Revision of Lecture 2

- Pulse shaping Tx/Rx filter pair
  - Design of Tx/Rx filters (pulse shaping): to achieve zero ISI and to maximise received signal to noise ratio
  - Combined Tx/Rx filters: Nyquist system (regular zero-crossings at symbol-rate spacings except at $t = 0$), and Rx filter matched (identical) to Tx filter
  - Nyquist criterion for zero ISI; to transmit at symbol rate f_s requires at least a baseband bandwidth of $f_s/2$
  - Raised cosine pulse, roll-off factor, and required baseband transmission bandwidth $B = \frac{f_s}{2}(1 + \gamma)$

MODEM components

pulse shaping Tx/Rx filter pair

modulator/demodulator

bits $\overset{map}{\leftrightarrow}$ symbols

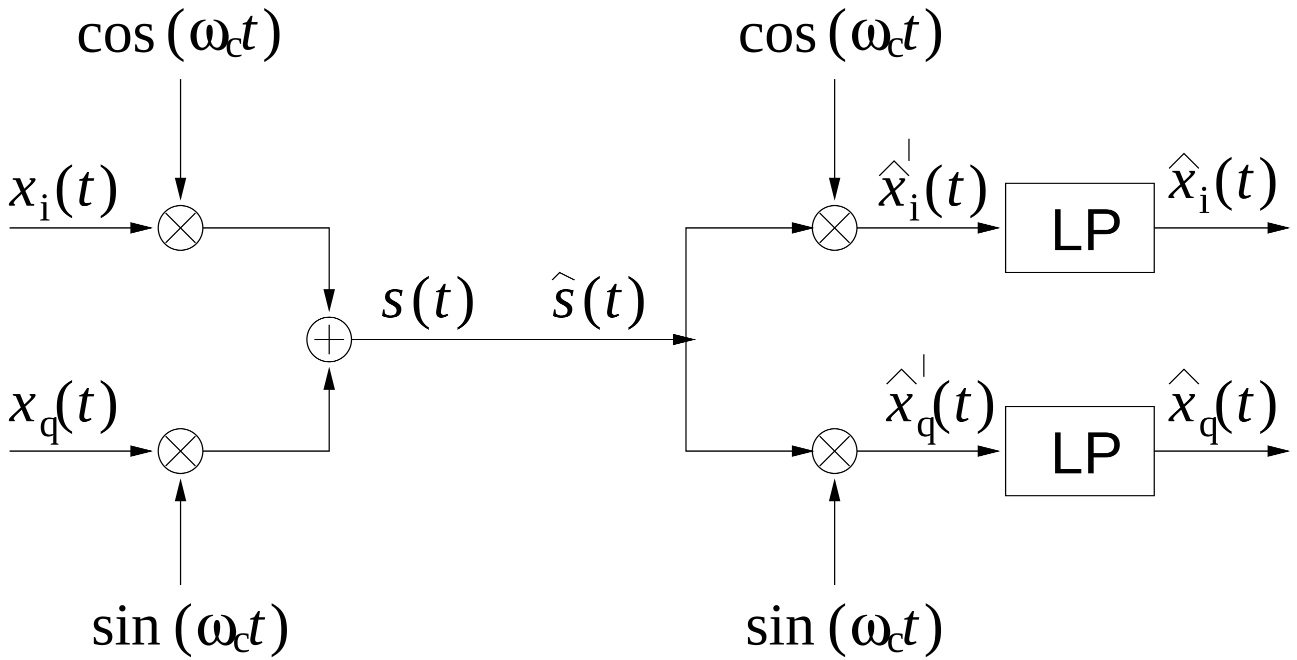
equalisation (distorting channel)

bit error rate and other issues

This lecture: **modulator/demodulator** and system design issues, such as carrier recovery and timing recovery

QAM Modulator / Demodulator

- Recall modulator and demodulator of the QAM scheme (slides **5** and **6**):



- **Carrier modulation** at transmitter: low-frequency or baseband analogue signals $x_\mathrm{i}(t)$ and $x_\mathrm{q}(t)$ are modulated by two carriers
- **Carrier demodulation** at receiver: two transmitted baseband signals are demodulated or obtained from received carrier signal
  - **Inphase** and **quadrature** carriers are orthogonal, and they can be separated at receiver
  - Inphase or quadrature rate is half of original transmission rate, meaning half of bandwidth

