Revision of Lecture 2

- Pulse shaping Tx/Rx filter pair
  - Design of Tx/Rx filters (pulse shaping): to achieve zero ISI and to maximise received signal to noise ratio
  - Combined Tx/Rx filters: Nyquist system (regular zero-crossings at symbol-rate spacings except at $t = 0$), and Rx filter matched (identical) to Tx filter
  - Nyquist criterion for zero ISI; to transmit at symbol rate f_s requires at least a baseband bandwidth of $f_s/2$
  - Raised cosine pulse, roll-off factor, and required baseband transmission bandwidth $B = \frac{f_s}{2}(1 + \gamma)$

MODEM components

pulse shaping Tx/Rx filter pair

modulator/demodulator

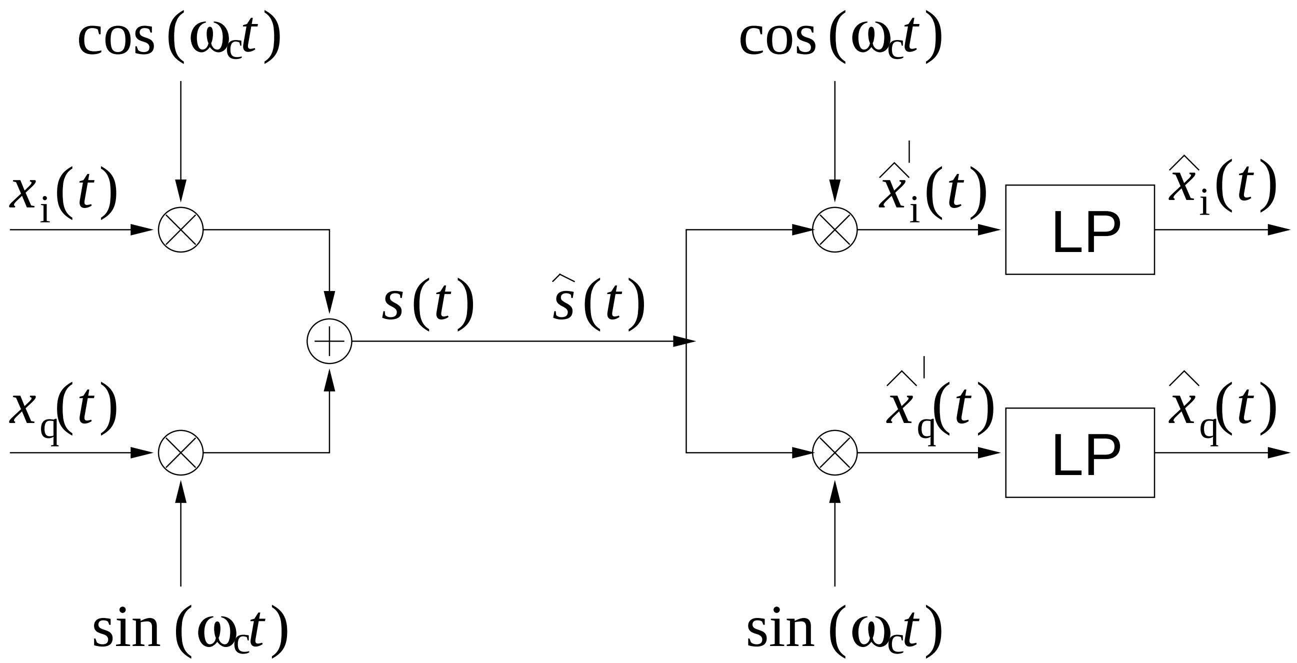
bits $\overset{map}{\leftrightarrow}$ symbols

equalisation (distorting channel)

This lecture: **modulator/demodulator**

QAM Modulator / Demodulator

- Recall modulator and demodulator of the QAM scheme (slides **4** and **5**):



- All signals here are analogue.

Why **carrier modulation** – "low-frequency" or baseband signals $x_{\rm i}(t)$ and $x_{\rm q}(t)$ cannot travel far in most channels (transmission media)

Why **inphase/quadrature** – inphase and quadrature carriers are orthogonal and go through same channel, meaning they can be separated; inphase or quadrature rate is half of original transmission rate, meaning half of bandwidth

QAM — Modulation

- **Modulation** of "in-phase" and "quadrature" components to carrier frequency ω_c:

$$
\begin{aligned}
x_1(t) &= x_{\rm i}(t)\cdot\cos(\omega_c t)\\
x_2(t) &= x_{\rm q}(t)\cdot\sin(\omega_c t)
\end{aligned}
$$

