Revision of Lecture 3

- Modulator/demodulator
  - Basic operations of modulation and demodulation
  - Complex notations for modulation and demodulation
  - Carrier recovery and timing recovery

This lecture: **bits** $\stackrel{map}{\leftrightarrow}$ **symbols**

MODEM components

pulse shaping Tx/Rx filter pair

modulator/demodulator

bits $\stackrel{map}{\leftrightarrow}$ **symbols**

equalisation (distorting channel)

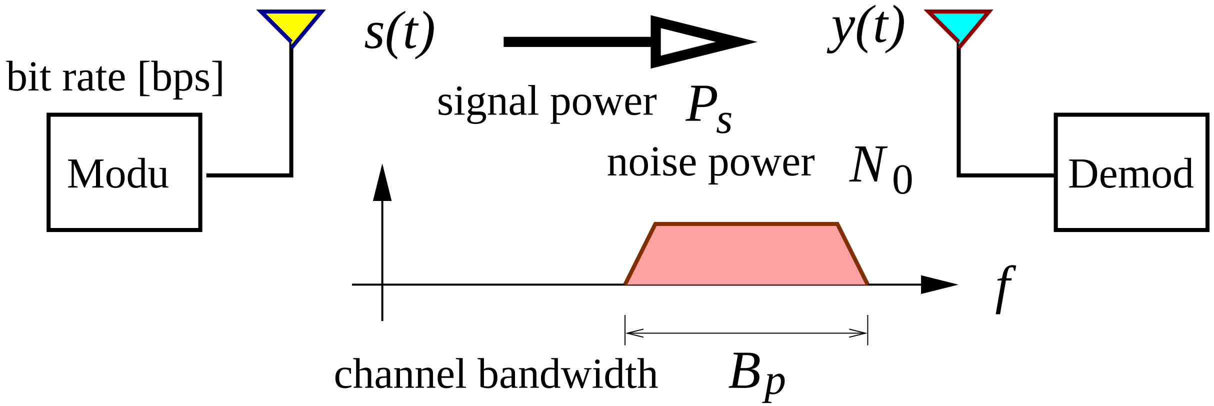
bit error rate and other issues

Recall that to transmit at a rate f_s requires at least baseband bandwidth of $\frac{f_s}{2}$

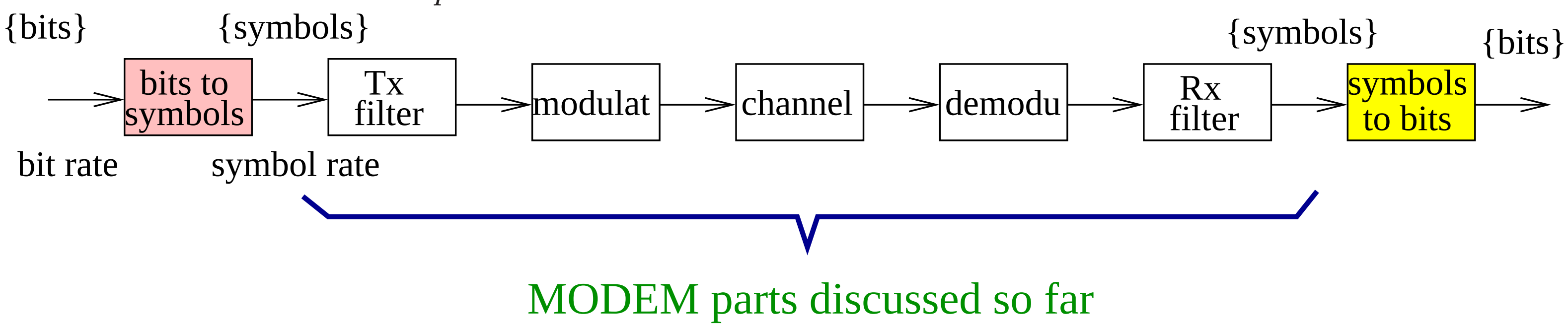
Can you see why do we want to group several bits into a symbol?

Motivations

- Recap MODEM aim and resource



- To transmit at bit rate R_b would require baseband bandwidth $B = \frac{R_b}{2}(1+\gamma)$ with rolloff factor γ, and channel bandwidth $B_p = 2B$