QAM — Modulation

- **Modulation** of "in-phase" and "quadrature" components to carrier frequency ω_c:

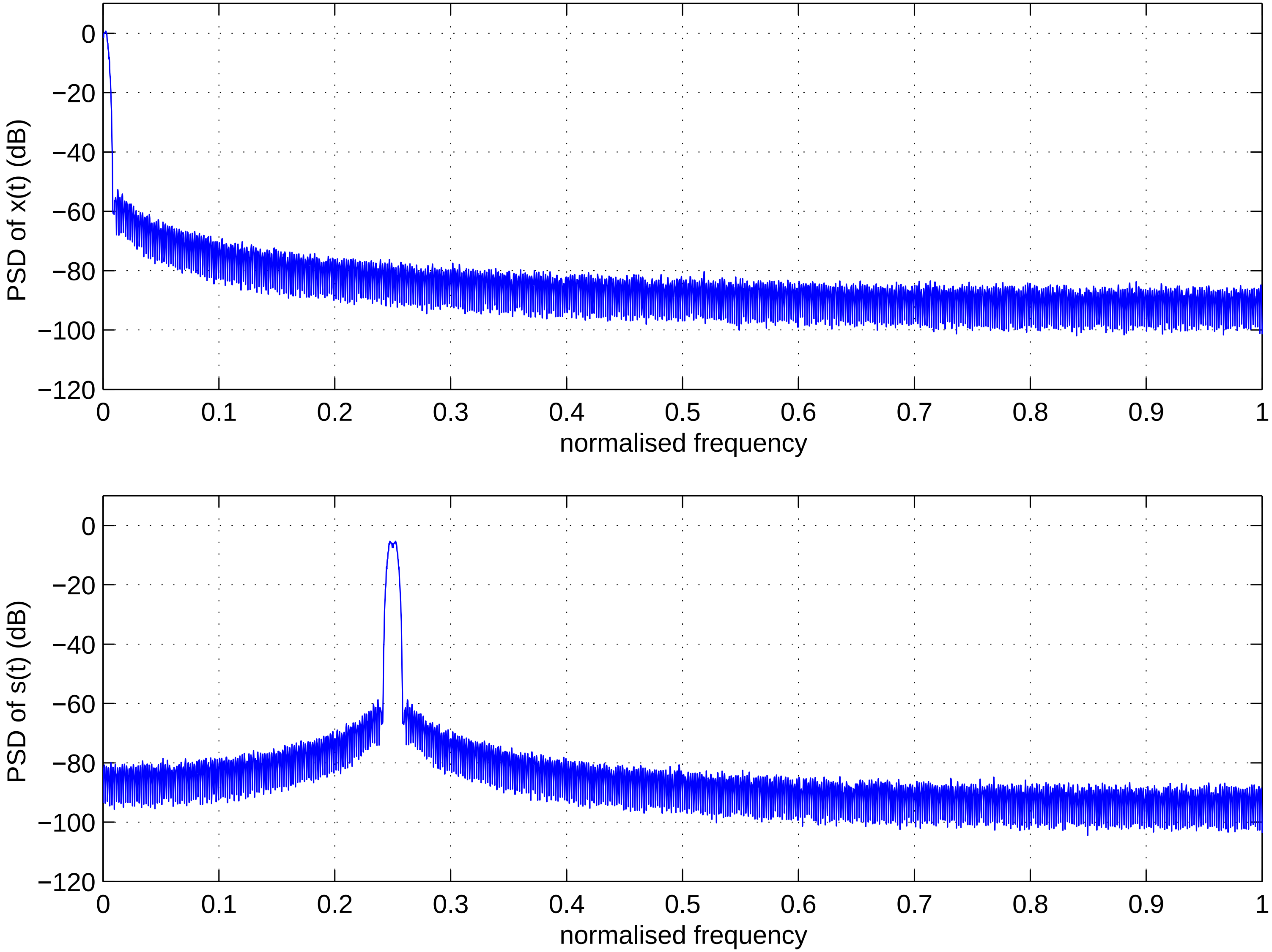
$$x_1(t) = x_{\mathrm{i}}(t) \cdot \cos(\omega_c t)$$
$$x_2(t) = x_{\mathrm{q}}(t) \cdot \sin(\omega_c t)$$

