Revision of Lecture 1

- Major blocks of digital communication system
- MODEM functions
- Channel has finite bandwidth and introduces noise: two main factors to consider in design
- Transmitted signal must have finite bandwidth – pulse shaping – enable correct recovering of transmitted data symbols

MODEM components

pulse shaping Tx/Rx filter pair

modulator/demodulator

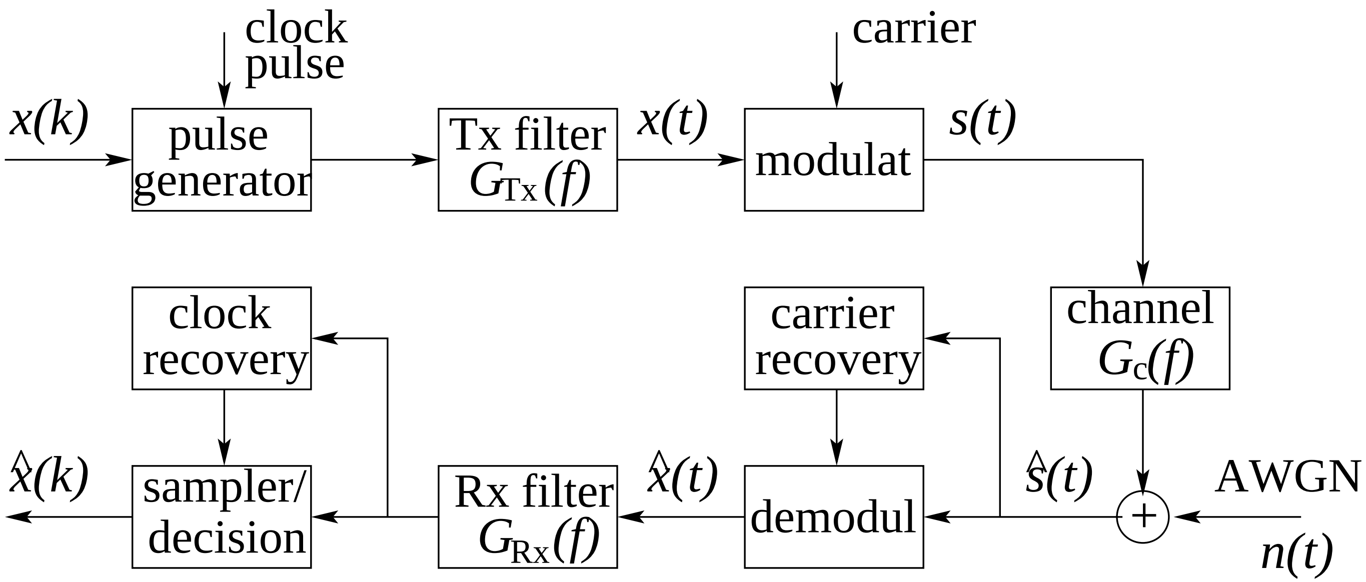
bits $\overset{map}{\leftrightarrow}$ symbols

equalisation (distorting channel)

Since pulse shaping is so fundamental to digital communication, this lecture we will again go through **pulse shaping and Tx/Rx filter pair**, but in more depth with both theoretical and practical considerations

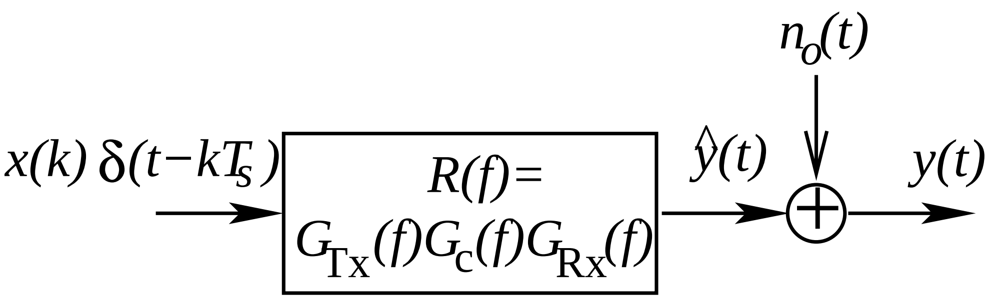
Baseband System

- Redraw I or Q branch in details:



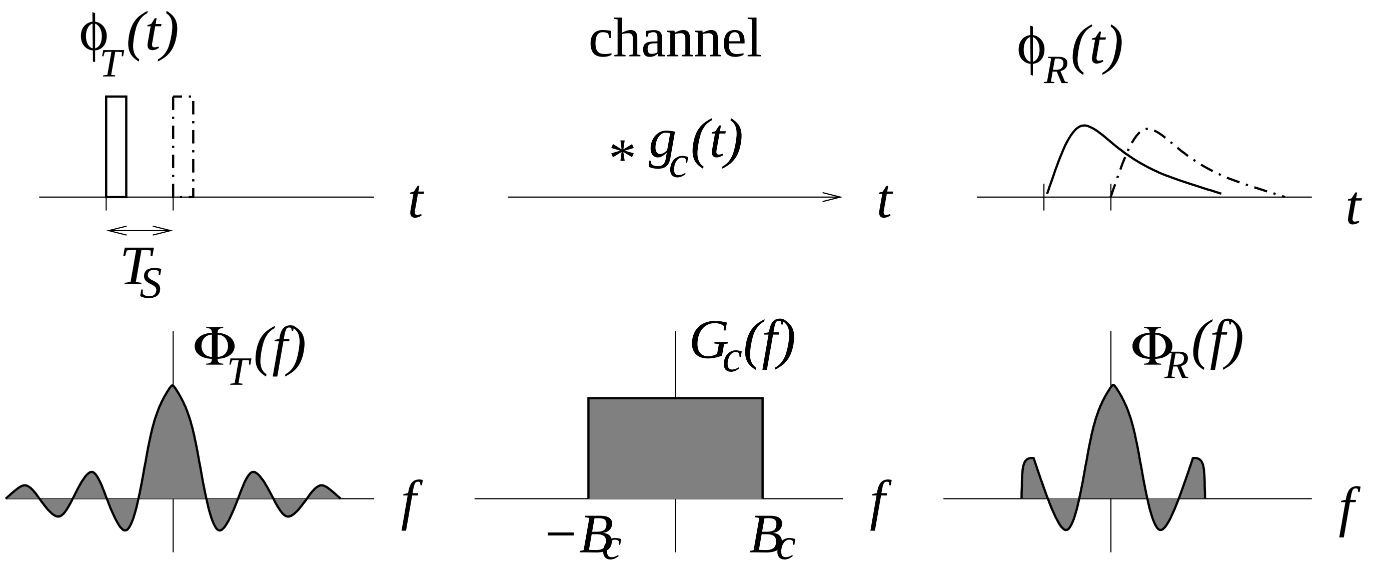
- Assuming a perfect demodulation, we can consider the **baseband** system:

$G_c(f)$ is equivalent baseband channel



Pulse Shaping Revisit — I

- Why we **cannot** transmit $x(k)$ as narrow-width (or rectangular) pulses:

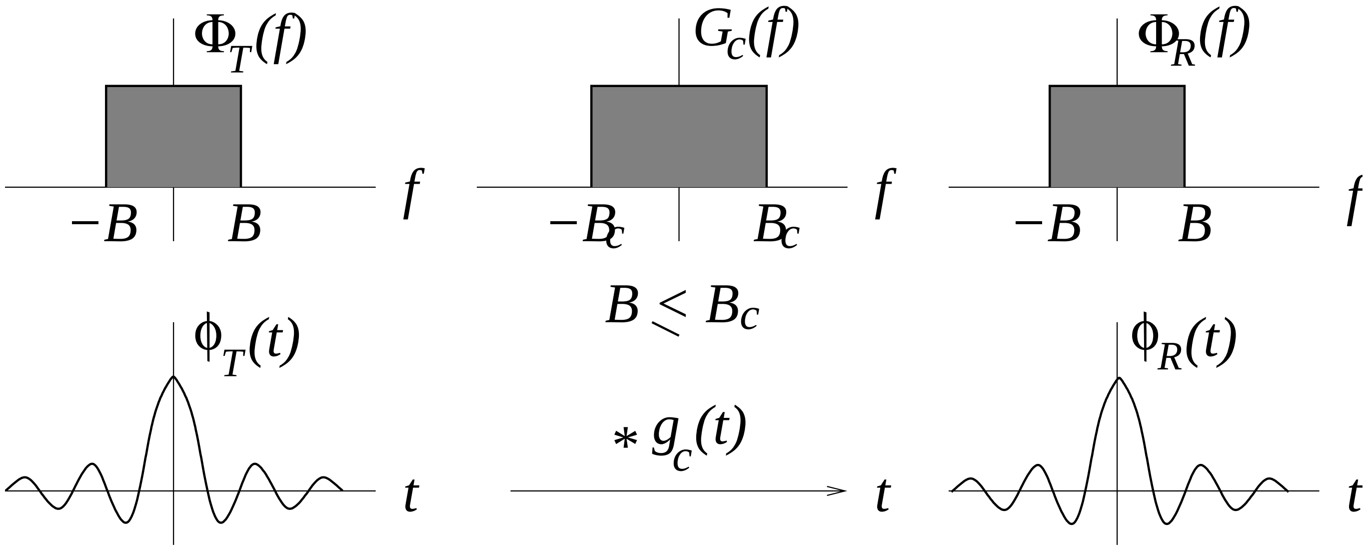


- Remember that the channel has finite bandwidth, and the bandwidth of a narrow pulse is not finite

- The pulses will spread out as their high-frequency components are suppressed, causing interference with neighbouring pulses (symbols)

Pulse Shaping Revisit — II

- Transmitted pulses should have a bandwidth $\leq$ channel bandwidth:



- Then a pulse (symbol) will be passed through channel without distortion

- But **finite bandwidth means infinite time waveform**, and this would cause interference in time domain with neighbouring pulses, unless:

  The pulse waveform has **regular zero-crossing at symbol-rate spacings**

InterSymbol Interference (ISI) — I

- Let a symbol $x(k)$ be transmitted as $x(k)\phi_T(t - kT_s)$ with T_s the symbol period
- Assume that the Tx pulse $\phi_T(t)$ peaks at $t = 0$ with $\phi_T(0) = 1$
- The transmitter output waveform is:

$$x(t) = \sum_{k=-\infty}^{\infty} x(k)\phi_T(t - kT_s)$$

- Assume that the Tx pulse $\phi_T(t)$ arrives at the receiver as $\phi_R(t - t_d)$, where the Rx pulse $\phi_R(t)$ peaks at $t = 0$ with $\phi_R(0) = 1$
- The receiver output waveform is:

$$y(t) = \sum_{k=-\infty}^{\infty} x(k)\phi_R(t - kT_s - t_d) + n_o(t)$$

InterSymbol Interference (ISI) — II

- The sampled receiver output at $t_k = kT_s + t_d$ is:

$$y(t_k) = x(k) + \sum_{m \neq k} x(m)\phi_R((k-m)T_s) + n_o(t_k)$$

- First term: correct symbol, second term: ISI, third term: due to channel noise

- In oder to eliminate ISI, the received pulse should satisfy:

$$\phi_R(kT_s) = \begin{cases} 1, & \text{for } k = 0 \\ 0, & \text{for } k \neq 0 \end{cases}$$

