transition diagram, indicating the resultant encoded bits for all transitions.

(*8 marks*)

c) Consider the received sequence of **01**, 00, 01, 00, 00, 00, 00, 00, where the left-most two-bit codeword printed in bold font was received last and hence appears at the left hand side of the corresponding decoding trellis. Determine the transmitted information sequence by drawing the trellis diagram and clearly indicate the "winning" path through the trellis.

(*15 marks*)

d) Assuming that the decoded bit sequence is identical to the transmitted one, determine the number of transmission errors induced by the channel during the specified received sequence.

(*4 marks*)

END OF PAPER