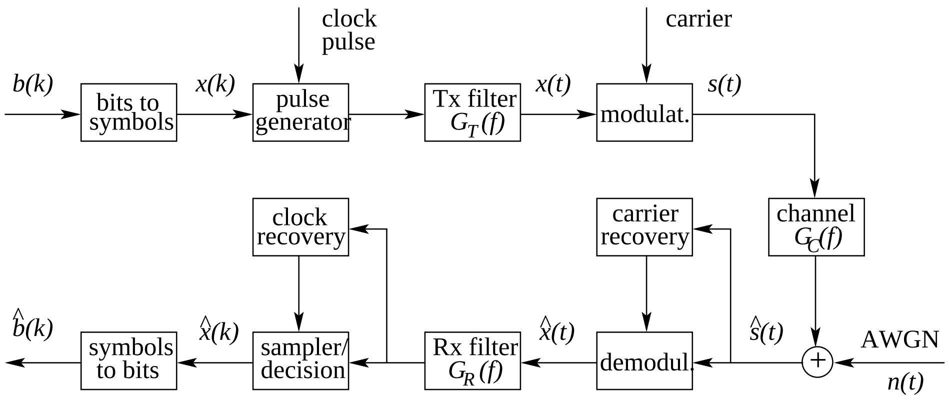
Digital Modulation Overview

- Schematic of **MODEM** (modulation and demodulation) with **basic components**:



- This is for the **ideal** AWGN channel. If the channel is **dispersive**, then **equaliser** is required at receiver

Coherent Receiver

(a) **Carrier recovery** for demodulation

- Receiver signal $\hat{S}(t) = A\cos\left(\omega_c t + \varphi\right) + N(t)$
- Local carrier $\cos\left(\omega_c t + \bar{\varphi}\right)$
- Carrier recovery (e.g. phase lock loop) circuit

$$\Delta\varphi = \varphi - \bar{\varphi} \rightarrow 0 \quad \text{i.e.} \quad \bar{\varphi} \rightarrow \varphi$$

- Demodulation leads to recovered baseband signal

$$Y(t) = X(t+\tau) + N(t)$$

where $X(t)$ is transmitted baseband signal

(b) **Timing recovery** for sampling

- Align receiver clock with transmitter clock, so that sampling $\Rightarrow$ no ISI

$$Y_k = X_k + N_k$$

where X_k are transmitted symbols, and N_K noise samples

Non-coherent Receiver

(a) **No carrier recovery** for demodulation

- Receiver signal $\hat{S}(t) = A\cos\left(\omega_c t + \varphi\right) + N(t)$
- Local carrier $\cos\left(\omega_c t + \bar{\varphi}\right)$
- No carrier recovery,

$$\phi = \Delta\varphi = \varphi - \bar{\varphi} \neq 0 \quad \text{i.e.} \quad \bar{\varphi} \neq \varphi$$

- Demodulation leads to recovered baseband signal

$$Y(t) = X(t+\tau)e^{j\phi} + N(t)$$

(b) **Timing recovery** for sampling

- Align receiver clock with transmitter clock, sampling results in

$$Y_k = X_k e^{j\phi} + N_k$$

Could not recover transmitted symbols properly from Y_k !

Differential Detection

(a) **Differential encoding** for transmission

- Symbols $\{C_k\} \Rightarrow \{X_k\}$ for transmission

$$X_k = C_k \cdot X_{k-1}$$

- As $X_k \cdot X_{k-1}^* = C_k \cdot \left(X_{k-1} \cdot X_{k-1}^*\right)$,

$$C_k = \frac{X_k \cdot X_{k-1}^*}{|X_{k-1}|^2} \qquad (1)$$

(b) **Non-coherent** detection

- Receiver samples $Y_k = X_k \cdot |H| \cdot e^{j\phi} + N_k$

  $|H|$: magnitude of combined channel tap, $\phi \neq 0$: unknown phase
- Differential decoding

$$\hat{C}_k = \frac{Y_k \cdot Y_{k-1}^*}{|Y_{k-1}|^2} \qquad (2)$$

Differential Detection (derivation)