- In-phase and quadrature signals are mixed and transmitted as $s(t) = x_1(t) + x_2(t)$
- To explain demodulation, we assume perfect transmission $\hat{s}(t) = s(t)$
- Real carriers are in hundreds of MHz or in GHz
  - In next slide we have a baseband transmission bandwidth 10 kHz and carrier $f_c = 250$ kHz, normalised by 1 MHz in plots
  - Thus, carrier $f_c = 250$ kHz equals to normalised frequency 0.25, and a bandwidth of 10 KHz is equal to a width of 0.01 in normalised frequency

You may like to pause and think: digital communications is about — make analogue signal digital $\longrightarrow$ back to analogue for transmission $\longrightarrow$ digital again $\longrightarrow$ restore to original analogue signal

Why go such a length? or what are the advantages of digital communications as opposed to the original analogue communications?

I or Q Branch Modulation Example



QAM — Demodulation

- **Demodulation** for the in-phase component:

$$\begin{aligned}\hat{x}'_{\rm i}(t) =& s(t)\cdot\cos(\omega_c t) = (x_{\rm i}(t)\cdot\cos(\omega_c t) + x_{\rm q}(t)\cdot\sin(\omega_c t))\cdot\cos(\omega_c t)\\ =& x_{\rm i}(t)\cdot\cos^2(\omega_c t) + x_{\rm q}(t)\cdot\sin(\omega_c t)\cos(\omega_c t)\\ =& x_{\rm i}(t)\cdot\frac{1}{2}\cdot\big(1+\cos(2\omega_c t)\big) + x_{\rm q}(t)\cdot\frac{1}{2}\cdot\sin(2\omega_c t)\end{aligned}$$

  – If lowpass filter LP in slide **31** is selected appropriately (cut-off frequency $\leq \omega_c$), the components modulated at frequency $2\omega_c$ can be filtered out, and hence:

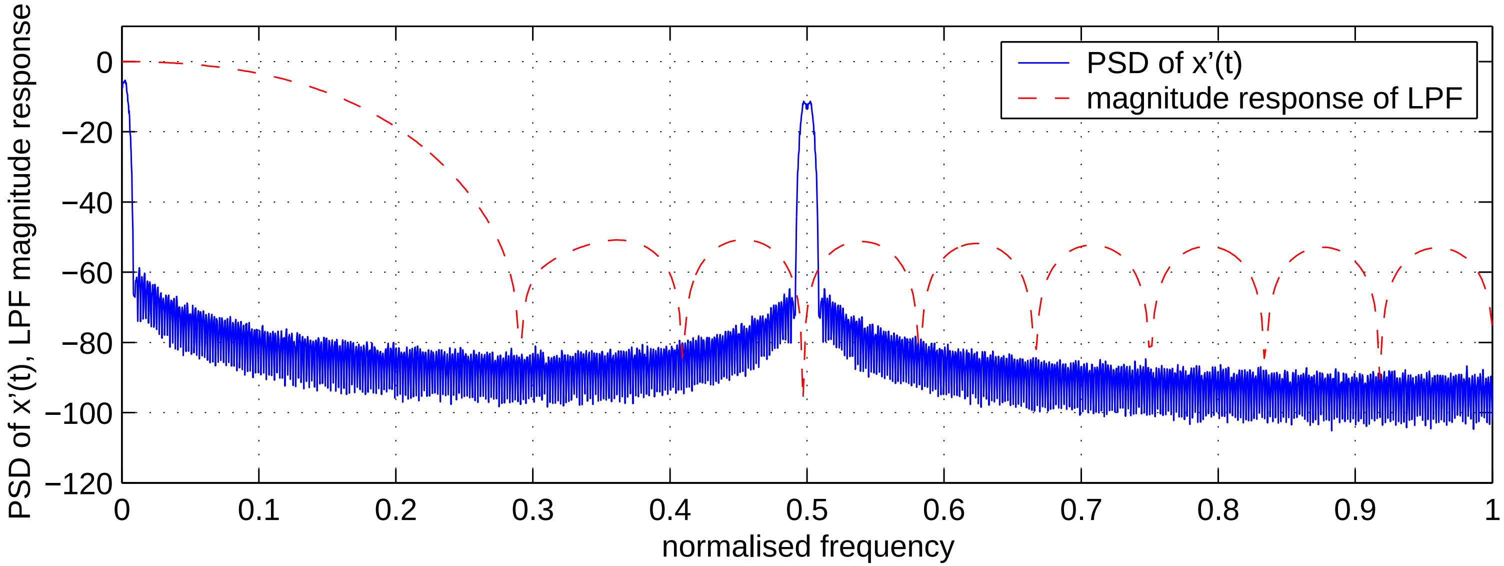
$$\hat{x}_{\rm i}(t) = \mathsf{LP}\big(\hat{x}'_{\rm i}(t)\big) = \frac{1}{2}x_{\rm i}(t)$$