- That is, $\phi_R(t)$ has zero-crossing at symbol-rate spacings, leading to:

$$y(t_k) = x(k) + n_o(t_k)$$

- **Pulse shaping**: achieve a desired finite transmission bandwidth and eliminate ISI

Nyquist Criterion for Zero ISI

- Recall symbol rate f_s and symbol period T_s; if $\Phi_R(f) = \mathcal{F}[\phi_R(t)]$ satisfies:

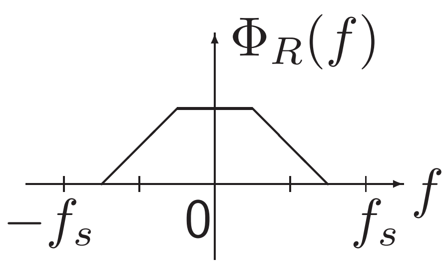
$$\sum_{k=-\infty}^{\infty} \Phi_R(f - kf_s) = \text{constant} \quad \text{for} \quad |f| \le f_s/2$$

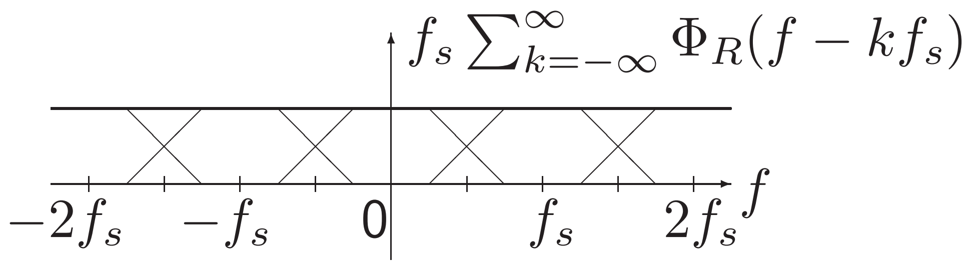
then

$$\phi_R(kT_s) = \begin{cases} 1, & \text{for } k = 0 \\ 0, & \text{for } k \neq 0 \end{cases}$$

- Filter (system) generates such a pulse is a **Nyquist** system

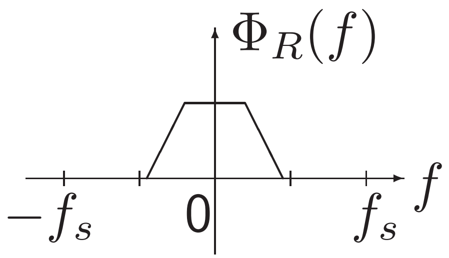
- Illustration of condition for zero ISI, seeing from frequency domain:

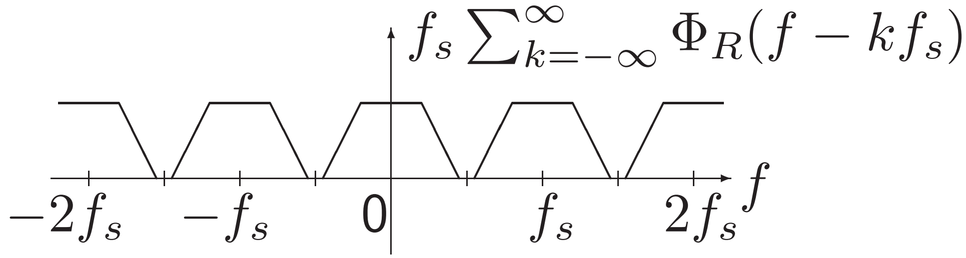




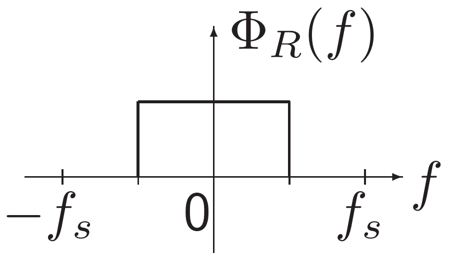
Minimum Transmission Bandwidth

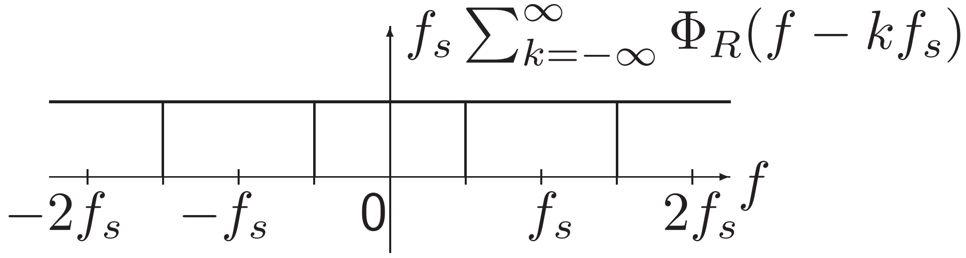
- ISI cannot be removed if the bandwidth of $\Phi_R(f)$ is less than $f_s/2$:





- The **minimum required bandwidth** of $\Phi_R(f)$ for zero ISI is $f_s/2$:



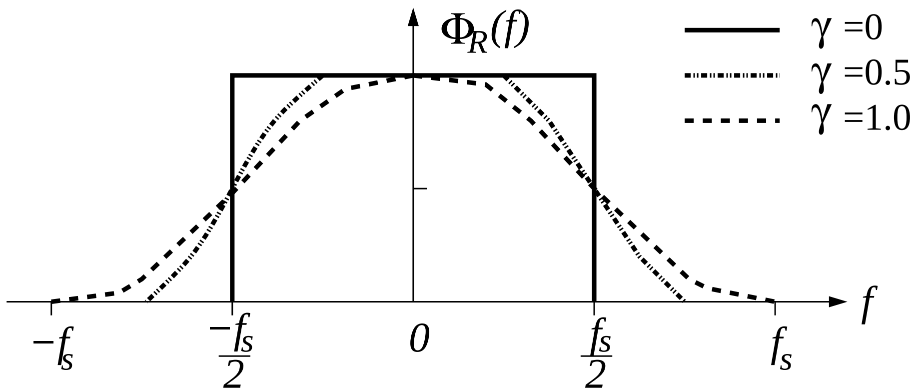


- This is in fact the sinc pulse: $\mathrm{sinc}(f_s t) = \frac{\sin(\pi f_s t)}{\pi f_s t}$, the only Nyquist system with bandwidth $f_s/2$

Raised Cosine Nyquist System

- The required baseband bandwidth: $f_s/2 \le B \le f_s$, and the spectrum:

$$\Phi_R(f) = \begin{cases} 1 & |f| \le \frac{f_s}{2} - \beta \\ \cos^2\left(\frac{\pi}{4\beta}|f| - \frac{f_s}{2} + \beta\right) & \frac{f_s}{2} - \beta < |f| \le \frac{f_s}{2} + \beta \\ 0 & |f| > \frac{f_s}{2} + \beta \end{cases}$$



- β: the extra bandwidth over the minimum $f_s/2$, and roll-off factor γ:

$$\gamma = \frac{\beta}{f_s/2} = \frac{B - f_s/2}{f_s/2} \quad \text{or} \quad B = \frac{f_s}{2}(1+\gamma)$$