$$\begin{aligned} Y_k \cdot Y_{k-1}^* &= \left(X_k \cdot |H| \cdot e^{j\phi} + N_k\right) \cdot \left(X_{k-1}^* \cdot |H| \cdot e^{-j\phi} + N_{k-1}^*\right) \\ &= X_k \cdot X_{k-1}^* \cdot |H|^2 \cdot e^{j(\phi-\phi)} \\ &\quad + N_k \cdot N_{k-1}^* + X_k \cdot |H| \cdot e^{j\phi} \cdot N_{k-1}^* + N_k \cdot X_{k-1}^* \cdot |H| \cdot e^{-j\phi} \end{aligned}$$

$$\begin{aligned} |Y_{k-1}|^2 &= X_{k-1} \cdot X_{k-1}^* \cdot |H|^2 + N_{k-1} \cdot N_{k-1}^* \\ &\quad + X_{k-1} \cdot |H| \cdot e^{j\phi} \cdot N_{k-1}^* + N_{k-1} \cdot X_{k-1}^* \cdot |H| \cdot e^{-j\phi} \end{aligned}$$

When noise N_k is very small

$$Y_k \cdot Y_{k-1}^* \approx X_k \cdot X_{k-1}^* \cdot |H|^2 \quad \text{and} \quad |Y_{k-1}|^2 \approx |X_{k-1}|^2 \cdot |H|^2$$

- Thus,
$$\hat{C}_k = \frac{Y_k \cdot Y_{k-1}^*}{|Y_{k-1}|^2} \approx C_k + \bar{N}_k$$
where power of enhanced noise $\bar{N}_k$ is **larger** than that of N_k
- Note that influence of channel phase ϕ has been removed

Comparison

- Coherent detection
  - Require expensive and complex carrier recovery circuit
  - Better bit error rate of detection

$$\hat{X}_k = X_k + N_k$$

- Non-coherent detection
  - Do not require expensive and complex carrier recovery circuit
  - Poorer bit error rate of detection

$$\hat{C}_k = C_k + \bar{N}_k$$

- Differential systems have important advantages and are widely used in practice

Differential PSK

(a) For differential **phase shift keying**, $|X_{k-1}|^2 = \text{con}$ and $C_k = \frac{X_k \cdot X_{k-1}^*}{\text{con}}$
- $C_k \leftarrow$ phase of $X_k \cdot X_{k-1}^*$
- $\hat{C}_k \leftarrow$ phase of $Y_k \cdot Y_{k-1}^*$

(b) At receiver, **differential decoding** (2) becomes

$$\hat{C}_k = \frac{Y_k \cdot Y_{k-1}^*}{|X_{k-1}|^2} = \frac{Y_k \cdot Y_{k-1}^*}{\text{con}} \tag{3}$$

- For convenience, assuming $|H|^2 = 1$ (or $|H|^2$ is known), then

$$\frac{Y_k \cdot Y_{k-1}^*}{\text{con}} = \frac{X_k \cdot X_{k-1}^*}{\text{con}} + \frac{N_k \cdot N_{k-1}^*}{\text{con}} + \frac{X_k}{\text{con}} \cdot e^{j\phi} \cdot N_{k-1}^* + N_k \cdot e^{-j\phi} \cdot \frac{X_{k-1}^*}{\text{con}}$$

- Noting magnitudes of $\frac{X_k}{\text{con}}$ and $\frac{X_{k-1}^*}{\text{con}}$ are 1, $\frac{N_k \cdot N_{k-1}^*}{\text{con}}$ is much smaller than the last two terms, while $e^{j\phi} \cdot N_{k-1}^*$ and $N_k \cdot e^{-j\phi}$ have the same variance as N_k,

$$\hat{C}_k \approx C_k + 2N_k$$

- Compared with coherent detection, noise is **doubled** or 3 dB worse off

Error Probability: Introduction

- Recall **pulse shaping** purposes: i) achieve zero ISI, and ii) maximise receive signal to noise ratio (SNR)
- Why maximise receive SNR? $\longrightarrow$ best **detection accuracy**
- BPSK transmitter: bit 0: $a = +d$, bit 1: $a = -d$
- BPSK receiver: the received sample is

$$r = a + n, \ a \in \{\pm d\} \quad \text{and} \quad n \in N(0, \sigma^2)$$

Decision boundary is $r = 0$, and **decision rule** is

$$r > 0 \rightarrow \hat{a} = d \text{ or bit 0}, \ r \leq 0 \rightarrow \hat{a} = -d \text{ or bit 1}$$