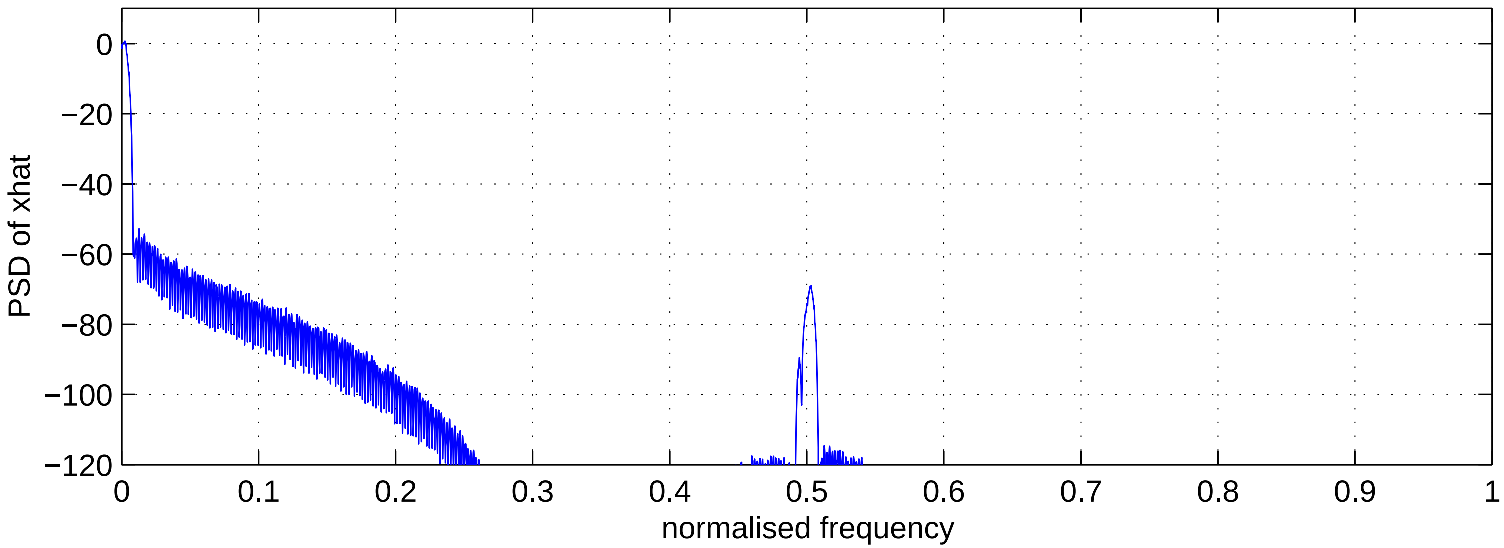
- A similar calculation can be performed for the demodulation of $\hat{x}_{\rm q}(t)$:

$$\hat{x}'_{\rm q}(t) = \cdots = x_{\rm i}(t)\cdot\frac{1}{2}\cdot\sin(2\omega_c t) + x_{\rm q}(t)\cdot\frac{1}{2}\cdot\big(1-\cos(2\omega_c t)\big)$$