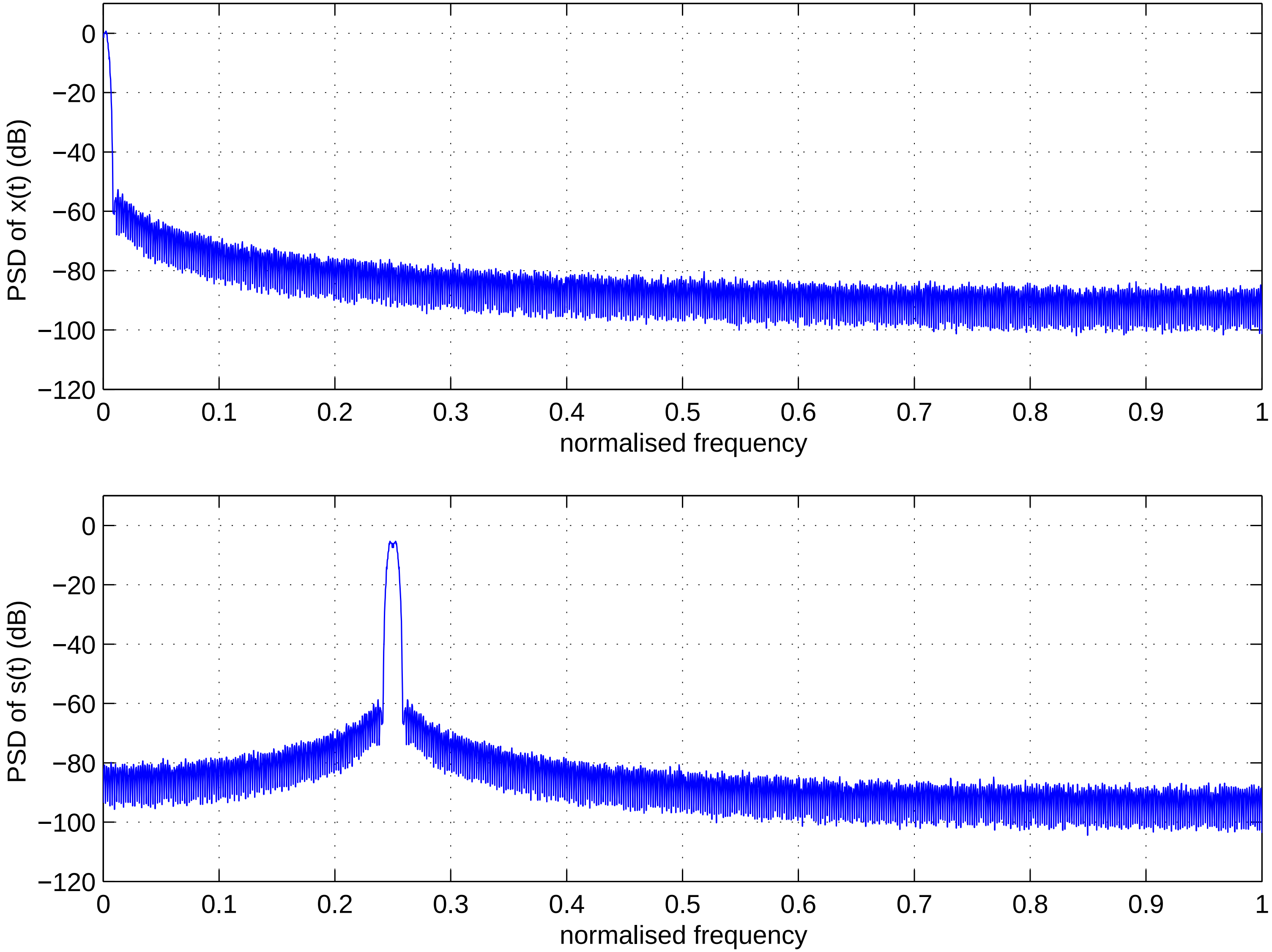
- Transmitted signal is $s(t) = x_1(t) + x_2(t)$

- To explain demodulation, we assume perfect transmission $\hat{s}(t) = s(t)$

  In next slide we have a baseband transmission bandwidth 10 kHz and carrier $f_c = 250$ kHz, normalised by 1 MHz in plots

You may like to pause and think this: digital communications – make analogue signal digital → back to analogue for transmission → digital again → restore to original analogue signal. Why? or what are the advantages of digital communications as opposed to the original analogue communications?

