UNIVERSITY OF SOUTHAMPTON ELEC3028W1

SEMESTER 1 EXAMINATION 2009/2010

DIGITAL TRANSMISSION

Duration: 120 mins

Answer THREE questions,
at least ONE from EACH of the two sections

University approved calculators MAY be used
An approximate marking scheme is indicated.

$$\log_2 x = \frac{\log_{10} x}{\log_{10} 2} \approx 3.322 \cdot \log_{10} x$$

$$x \log_y x = 0 \quad \text{for} \quad x = 0$$

$$e^{j\varphi} = \cos\varphi + j\sin\varphi$$

$$\cos\varphi = \frac{1}{2}(e^{j\varphi} + e^{-j\varphi}) \qquad \sin\varphi = \frac{1}{2j}(e^{j\varphi} - e^{-j\varphi})$$

$$\sin(2\varphi) = 2\sin(\varphi)\cos(\varphi)$$

$$\cos(2\varphi) = 2\cos^2(\varphi) - 1 = 1 - 2\sin^2(\varphi)$$

$$\frac{1}{1-a} = \sum_{i=0}^{\infty} a^i \qquad \text{for} \quad |a| < 1$$

SECTION A

1. Source Encoding

(a) During the digitisation of analogue telephone network in 1960s a 64 kbps speech coder standard was formulation in which the speech waveform is first filtered to limit its bandwidth to approximately 3.8 kHz and a sampling rate of 8 kHz is then applied. The resultant samples are first passed through an A-law compressor and the compressed samples are then quantised to 256 levels with each sample being represented by 8 bits.

What will be your main criticism of this scheme? You should explain your view clearly according to the information theory you have learnt.

(*6 marks*)

(b) A memoryless source emits symbols X_i, $1 \leq i \leq 8$, in binary coded decimal (BCD) format with probabilities $P(X_i)$ as given in Table 1, at a rate $R_\mathrm{s} = 100$ kBaud (Baud=symbol/s).

Table 1.

X_i	$P(X_i)$	BCD word
A	0.51	000
B	0.17	001
C	0.10	010
D	0.09	011
E	0.06	100
F	0.04	101
G	0.02	110
H	0.01	111

(b.i) Calculate the information rate of the source and the data rate of the uncoded BCD signal.

(*4 marks*)

(b.ii) Apply Huffman coding to the source characterised in Table 1.

(*7 marks*)

Question 1 continues on the next page

1. Source Encoding continued...

(b.iii) What is the data rate of the signal after Huffman coding? What compression ratio has been achieved?

(*3 marks*)

(c) A binary memoryless source emits binary digits at a rate of 10^6 symbols/s. It is also known that the source emits the bit **0** with probability 0.99 and the bit **1** with probability 0.01.

(c.i) What is the bit rate of this source? What is the information rate of this source?

(*3 marks*)

(c.ii) The bit sequence generated by this source is encoded by a run length encoder (RLC) with a codeword length of $n = 5$ bits. What is the bit rate after the RLC encoding?

(*5 marks*)

(c.iii) Determine the RLC encoder input patterns that produce the following encoder output codewords

$$11111 \quad 11110 \quad 11101 \quad 11100 \quad 11011 \quad \cdots \quad 00011 \quad 00010 \quad 00001 \quad 00000$$

(*5 marks*)

Section A continues on the next page

2. Information Transmission Across Channels

(a) Consider the binary symmetric channel (BSC) shown in Figure 1.

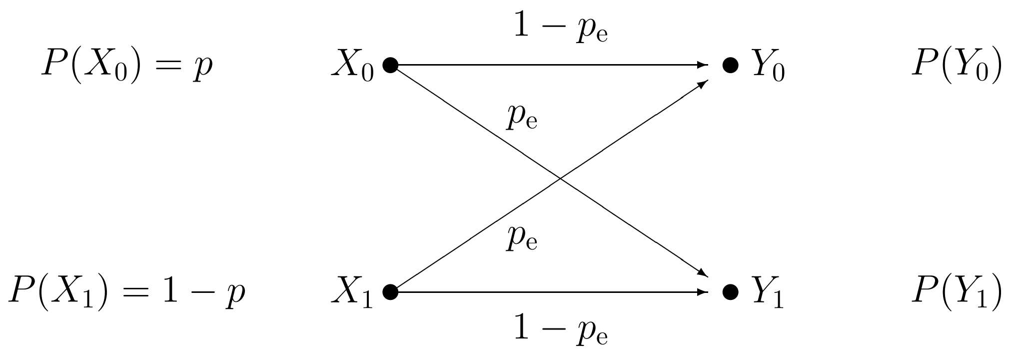

Figure 1.