  – and hence $\hat{x}_{\rm q}(t) = \mathsf{LP}\big(\hat{x}'_{\rm q}(t)\big) = \frac{1}{2}x_{\rm q}(t)$

I or Q Branch Demodulation Example





Complex Notation Representation

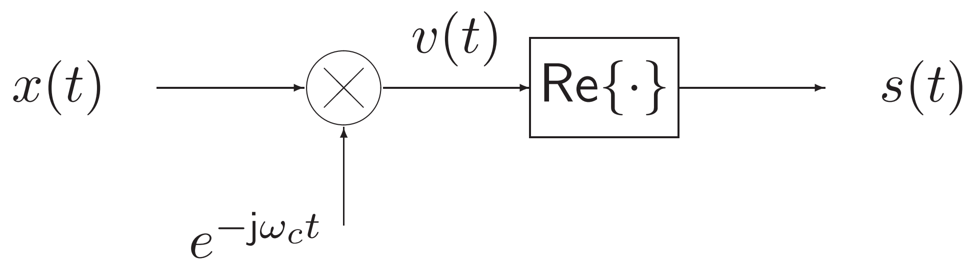
- The modulation/demodulation scheme is often expressed in complex notation
  - in-phase and quadrature components are considered to be real and imaginary parts of the complex signal, in which $\mathsf{j} = \sqrt{-1}$ represents imaginary axis

$$x(t) = x_{\mathrm{i}}(t) + \mathsf{j} \cdot x_{\mathrm{q}}(t)$$

- Carrier modulation is viewed as modulating a complex carrier $e^{-\mathsf{j}\omega_c t}$ by $x(t)$, where angular frequency $\omega_c = 2\pi f_c$
  - The transmitted signal is obtained by taking the real part of modulated carrier $x(t)e^{-\mathsf{j}\omega_c t}$

$$s(t) = \mathrm{Re}\{x(t) \cdot e^{-\mathsf{j}\omega_c t}\}$$

- Flow graph of modulator in complex notation



Modulation — Complex Notation

- Modulation:

$$
\begin{aligned}
v(t) &= e^{-\mathrm{j}\omega_c t} \cdot x(t) \\
&= \big(\cos(\omega_c t) - \mathrm{j}\sin(\omega_c t)\big) \cdot \big(x_{\mathrm{i}}(t) + \mathrm{j} \cdot x_{\mathrm{q}}(t)\big) \\
&= \underbrace{x_{\mathrm{i}}(t) \cdot \cos(\omega_c t) + x_{\mathrm{q}}(t) \cdot \sin(\omega_c t)}_{\text{real}} \underbrace{- \mathrm{j}x_{\mathrm{i}}(t) \cdot \sin(\omega_c t) + \mathrm{j}x_{\mathrm{q}}(t) \cdot \cos(\omega_c t)}_{\text{imaginary}}
\end{aligned}
$$

- Transmitted signal:

$$
s(t) = \mathrm{Re}\{v(t)\} = x_{\mathrm{i}}(t) \cdot \cos(\omega_c t) + x_{\mathrm{q}}(t) \cdot \sin(\omega_c t)
$$

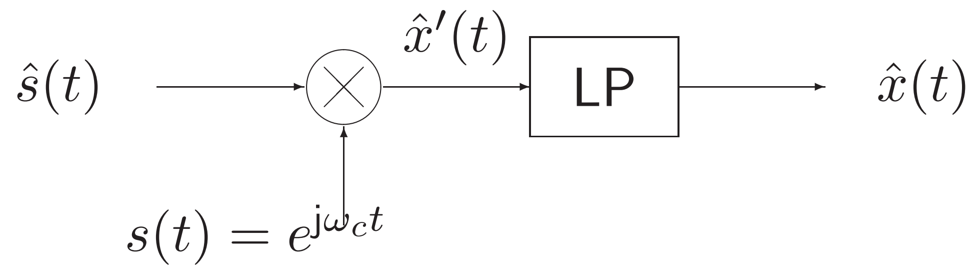
  – This is identical to the signal $s(t)$ on slide **32**

- In real world, signals are always real-valued

  – Theoretical analysis and design are often easier and more insights can be gained by adopting complex representations