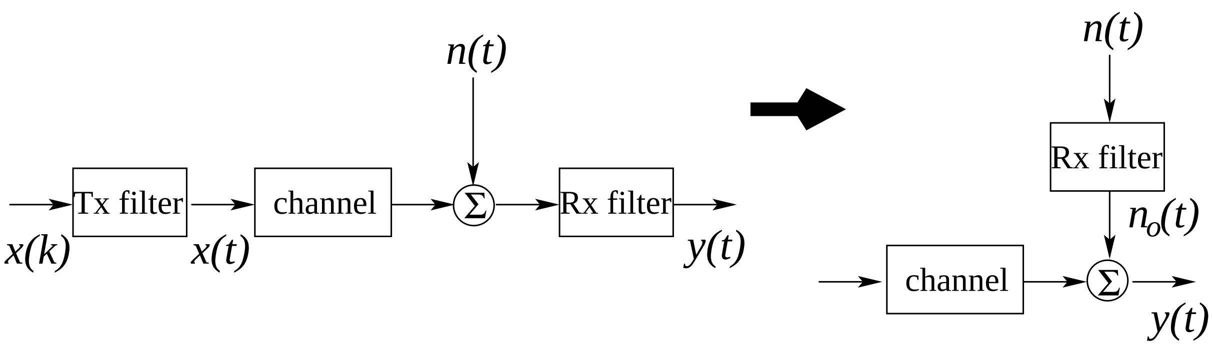
Optimal Transmit and Receive Filters

- **Task 1.** Combined Tx/Rx filters provide desired spectrum shape; note:

$$R(f) = G_{Tx}(f)G_c(f)G_{Rx}(f) = \Phi_R(f)$$

- Assume that the (baseband) channel bandwidth $B_c \geq B$, then $G_c(f) = 1$ and $R(f) = G_{Tx}(f)G_{Rx}(f)$

- **Task 2.** Maximise the receiver output signal to noise ratio (SNR); note:

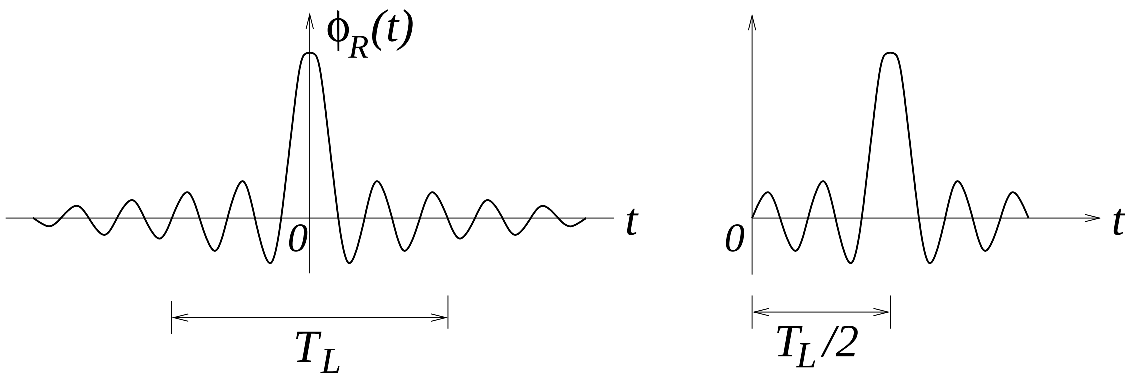


- This leads to: $G_{Tx}(f) = G_{Rx}(f)$, that is, Tx and Rx filters are identical (matched to each other) as a square-root of Nyquist system $\Phi_R(f)$

Practical Implementation

- A true Nyquist system (e.g. raised cosine) has absolute finite bandwidth but the corresponding time waveform is non-causal and lasts infinite long; the pulse shaping filters to realize such a true Nyquist system cannot be constructed physically

- A practical way: truncate the pulse to a finite but sufficient length and delay the truncated pulse as shown:

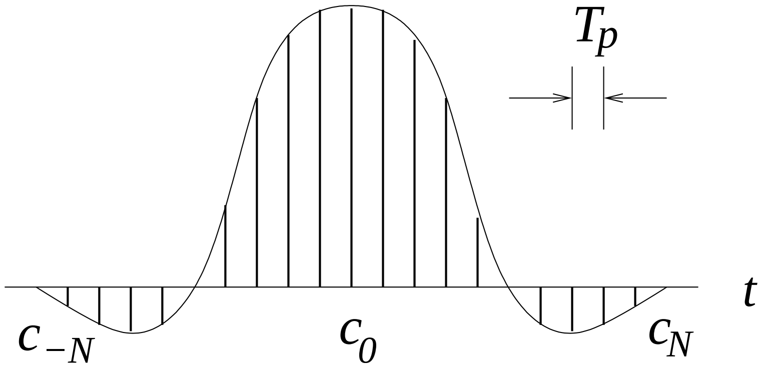


(a) truncate the pulse to length T_L (b) delay the truncated pulse

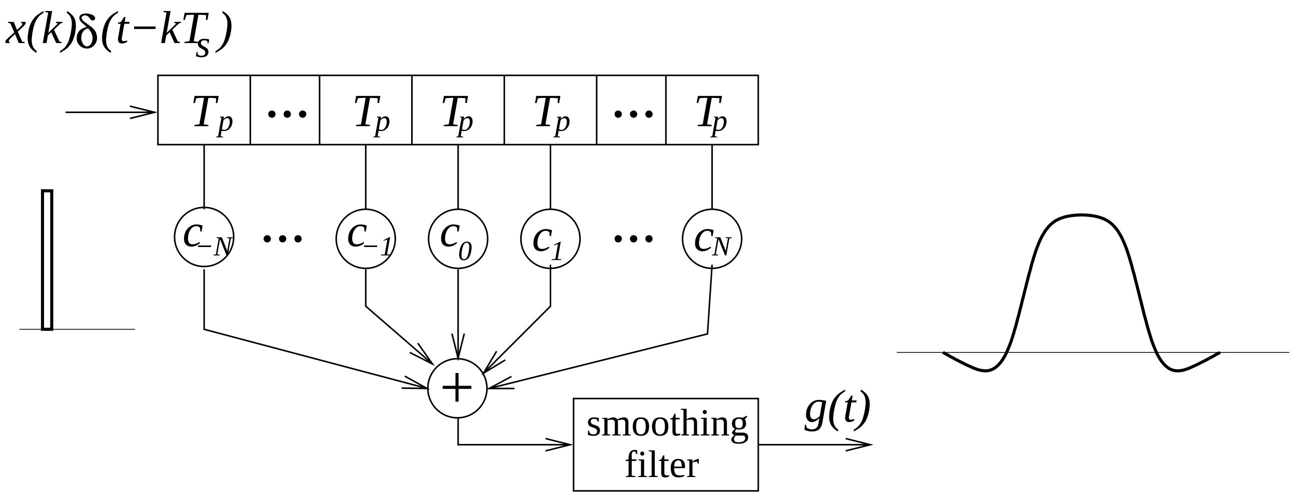
- The bandwidth of the truncated pulse is no longer finite (e.g. the truncated raised cosine pulse in slide **10**); but if the pulse is selected to decaying rapidly, the resulting ISI can be made sufficiently small

Tx / Rx filters Realization

- Sampled values are obtained from the waveform of $g(t)$:



- FIR or transversal filter is used to realize the required Tx / Rx filters:



Summary

- Design of transmit and receiver filters (pulse shaping): to achieve zero ISI and to maximise the received signal to noise ratio

- The combined Tx / Rx filters should form a Nyquist system (regular zero-crossings at symbol-rate spacings except at $t = 0$), and Rx filter should be identical (matched) to Tx filter as a square-root Nyquist system

- Nyquist criterion for zero ISI, to transmit at symbol rate f_s requires at least a baseband bandwidth of $f_s/2$;

- The raised cosine pulse, roll-off factor, and the required baseband transmission bandwidth:

$$B = \frac{f_s}{2}(1 + \gamma)$$

  (passband bandwidth is doubled)

- Practical considerations for implementing pulse shaping filters