(a.i) For what values of p and p_e are the source and destination entropies, $H(X)$ and $H(Y)$, identical? In general, is $H(Y) \geq H(X)$ or $H(Y) \leq H(X)$ true? Justify your answer.

(*8 marks*)

(a.ii) State and justify the relation ($\leq$ or $\geq$) between the average information lost per symbol $H(X|Y)$ and the error entropy $H(Y|X)$.

(*4 marks*)

(b) For the BSC in Figure 1, we now have $p = \frac{1}{2}$ and a channel error probability of $p_e = \frac{1}{16}$.

(b.i) Calculate all the probabilities $P(X_i, Y_j)$ as well as $P(X_i|Y_j)$, and derive the numerical value for the mutual information $I(X, Y)$.

(*8 marks*)

(b.ii) Determine $H(X|Y)$ using the values calculated in (b.i). What is the value of $H(Y|X)$?

(*5 marks*)

Question 2 continues on the next page

2. Information Transmission Across Channels continued...

(c) A digital communication system uses a 4-ary signalling scheme. The probabilities of occurrence for the four symbols are respectively

$$P(X_1) = 0.1, \;\; P(X_2) = 0.2, \;\; P(X_3) = 0.3, \;\; P(X_4) = 0.4,$$

and this symbol source can be modelled as a first-order Markov process with the known transition probability matrix

$$\mathbf{\Gamma} = [p_{ij}] = \begin{bmatrix} 0.2 & 0.1 & 0.1 & 0.6 \\ 0.1 & 0.2 & 0.1 & 0.6 \\ 0.1 & 0.1 & 0.8 & 0.0 \\ 0.0 & 0.1 & 0.1 & 0.8 \end{bmatrix},$$

where $p_{ij} = P(X_j|X_i)$ for $1 \leq i, j \leq 4$ are the transition probabilities.

The channel is an ideal additive white Gaussian noise (AWGN) channel, the transmission rate is 42 MBaud (42×10^6 symbols/s) and the channel's signal to noise ratio is known to be 127.

Determine the minimum channel bandwidth required to achieve error-free transmission.

(*8 marks*)

Section A continues on the next page

3. Modem

(a) Draw the block diagram of a typical digital communication modem, and briefly explain the purpose of each component.

(*10 marks*)

(b) Based on a QAM signal $x(t) = x_i(t) + jx_q(t)$, the transmitted signal in Figure 2 is given by $s(t) = x_i(t)\cos\omega_c t + x_q(t)\sin\omega_c t$.

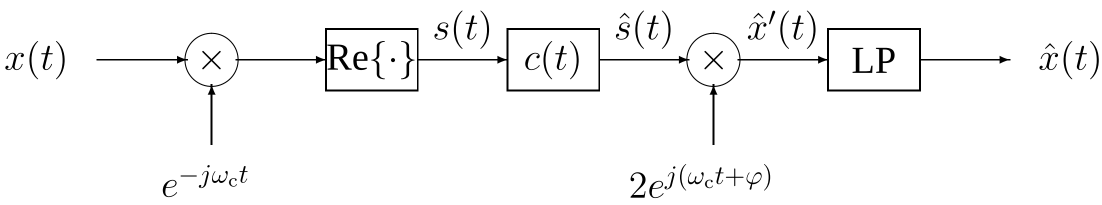

Figure 2.

With a channel impulse response $c(t) = \delta(t - 0.2T_s)$ where T_s is the symbol period, and a suitably selected lowpass filter LP, show that the receiver output is given by

$$\hat{x}(t) = x(t - 0.2T_s) \cdot e^{j(\varphi + \omega_c 0.2T_s)}$$

Sketch the magnitude response of the lowpass filter LP.