Demodulation — Complex Notation

- Flow graph for the complex representation of the demodulation scheme:



- The demodulated signal: $\hat{x}'(t) = e^{j\omega_c t} \cdot s(t)$, yielding

$$
\begin{aligned}
\hat{x}'(t) =& (\cos(\omega_c t) + j\sin(\omega_c t)) \cdot (x_{\mathrm{i}}(t) \cdot \cos(\omega_c t) + x_{\mathrm{q}}(t) \cdot \sin(\omega_c t)) \\
=& x_{\mathrm{i}}(t) \cdot \frac{1}{2}\big(1 + \cos(2\omega_c t) + j\sin(2\omega_c t)\big) + \\
& j x_{\mathrm{q}}(t) \cdot \frac{1}{2}\big(1 - \cos(2\omega_c t) - j\sin(2\omega_c t)\big)
\end{aligned}
$$

- Lowpass filter (LP) will again remove components modulated at $2\omega_c$

$$
\mathrm{LP}[\hat{x}'(t)] = \frac{1}{2}x_{\mathrm{i}}(t) + j\frac{1}{2}x_{\mathrm{q}}(t)
$$

which is equivalent to slide **34**

Carrier Recovery — Phase Offset

- Previously, we assume $\hat{s}(t) = x_{\rm i}(t)\cdot\cos(\omega_c t) + x_{\rm q}(t)\cdot\sin(\omega_c t)$, and we can use $e^{{\rm j}\omega_c t} = \cos(\omega_c t) + {\rm j}\sin(\omega_c t)$ to remove carrier — What we **really** assume:
  1. Received carrier signal is $\hat{s}(t) = x_{\rm i}(t)\cdot\cos(\omega_c t) + x_{\rm q}(t)\cdot\sin(\omega_c t)$
  2. At receiver we can generate a **local** carrier $\tilde{s}(t) = \cos(\omega_c t) + {\rm j}\sin(\omega_c t) = e^{{\rm j}\omega_c t}$
  3. Hence we can carry out demodulation by $\hat{s}(t)\cdot\tilde{s}(t) \Rightarrow x_{\rm i}(t) + {\rm j}x_{\rm q}(t) = x(t)$

- Most likely, transmitted signal having travelled to receiver will accumulate a **random** and **unknown** **phase** φ:

$$\hat{s}(t) = x_{\rm i}(t)\cdot\cos(\omega_c t + \varphi) + x_{\rm q}(t)\cdot\sin(\omega_c t + \varphi)$$

  – At receiver we may generate a local carrier

$$\tilde{s}(t) = \cos(\omega_c t + \tilde{\varphi}) + {\rm j}\sin(\omega_c t + \tilde{\varphi}) = e^{{\rm j}(\omega_c t + \tilde{\varphi})}$$

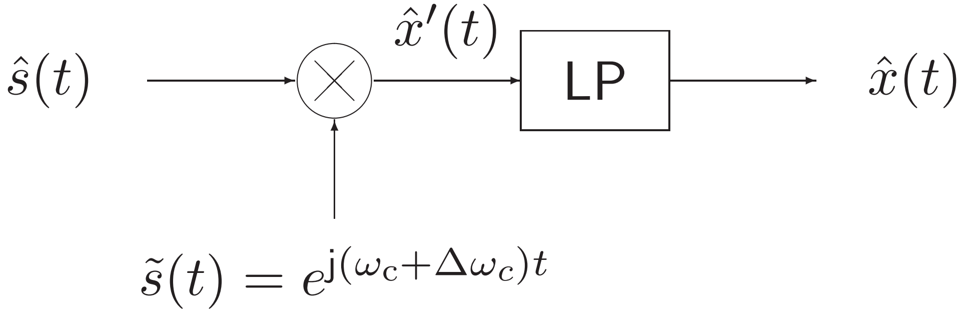
  – Demodulation by $\hat{s}(t)\cdot\tilde{s}(t) \Rightarrow x(t)\cdot e^{{\rm j}\Delta\varphi}$, where phase offset

$$\Delta\varphi = \varphi - \tilde{\varphi}$$

- Unless local carrier $\tilde{s}(t)$ happens to have same phase as incoming carrier signal $\hat{s}(t)$, i.e. **phase offset** $\Delta\varphi = 0$, you cannot recover $x(t)$!