I or Q Branch Modulation Example



QAM — Demodulation

- **Demodulation** for the "in-phase" component:

$$\begin{aligned}\hat{x}'_{\mathrm{i}}(t) &= s(t)\cdot\cos(\omega_c t) = (x_{\mathrm{i}}(t)\cdot\cos(\omega_c t) + x_{\mathrm{q}}(t)\cdot\sin(\omega_c t))\cdot\cos(\omega_c t) \\ &= x_{\mathrm{i}}(t)\cdot\cos^2(\omega_c t) + x_{\mathrm{q}}(t)\cdot\sin(\omega_c t)\cos(\omega_c t) \\ &= x_{\mathrm{i}}(t)\cdot\frac{1}{2}\cdot\big(1+\cos(2\omega_c t)\big) + x_{\mathrm{q}}(t)\cdot\frac{1}{2}\cdot\sin(2\omega_c t)\end{aligned}$$

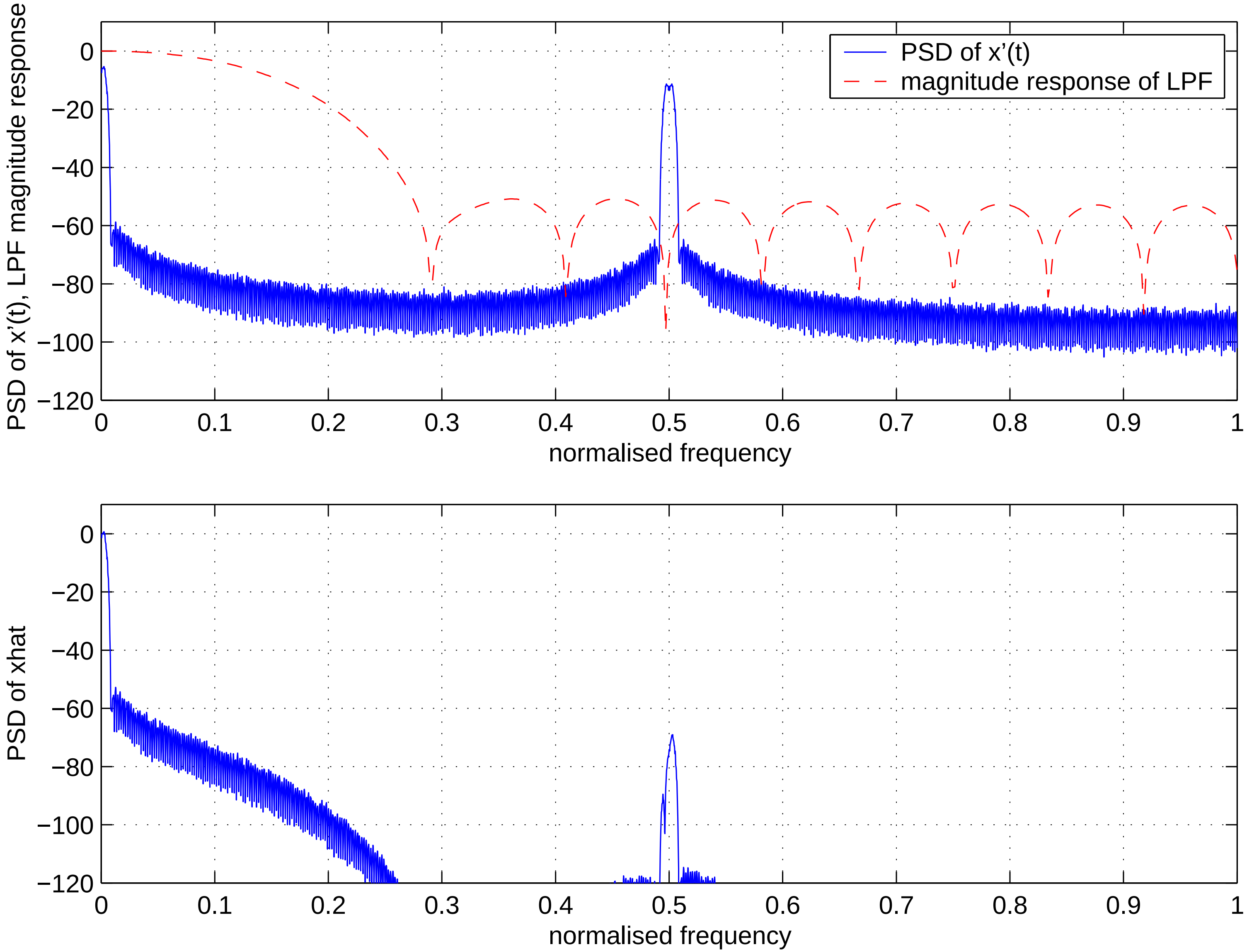
- If the lowpass filter LP in slide **28** is selected appropriately (cut-off frequency $\leq \omega_c$), the components modulated at frequency $2\omega_c$ can be filtered out; hence:

$$\hat{x}_{\mathrm{i}}(t) = \mathsf{LP}\big(\hat{x}'_{\mathrm{i}}(t)\big) = \frac{1}{2}x_{\mathrm{i}}(t)$$