(*9 marks*)

(c) A communication channel has a passband bandwidth of 30 MHz. The combined transfer function of the transmit and receive filters has a raised-cosine characteristic with a roll-off factor of 0.2. Design a modulation scheme so that you can transmit at the bit rate of 50 Mbits/s over this channel. You must clearly explain your design.

(*6 marks*)

(d) If the channel exhibits severe amplitude and phase distortions, an equaliser is required at the receiver.

Draw the schematic of adaptive baseband channel equalisation. Explain the two operational modes of the adaptive equaliser.

(*8 marks*)

SECTION B

4. Video Compression

a) Calculate the uncompressed bitrate required for the transmission of both 288 $\times$ 352 pixel Common Intermediate Format (CIF) and 144 $\times$ 176 pixel Quarter CIF (QCIF) black and white video sequences using the standard 8 bit/pixel grey-scale representation at a frame-scanning rate of 30 frames/s.

(*3 marks*)

b) Sketch the schematic of a simple video codec using frame differencing, but no motion vectors and explain its operation.

(*3 marks*)

c) Sketch the stylised autocorrelation function and the probability density function of both the original video signal and that of the frame difference signal.

(*3 marks*)

d) Assuming that the frame-difference signal was encoded using 3 bits/pixel, calculate the required bitrate of both the CIF and QCIF frame formats of Question 4 a) at 30 frames/s.

(*3 marks*)

e) Sketch the schematic of a more advanced video codec using motion compensation, rather than simple frame-differencing.

(*3 marks*)

f) Highlight the benefits and disadvantages of simple frame-differencing versus Motion Compensation (MC) using Motion Vectors (MV) in terms of their video quality, bit-rate, implementational complexity, etc.

(*4 marks*)

Question 4 continues on the next page

4. Video Compression continued...

g) Calculate the number of 8x8-pixel video tiles in a QCIF frame and assuming that only 10 % of the tiles is deemed to be motion-active, calculate the minimum number of bits required for signalling their position within the QCIF frame to the decoder. In this context, explain the rationale of 'cost-gain' motivated motion compensation.

(*4 marks*)

h) Assuming that each of these motion-active tiles has a MC search-scope of ± 4 pixels in both the horizontal and vertical directions, calculate the number of bits/video frame required for signalling all the MVs to the decoder. Recalculate the number of bits for a reduced-scope MC scheme searching within a range of ± 2 pixels and comment on the inter-play of the video quality, bit-rate, reduced-scope versus sub-sampled spatial motion-search and implementational complexity.

(*4 marks*)

i) Design at least two different Motion Compensated Error Residual (MCER) encoding schemes, assuming that only 10 % of the 8x8-pixel QCIF tiles have a significant MCER value, while the remaining 90 % of block are deemed to have a near-zero MCER and hence are set to zero. Calculate the number of bits/QCIF frame required for MCER-encoding. Contrast the two MCER encoding schemes' video quality, bit-rate as well as implementational complexity.

(*6 marks*)

Section B continues on the next page

5. Channel Coding

a) Let us consider the systematic BCH(7,4,3) code, which has a generator polynomial of $g(x) = x^3 + x + 1$. Draw the corresponding BCH encoder's schematic and explain its operation.

(*10 marks*)

b) Given the original data bits D=1011, where the right-most bit is entered into the encoder first, calculate the entire encoded sequence, explicitly showing the encoder's action and the shift-register's contents for each new clock-cycle.

(*6 marks*)

c) Draw the encoder's state transition diagram, indicating the resultant encoded bits for all transitions.

(*4 marks*)

d) Draw the encoder's trellis diagram, indicating the bits associated with all trellis transitions.

(*4 marks*)

e) Assuming that the 7-bit string of 1000 000 was received at the output of the channel, draw the associated decoder trellis, indicating all the branch-metric and path-metric values and hence deduce, what the most likely 7-bit transmitted codeword and 4-bit information word are.

(*5 marks*)

f) Assuming now that an additional error was caused by the channel and hence the 7-bit string of 1010 000 was received, redraw the trellis and deduce the most likely 4-bit information word.

(*4 marks*)

END OF PAPER