Carrier Recovery — Frequency Offset

- Tx and Rx frequency generators are unlikely to match exactly, and consider demodulation with a Rx local "carrier" having a **frequency offset** $\Delta\omega_c$:



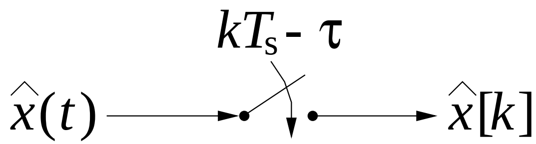
  - Even assuming $\hat{s}(t) = s(t)$, demodulated signal prior to sampling is $\hat{x}(t) = x(t) \cdot e^{\mathrm{j}\Delta\omega_c t}$, not $\hat{x}(t) = x(t)$!
  - The effect of carrier frequency mismatch $\Delta\omega_c t$, like the **phase offset** $\Delta\varphi$, has to be compensated at receiver to recover $x(t)$
  - $\Delta\omega_c t + \varphi$ is called **carrier offset** between actual carrier and Rx local carrier

- Thus, the receiver has to "recover" the actual carrier $e^{\mathrm{j}(\omega_c t + \varphi)}$ (in fact the phase φ) in order to demodulate the signal correctly

  - Usually, this is done by means of some phase lock loop based carrier recovery

Synchronisation

- The process of selecting the correct sampling instances is called synchronisation (timing or clock recovery)
  - Tx and Rx clocks likely to mismatch, clock recovery synchronises receiver clock with transmitter clock to obtain samples at appropriate instances
  - Sampling demodulated signal $\hat{x}(t)$ at appropriate sampling instances is vital for recovering transmitted symbols $\{x[k]\}$, as this is condition for avoiding ISI
  - This is equivalent to replacing sampling impulse train $\sum \delta(t - kT_{\mathrm{s}})$ in slide **6** by $\sum \delta(t - kT_{\mathrm{s}} - \tau)$, where $0 \leq \tau \leq T_{\mathrm{s}}$, with correct τ value

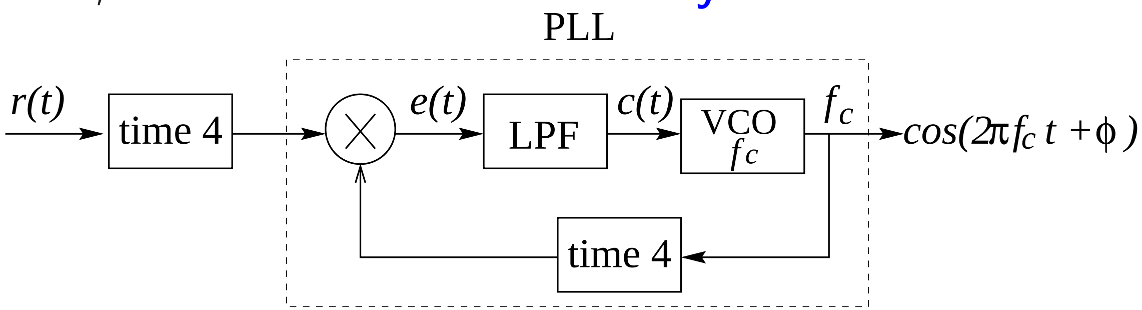


- During link initialisation and between data frames, transmitter sends **preamble** which contains known training pseudo noise (PN) sequence
  - Receiver generate local PN sequence, and by **oversampling** matches it with incoming PN sequence to obtain correct sampling information

- During data transmission, timing recovery has to rely on demodulated baseband signal $\hat{x}(t)$ only