- A similar calculation can be performed for the demodulation of $\hat{x}_{\mathrm{q}}(t)$:

$$\hat{x}'_{\mathrm{q}}(t) = \cdots = x_{\mathrm{i}}(t)\cdot\frac{1}{2}\cdot\sin(2\omega_c t) + x_{\mathrm{q}}(t)\cdot\frac{1}{2}\cdot\big(1-\cos(2\omega_c t)\big)$$

I or Q Branch Demodulation Example



Modulation — Complex Notation I

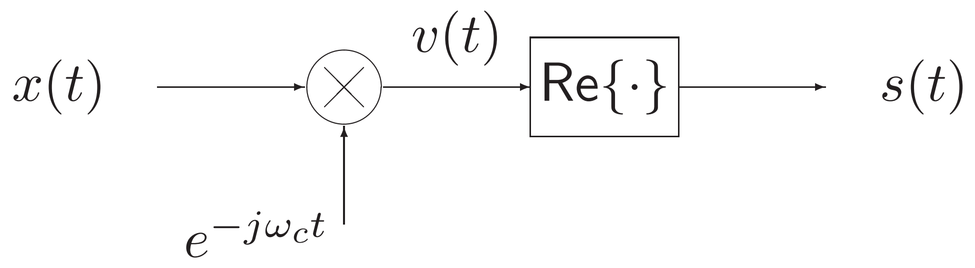
- The modulation/demodulation scheme can also be expressed in complex notation: "in-phase" and "quadrature" components are real and imaginary part of the signal:

$$x(t) = x_{\rm i}(t) + j \cdot x_{\rm q}(t)$$

- The transmitted signal is obtained by taking the real part only of a complex carrier ($e^{-j\omega_c t}$) modulated signal:

$$s(t) = \text{Re}\{x(t) \cdot e^{-j\omega_c t}\}$$

- Flow graph:



Modulation — Complex Notation II

- Modulation:

$$
\begin{aligned}
v(t) &= e^{-j\omega_c t} \cdot x(t) \\
&= \big(\cos(\omega_c t) - j\sin(\omega_c t)\big) \cdot \big(x_\mathrm{i}(t) + j \cdot x_\mathrm{q}(t)\big) \\
&= \underbrace{x_\mathrm{i}(t) \cdot \cos(\omega_c t) + x_\mathrm{q}(t) \cdot \sin(\omega_c t)}_{\text{real}} - \underbrace{j x_\mathrm{i}(t) \cdot \sin(\omega_c t) + j x_\mathrm{q}(t) \cdot \cos(\omega_c t)}_{\text{imaginary}}
\end{aligned}
$$

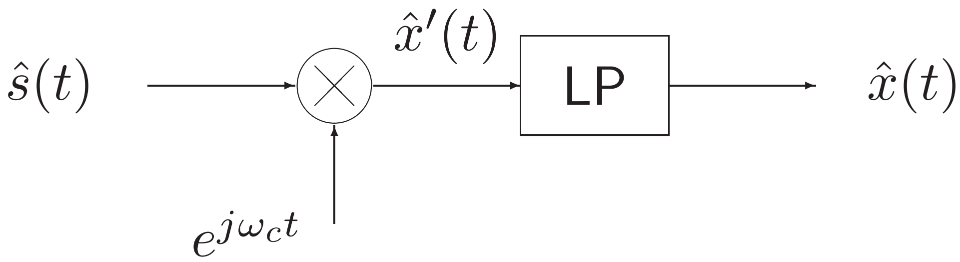
- Transmitted signal:

$$
s(t) = \mathrm{Re}\{v(t)\} = x_\mathrm{i}(t) \cdot \cos(\omega_c t) + x_\mathrm{q}(t) \cdot \sin(\omega_c t)
$$

- This is identical to the signal $s(t)$ on slide **29**

Demodulation — Complex Notation

- Flow graph for the complex demodulation scheme:



- The demodulated signal: $\hat{x}'(t) = e^{j\omega_c t} \cdot s(t)$

$$
\begin{aligned}
&= (\cos(\omega_c t) + j\sin(\omega_c t)) \cdot (x_\mathrm{i}(t) \cdot \cos(\omega_c t) + x_\mathrm{q}(t) \cdot \sin(\omega_c t)) \\
&= x_\mathrm{i}(t) \cdot \frac{1}{2}\big(1 + \cos(2\omega_c t) + j\sin(2\omega_c t)\big) \quad + \\
&\quad\; j x_\mathrm{q}(t) \cdot \frac{1}{2}\big(1 - \cos(2\omega_c t) - j\sin(2\omega_c t)\big)
\end{aligned}
$$

- Lowpass filter (LP) will again remove components modulated at $2\omega_c$

$$
\mathrm{LP}[\hat{x}'(t)] = \frac{1}{2}x_\mathrm{i}(t) + j\frac{1}{2}x_\mathrm{q}(t)
$$

Carrier Recovery — Phase Offset

- Previously, we assume

$$\hat{s}(t) = s(t) = x_{\mathrm{i}}(t) \cdot \cos(\omega_c t) + x_{\mathrm{q}}(t) \cdot \sin(\omega_c t)$$

  so that we can use $e^{j\omega_c t}$ $(\cos(\omega_c t)$ and $\sin(\omega_c t))$ to remove carrier in demodulator

- Most likely, the transmitted signal having traveled to the receiver will accumulate a phase offset φ:

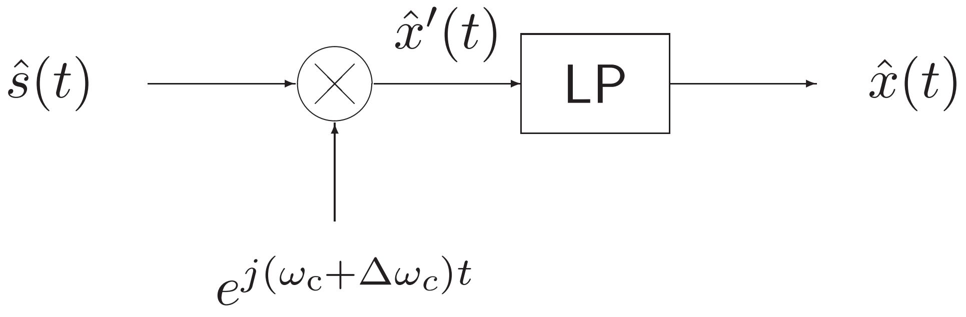
$$\hat{s}(t) = x_{\mathrm{i}}(t) \cdot \cos(\omega_c t + \varphi) + x_{\mathrm{q}}(t) \cdot \sin(\omega_c t + \varphi)$$

- Thus, the receiver has to "recover" the carrier $e^{j(\omega_c t + \varphi)}$ (in fact the phase φ) in oder to demodulate the signal correctly

- Usually, this is done by means of some phase lock loop based carrier recovery

Carrier Recovery — Frequency Offset

- Tx and Rx frequency generators are unlikely to match exactly; Consider demodulation with a Rx local "carrier" having a frequency offset $\Delta\omega_c$:



- Even assuming $\hat{s}(t) = s(t)$, the demodulated signal prior to sampling is $\hat{x}(t) = x(t) \cdot e^{j\Delta\omega_c t}$, not $\hat{x}(t) = x(t)$

- The effect of carrier frequency mismatch is $\Delta\omega_c t$ and, like the phase difference φ, it has to be compensated at the receiver

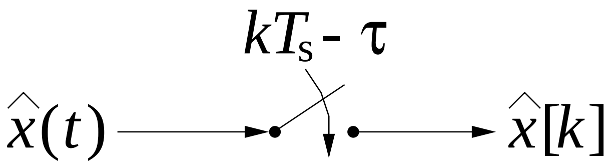
- $\Delta\omega_c t + \varphi$ is called carrier offset, and has to be "recovered" in order to demodulate the signal correctly

Synchronisation

- The process of selecting the correct sampling instances is called synchronisation (timing or clock recovery)

- Tx and Rx clocks are likely to have mismatch, clock recovery tries to synchronise the receiver clock with the symbol-rate transmitter clock to obtain samples at appropriate instances

- This is equivalent to replacing the impulse train $\sum \delta(t - kT_{\rm s})$ in slide **5** by $\sum \delta(t - kT_{\rm s} - \tau)$ with $0 \leq \tau \leq T_{\rm s}$:



- The demodulated signal can be *oversampled*, and from the distribution (histogram) of the sample sets for different τ, the one with the smallest deviation from the set of transmitted signal levels (symbols) is chosen

Summary

- Basic operations of modulation and demodulation
- Complex notations for modulation and demodulation
- Carrier recovery and timing recovery
- Appreciate advantages of digital communications as opposed to analogue communications
- Understand why carrier communications