Revision of Lecture 1

- Major blocks of digital communication system
  - transmitter and receiver (transceiver), and channel
  - CODEC, MODEM, channel
- MODEM responsible for transmission at required rate rate over channel reliably
  - Required rate with required quality $\leftrightarrow$ bandwidth and power requirements
- Channel has finite bandwidth and introduces noise: two main factors to consider in design
- Pulse shaping
  1. ensures transmitted signal has finite bandwidth, and
  2. enables correct recovering of transmitted data symbols

MODEM components

pulse shaping Tx/Rx filter pair

modulator/demodulator

bits $\overset{map}{\leftrightarrow}$ symbols

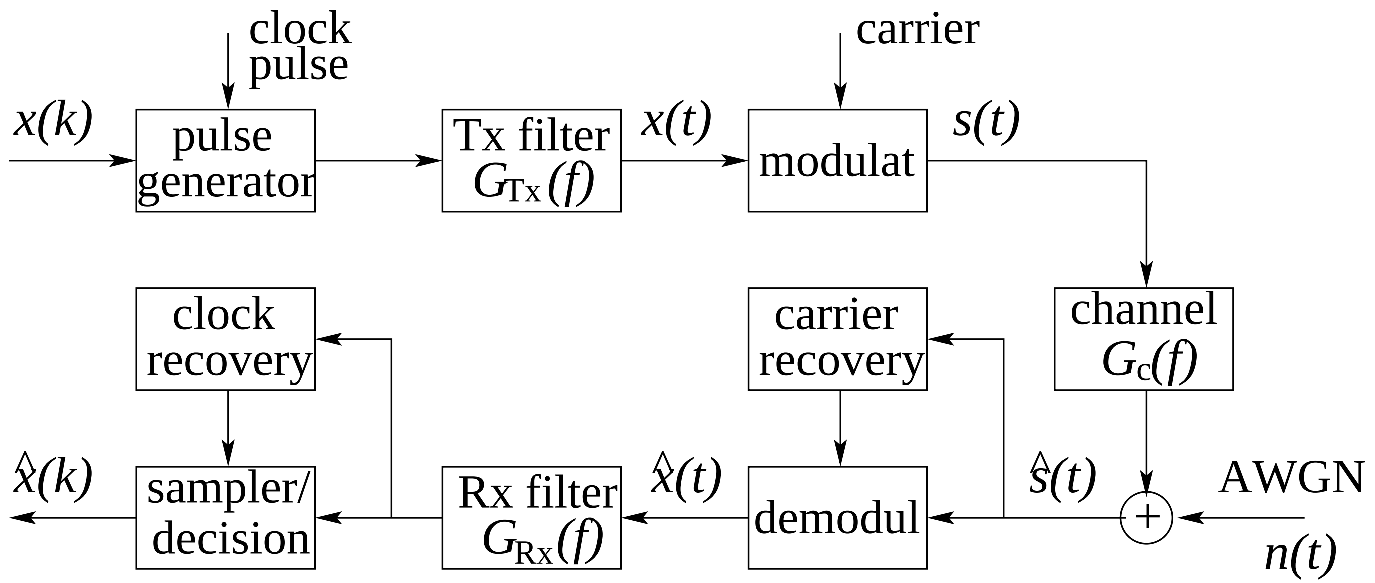
equalisation (distorting channel)

bit error rate and other issues

Since pulse shaping is so fundamental to digital communication, this lecture we will again go through **pulse shaping and Tx/Rx filter pair**, but in more depth with both theoretical and practical considerations

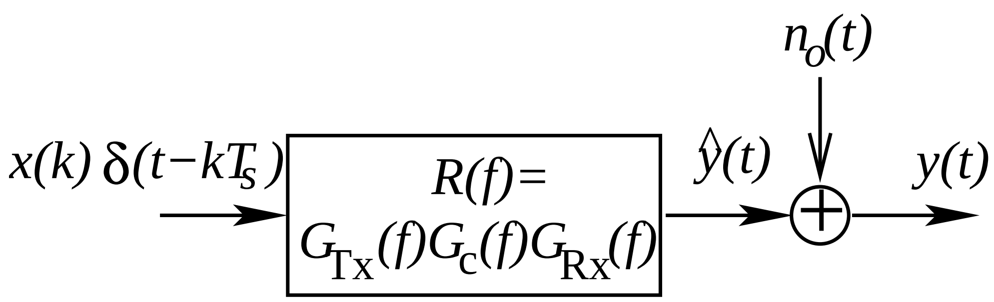
Baseband System

- Redraw I or Q branch in details:



- Assuming a perfect demodulation, we can consider the **baseband** system:

$G_c(f)$ is equivalent baseband channel

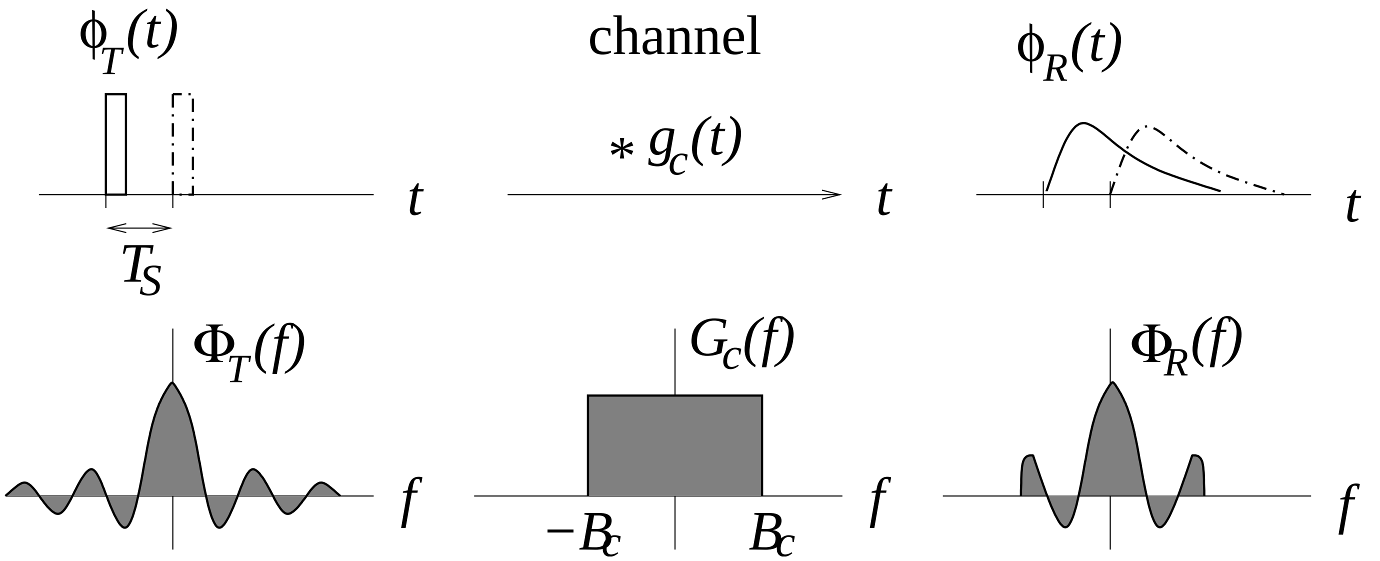


- Pulse shaping is about combined response $r(t) = g(-t) \star c(t) \star g(t)$ or $R(f)$

Pulse Shaping — Finite-Time Pulses

- Narrow-width or rectangular pulses with time support $\leq$ symbol period T_s
  - At $t = kT_s$, we transmit $x[k]$, and $\{x[k]\}$ in $\sum x[k]\Phi_T(t - kT_s)$ will not overlap, right? But $\{x[k]\}$ in $x(t) = \sum x[k]\Phi_R(t - kT_s)$ will overlap!