Implementation Notes

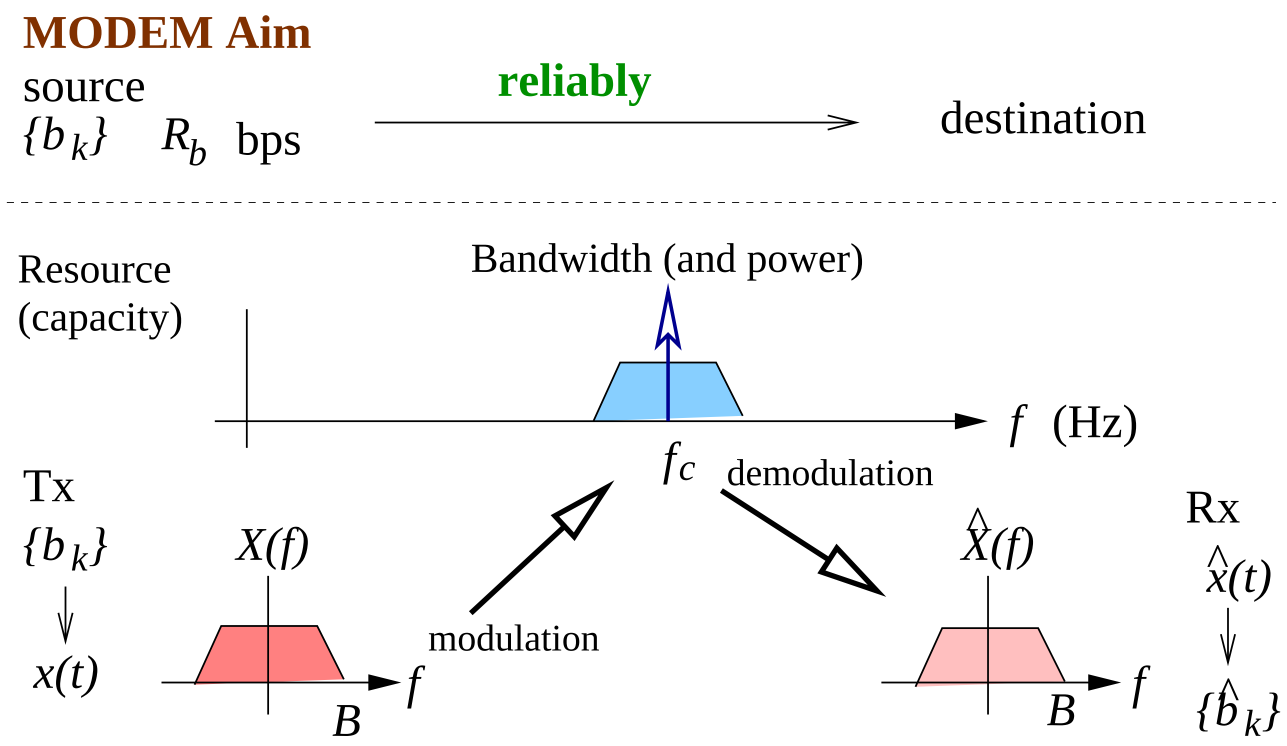
- Carrier recovery matches the phase $\tilde{\varphi}$ of local carrier to the unknown phase φ of incoming carrier, in order to demodulate, and a **time-4 carrier recovery** circuit:



  - Electronic circuit for carrier recovery operates at very high carrier frequency, and is expensive
  - Receiver with carrier recovery is called **coherent** receiver and performs much better, as it can correctly demodulate the baseband signal $x(t)$, but is more complicated and expensive
- Receiver without carrier recovery is called **non-coherent** receiver, and its performance is poorer but it is less complicated and cheaper
  - Using local carrier to demodulate without carrier recovery generates demodulated baseband signal $x(t) \cdot e^{j\Delta\varphi}$
  - Other means must be implement in order to remove unknown channel phase, e.g. differential encoding at transmitter and differential detection at receiver
- Timing recovery matches transmitter clock with receiver clock, in order to sample demodulated baseband signal at appropriate sampling instances
  - Timing recovery operates at much lower frequency baseband signal, and it is required for any transceiver, coherent or non-coherent

Summary

- This lecture explains basic operations of modulation and demodulation, including carrier recovery and timing recovery
- The MODEM lectures so far and key issues:



  - Shape the spectrum of $x(t)$ by pulse shaping
  - Carrier modulation $\leftrightarrow$ demodulation, carrier recovery
  - Timing recovery: Rx can then $\hat{x}(t) \longrightarrow \{\hat{b}_k\}$