- Average signal power or energy per bit is $E_b = d^2$, and receive SNR is

$$\text{SNR} = \frac{E_b}{\sigma^2} = \frac{d^2}{\sigma^2}$$

Error Probability: Derivation

- Using **Bayes** theorem, the error probability or BER is given by

$$\begin{aligned} P_e =& P(\hat{a} \neq a) = P(a = d \cap \hat{a} = -d) + P(a = -d \cap \hat{a} = d) \\ =& P(a = d)P(\hat{a} = -d | a = d) + P(a = -d)P(\hat{a} = d | a = -d) \end{aligned}$$

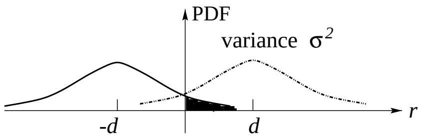
- As transmitted bit is equally likely to be 0 or 1, the two $a\ prior$ probabilities are

$$P(a = d) = P(a = -d) = \frac{1}{2}$$

- Given $a = -d$, the decision $\hat{a} = d$ means that $r = -d + n > 0$ or noise value $n > d$, and the **conditional probability** $P(\hat{a} = d | a = -d) =$

$$P(n > d) = \int_d^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx = \frac{1}{\sqrt{2\pi}} \int_{d/\sigma}^{\infty} \exp\left(-\frac{y^2}{2}\right) dy = Q\left(d/\sigma\right)$$

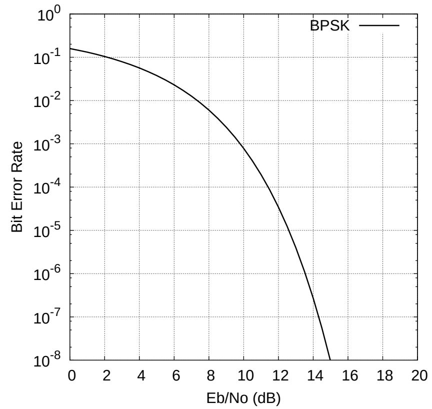
  - Interpretation of **conditional error probability** $P(\hat{a} = d | a = -d)$: Gaussian tail area over threshold $r = 0$



Error Probability: Results

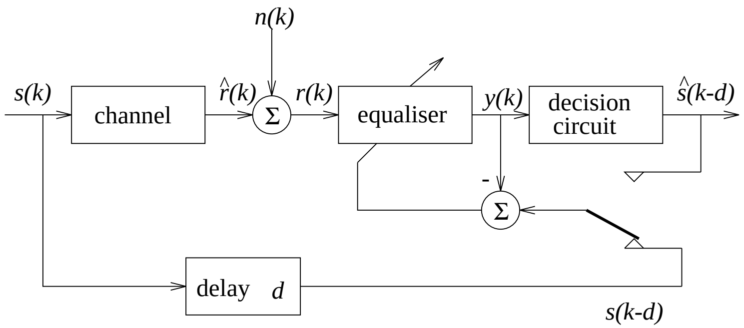
- Similarly, the other conditional error probability $P(\hat{a} = -d | a = d) = Q\left(d/\sigma\right)$
- Note signal power $E_s = \frac{1}{2}(d^2 + d^2) = d^2$ and noise power $\frac{N_0}{2} = \sigma^2$, BER is

$$P_e = \frac{1}{2} Q\left(d/\sigma\right) + \frac{1}{2} Q\left(d/\sigma\right) = Q\left(d/\sigma\right) = Q\left(\sqrt{\text{SNR}}\right) = Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$



Adaptive Equalisation

- For **dispersive** channel, adaptive **equalisation** is required



  - **Training mode**: During training, equaliser has access to the transmitted (training) symbols $s(k)$ and can use them as the desired response to adapt the equaliser's coefficients
  - **Decision-directed mode**: During data communication phase, equaliser's decisions $\hat{s}(k - d)$ are assumed to be correct and are used to substitute for $s(k - d)$ as the desired response to continuously track a time-varying channel