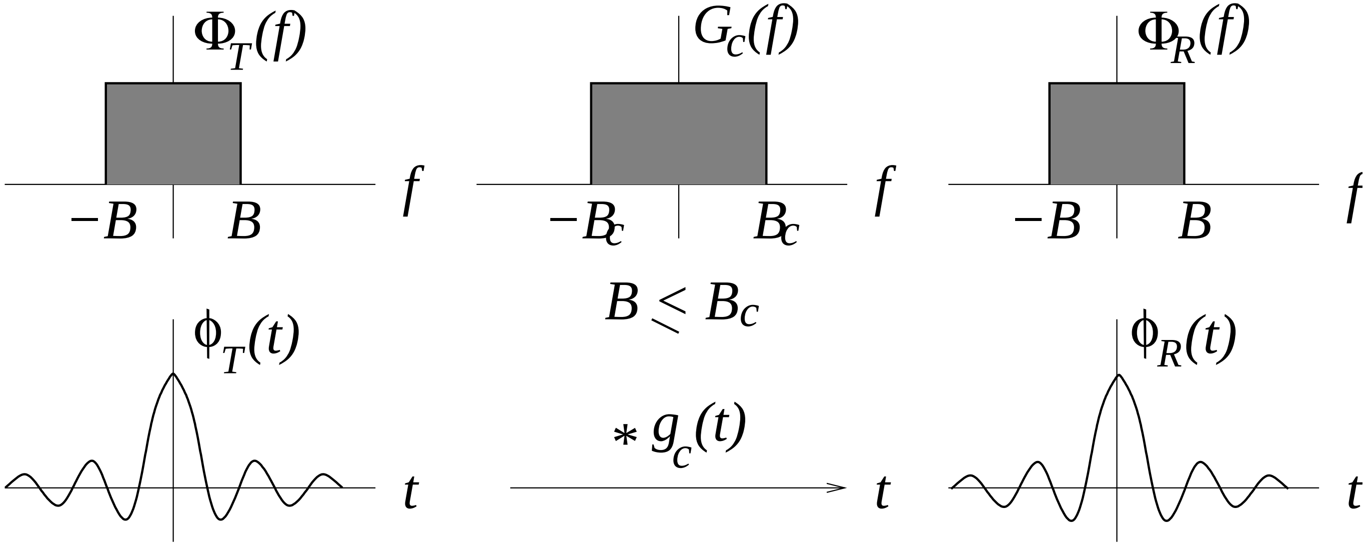
- Why we **cannot** transmit $\{x[k]\}$ as narrow-width (or rectangular) pulses:



  - Channel has finite bandwidth, and the bandwidth of a narrow pulse is not finite
  - The pulses will spread out as their high-frequency components are suppressed, causing interference with neighbouring pulses (symbols) in time

Pulse Shaping — Finite-Bandwidth Pulses

- I know! transmitted pulses should have a finite bandwidth $\leq$ channel bandwidth:

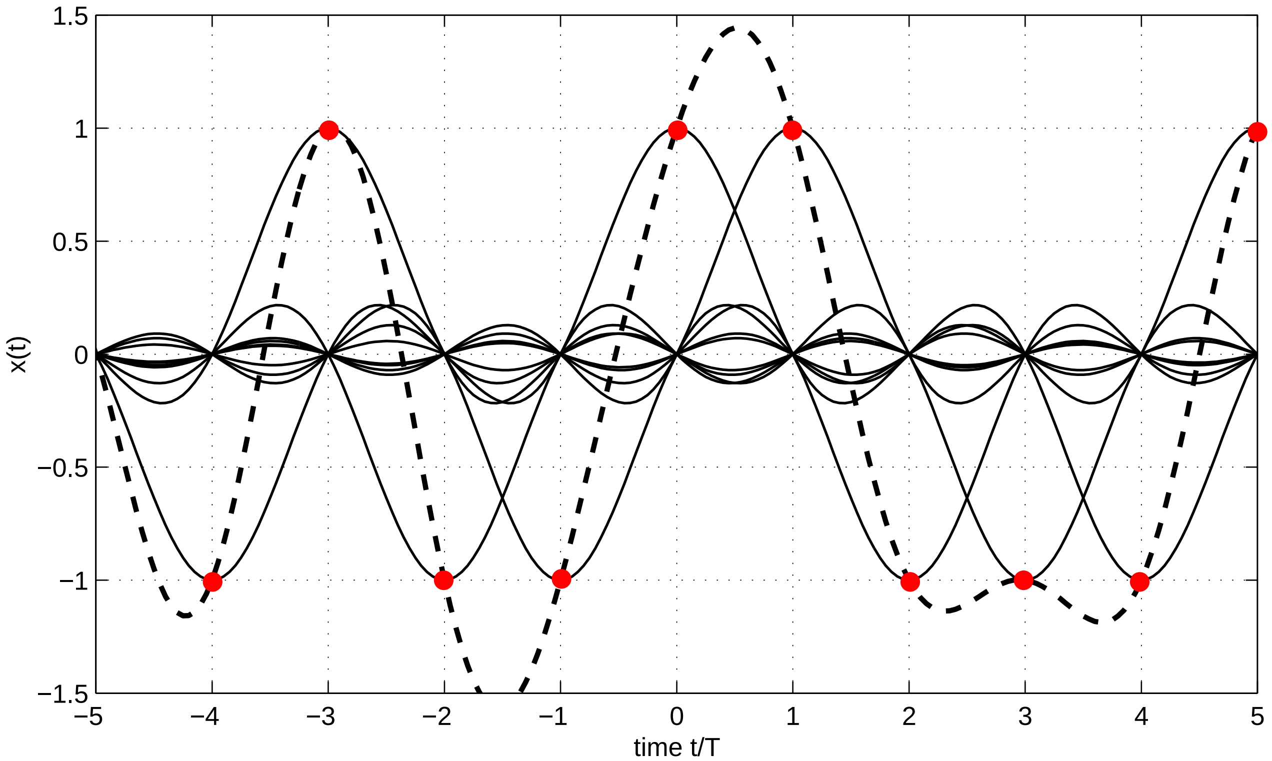


  - Then a pulse (symbol) will be passed through channel without distortion

- But **finite bandwidth means infinite time waveform**, and this would cause interference in time domain with neighbouring pulses
  - $\{x[k]\}$ in $x(t) = \sum x[k]\Phi_R(t - kT_s)$ will overlap, as each $x[k]\Phi_R(t - kT_s)$ lasts infinitely long in time
  - It seems no way out, or is it?

Pulse Shaping — Right Pulses

- Finite-bandwidth pulses are **actually** the **right** ones for transmitting $\{x[k]\}$
  - But how you avoid infinite long pulses interfere with each other in time?
  - You cannot!



- All we want is to recover transmitted symbols $\{x[k]\}$ from $x(t) = \sum x[k]\Phi_R(t - kT_s)$
  - Pulse waveform $\Phi_R(t)$ has regular **zero-crossing** at symbol-rate spacings except at $t = 0$
  - Interference from other pulses at $t = kT_s$ are all zero! i.e. $x(t = kT_s) = x[k]\Phi_R(0)$!

InterSymbol Interference (ISI)

- Let a symbol $x[k]$ be transmitted as $x[k]\phi_T(t - kT_s)$ with T_s the symbol period
- Assume that the Tx pulse $\phi_T(t)$ peaks at $t = 0$ with $\phi_T(0) = 1$, and it has zero-crossing symbol-rate spacings, i.e. it is a **Nyquist** filter
- The transmitter output waveform is:

$$x(t) = \sum_{k=-\infty}^{\infty} x[k]\phi_T(t - kT_s)$$

- Assume that the Tx pulse $\phi_T(t)$ arrives at the receiver as $\phi_R(t - t_d)$, where the Rx pulse $\phi_R(t)$ peaks at $t = 0$ with $\phi_R(0) = 1$, and it is also a **Nyquist** filter
- The receiver output waveform is:

$$y(t) = \sum_{k=-\infty}^{\infty} x[k]\phi_R(t - kT_s - t_d) + n_o(t)$$

  – Clearly, all transmitted symbols $\{x[k]\}$ in $y(t)$ are mixed up — **ISI**

Achieving Zero ISI

- The sampled receiver output at correct sampling instance $t_k = kT_s + t_d$ is:

$$y(t_k) = x[k] + \sum_{m \neq k} x[m]\phi_R((k-m)T_s) + n_o(t_k)$$

- First term: correct symbol, second term: ISI, third term: due to channel noise

- In order to **eliminate** ISI, the received pulse should satisfy:

$$\phi_R(kT_s) = \begin{cases} 1, & \text{for } k = 0 \\ 0, & \text{for } k \neq 0 \end{cases}$$

- That is, $\phi_R(t)$ has zero-crossing at symbol-rate spacings, leading to:

$$y(t_k) = x[k] + n_o(t_k)$$

- **Pulse shaping**: achieve a desired finite transmission bandwidth and eliminate ISI

Nyquist Criterion for Zero ISI

- Recall symbol rate f_s and symbol period T_s; if $\Phi_R(f) = \mathcal{F}[\phi_R(t)]$ satisfies:

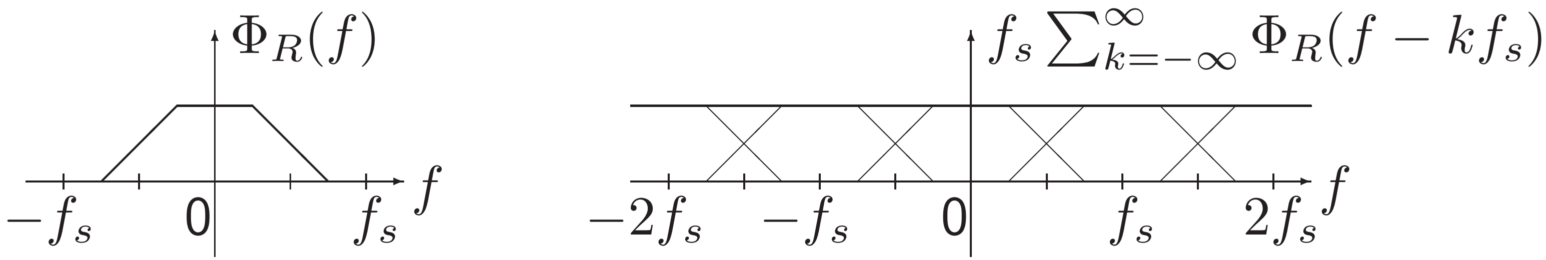
$$\sum_{k=-\infty}^{\infty} \Phi_R(f - kf_s) = \text{constant} \quad \text{for} \quad |f| \leq f_s/2$$

  then in time domain, the pulse waveform meets:

$$\phi_R(kT_s) = \begin{cases} 1, & \text{for } k = 0 \\ 0, & \text{for } k \neq 0 \end{cases}$$

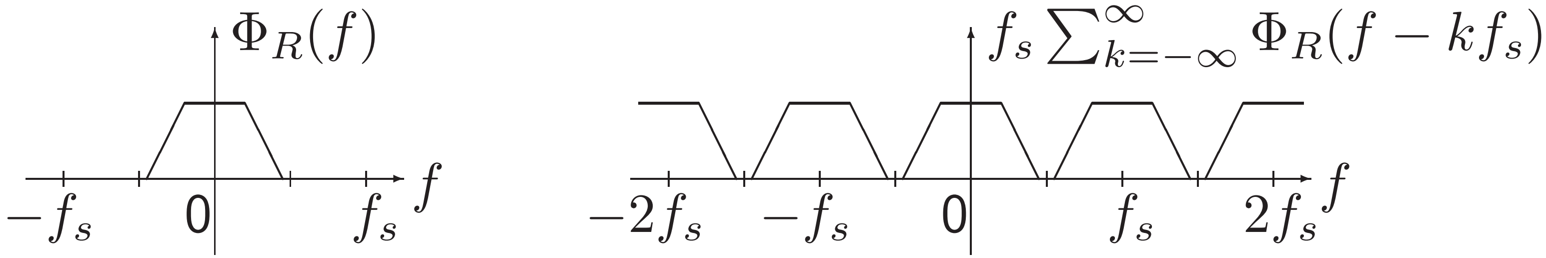
- Filter (system) generates such a pulse is a **Nyquist** system

- Illustration of condition for zero ISI, seeing from frequency domain:

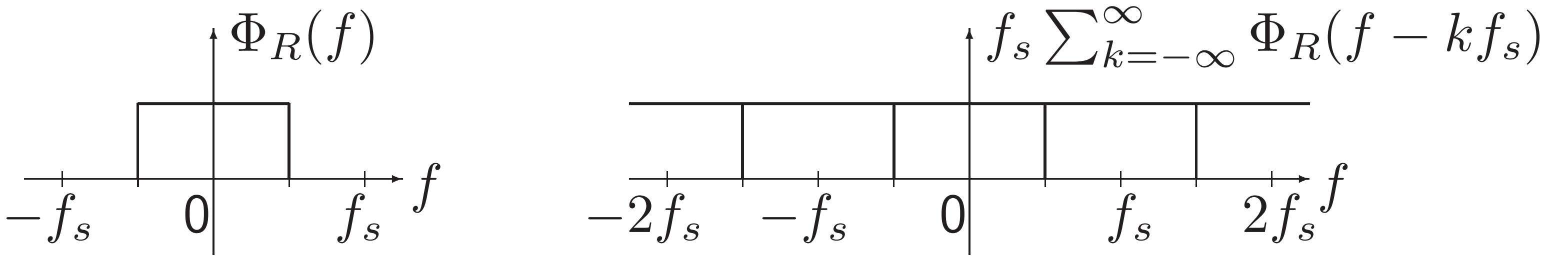


Minimum Transmission Bandwidth

- ISI cannot be removed if the bandwidth of $\Phi_R(f)$ is less than $f_s/2$:



- The **minimum required bandwidth** of $\Phi_R(f)$ required for zero ISI is $f_s/2$:



  – This is in fact the sinc pulse: $\mathrm{sinc}(f_s t) = \frac{\sin(\pi f_s t)}{\pi f_s t}$, the only Nyquist system with bandwidth $f_s/2$

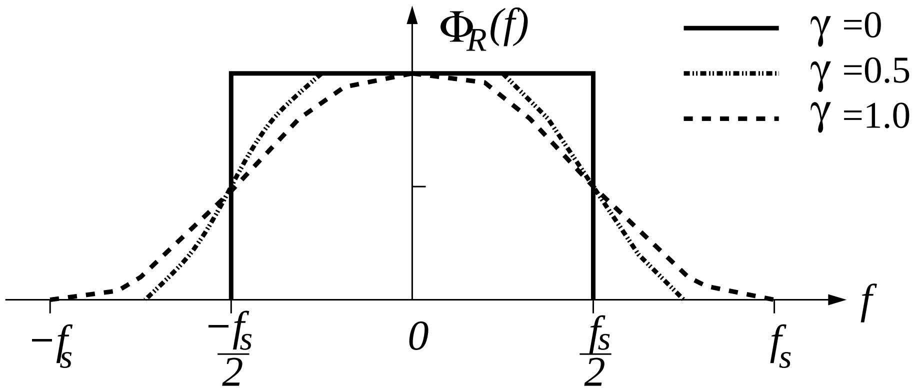
- From channel capacity: require **rate**,quality $\leftrightarrow$ **bandwidth**, power requirements

  – To transmit at symbol rate f_s, required transmission bandwidth must $B_T \geq f_s/2$

Raised Cosine Nyquist System

- The required baseband bandwidth: $f_s/2 \leq B \leq f_s$, and the spectrum:

$$\Phi_R(f) = \begin{cases} 1 & |f| \leq \frac{f_s}{2} - \beta \\ \cos^2\left(\frac{\pi}{4\beta}|f| - \frac{f_s}{2} + \beta\right) & \frac{f_s}{2} - \beta < |f| \leq \frac{f_s}{2} + \beta \\ 0 & |f| > \frac{f_s}{2} + \beta \end{cases}$$



- β: the extra bandwidth over the minimum $f_s/2$, and roll-off factor γ:

$$\gamma = \frac{\beta}{f_s/2} = \frac{B - f_s/2}{f_s/2} \quad \text{or} \quad B = \frac{f_s}{2}(1+\gamma)$$

- Raised cosine pulse is widely used pulse shaping, its time waveform of course satisfies
  - Regular **zero-crossing** at symbol rate spacing, i.e. $\phi_R(kT_s) = 0 \ \forall k \neq 0$
  - Its time waveform $\phi_R(t)$ decays from peak at $t = 0$ **much faster** than sinc pulse

Effect of Sampling Error

- It is clearly the ability of recovering transmitted symbols $\{x[k]\}$ all depends on sampling the received signal $y(t)$ at correct sampling instance
  - Sampling correctly at $t_k = kT_s + t_d$ leads to (omit noise and use $\phi_R(0) = 1$)

$$y(t_k) = x[k] + \sum_{m \neq k} x[m]\phi_R((k-m)T_s) = x[k]$$

  - In practice, small sampling error δt unavoidable, and sampling at $t_k = kT_s + t_d + \delta t$ leads to

$$y(t_k) = x[k]\phi_R(\delta t) + \sum_{m \neq k} x[m]\phi_R((k-m)T_s + \delta t) \neq x[k]\phi_R(\delta t)$$

  - Two neighbouring symbols $x[k-1]$ and $x[k+1]$ cause the biggest ISI to $x[k]$

$$y(t_k) = \cdots + x[k-1]\phi_R(T_s + \delta t) + x[k]\phi_R(\delta t) + x[k+1]\phi_R(-T_s + \delta t) + \cdots$$

- sinc pulse decays from peak at $t = 0$ at rate of $\frac{1}{t}$
  - Biggest ISI term due to small sampling error δt has magnitude of around $\frac{1}{T_s}$
- Raised cosine pulse with roll-off fact $\gamma = 1$ decays from peak at $t = 0$ at rate of $\frac{1}{t^3}$
  - Biggest ISI term due to small sampling error δt has magnitude of around $\frac{1}{T_s^3}$
- Raised cosine pulse costs more bandwidth but is less sensitive to sampling error and more practical

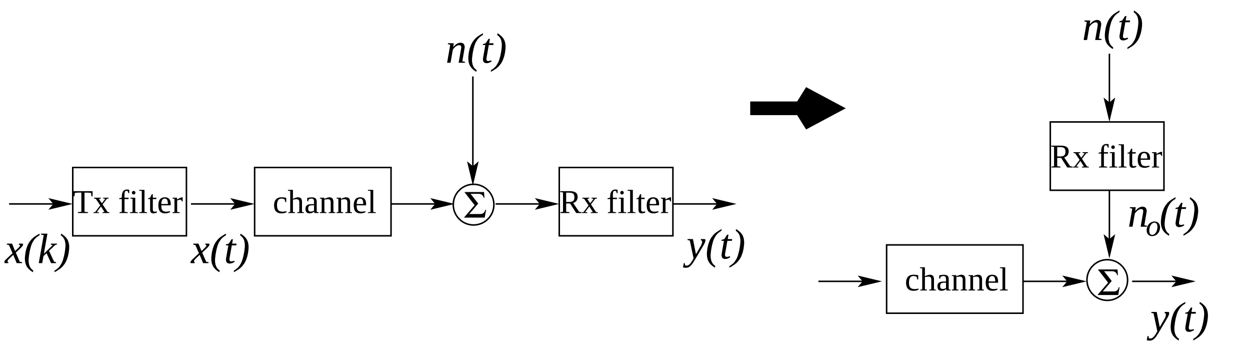
Optimal Transmit and Receive Filters

- **Task 1.** Combined Tx/Rx filters provide desired spectrum shape:

$$R(f) = G_{Tx}(f)G_c(f)G_{Rx}(f) = \Phi_R(f)$$

  - Assume the (baseband) channel bandwidth $B_c \geq B$, then $G_c(f) = 1$ and $R(f) = G_{Tx}(f)G_{Rx}(f) = \Phi_R(f)$, the required spectrum shape
  - Hence name 'pulse shaping' for transmit and receive filter pair $g(t)$ and $g(-t)$

- **Task 2.** Maximise the receiver output signal to noise ratio (SNR); note:

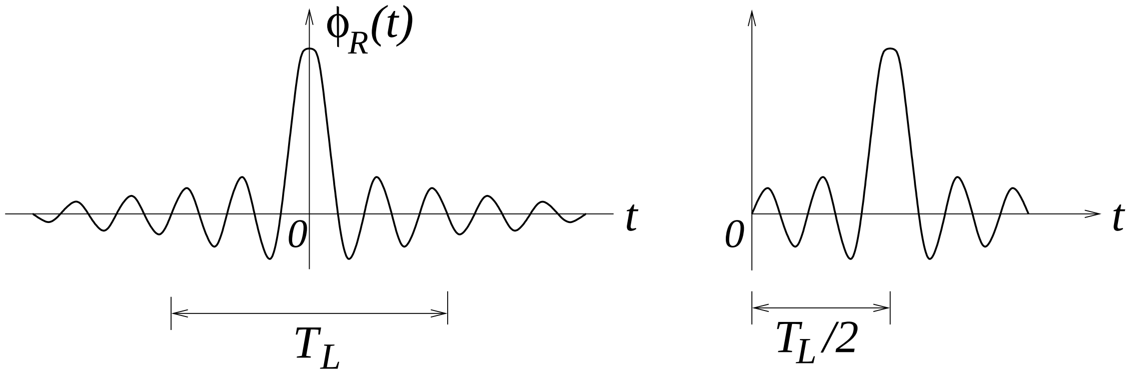


  - This leads to $G_{Tx}(f) = G_{Rx}(f)$, that is, identical as a square-root of Nyquist system $\Phi_R(f)$
  - Hence Rx filter $g(-t)$ matches Tx filter $g(t)$, this is, $g(-t)$ is 'mirror' of $g(t)$

Practical Implementation

- A true Nyquist system (e.g. raised cosine) has absolute finite bandwidth but the corresponding time waveform is non-causal and lasts infinite long
  - Pulse shaping filters to realize such a true Nyquist system cannot be constructed physically

- A practical way: truncate the pulse to a finite but sufficient length and delay the truncated pulse as shown:

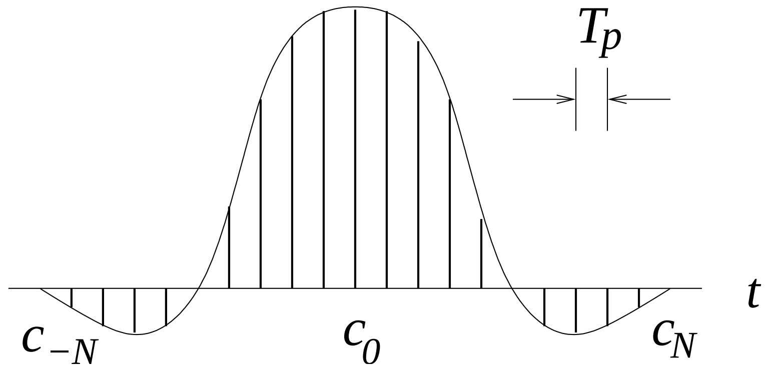


(a) truncate the pulse to length T_L (b) delay the truncated pulse

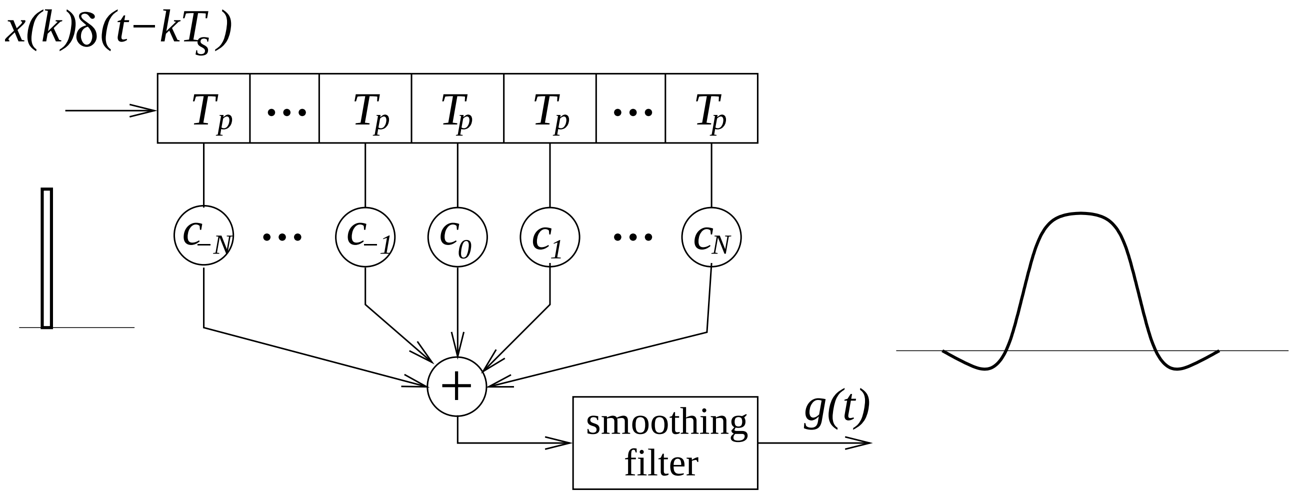
  - The bandwidth of the truncated pulse is no longer finite (e.g. the truncated raised cosine pulse in slide **10**)
  - but if the pulse is selected to decaying rapidly the resulting ISI can be made sufficiently small

Tx / Rx filters Realization

- Sampled values are obtained from the waveform of $g(t)$:



- FIR or transversal filter is used to realize the required Tx / Rx filters:



Summary

- Design of transmit and receiver filters (pulse shaping): to achieve zero ISI and to maximise the received signal to noise ratio
- The combined Tx / Rx filters should form a Nyquist system (regular zero-crossings at symbol-rate spacings except at $t = 0$), and Rx filter should be identical (matched) to Tx filter as a square-root Nyquist system
- Nyquist criterion for zero ISI, to transmit at symbol rate f_s requires at least a baseband bandwidth of $f_s/2$;
- The raised cosine pulse, roll-off factor, and the required baseband transmission bandwidth:

$$B = \frac{f_s}{2}(1 + \gamma)$$

(passband bandwidth is doubled)

- Practical considerations for implementing pulse shaping filters