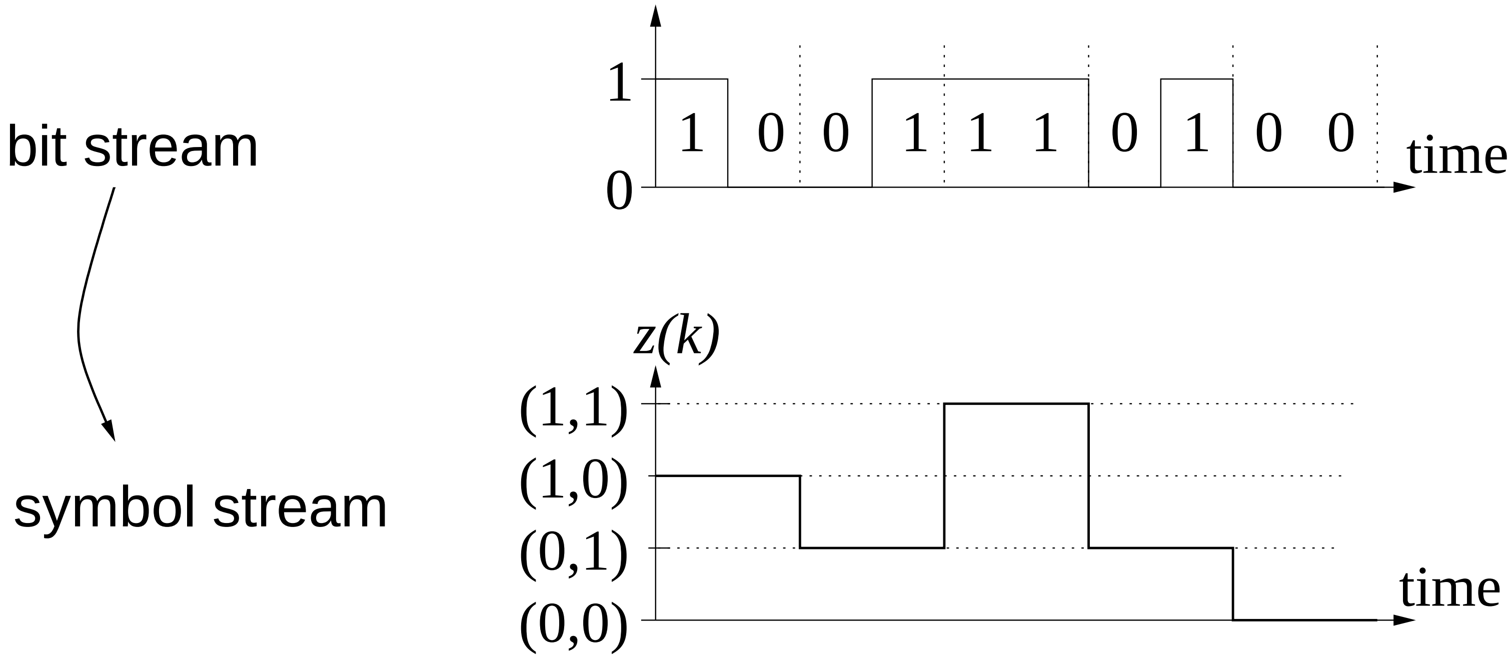


- Image grouping every q bits into a symbol, thus convert binary source into a digital source with symbol set of size 2^q
  - Transmitted symbol rate would be $f_s = \frac{R_b}{q}$, and required bandwidth is reduced by a factor of q
  - No free-lunch, you have to pay something (power) for this saving in bandwidth

Bits to Symbols

- The bit stream to be transmitted is serial to parallel multiplexed onto a stream of symbols with q bits per symbol (discrete 2^q levels)

- Example for $q = 2$ bits per symbol (4-ary modulation): symbol period T_s is twice of bit period T_b



- Symbol rate is half of bit rate; symbol stream is then pulse shaped, carrier modulated, ... (what happens to required bandwidth?)

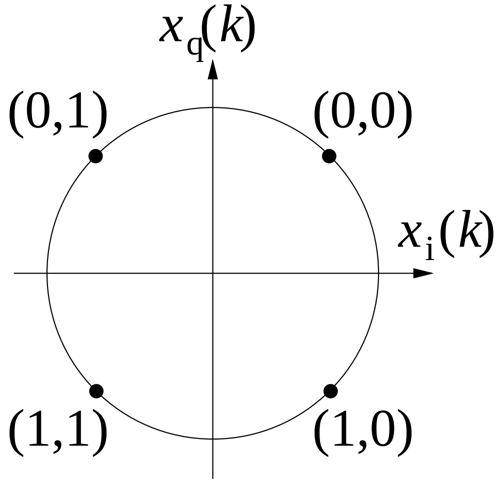
Mapping to Constellation Pattern

- It is typical practice to describe a symbol $x(k)$ by a point in *constellation diagram*, i.e. its in-phase and quadrature components, $x_{\mathrm{i}}(k)$ and $x_{\mathrm{q}}(k)$

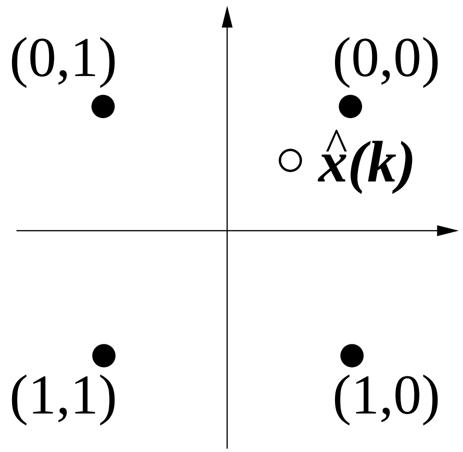
- Example for a case of $q = 2$ bits per symbol (QPSK):

- From the constellation pattern, the values $x_{\mathrm{i}}(k)$ and $x_{\mathrm{q}}(k)$ of symbol $x(k)$ are determined

- There is a **one-to-one** relationship between symbol set (constellation diagram) and modulation signal set (actually transmitted modulated signal)



- In the receiver, the constellation point and therefore the transmitted symbol value is determined from the received signal sample $\hat{x}(k)$



Phase Shift Keying (PSK)

- In PSK, carrier phase used to carry symbol information, and modulation signal set:

$$s_i(t) = A\cos(\omega_c t + \phi_i(t)),\ 0 \le t \le T_s,\ 1 \le i \le M = 2^q$$

where T_s: symbol period, A: constant carrier amplitude, M: number of symbol points in constellation diagram

- "Phase" carries symbol information, namely to transmit i-th symbol value (point), signal $s(t) = s_i(t)$ is sent, note:

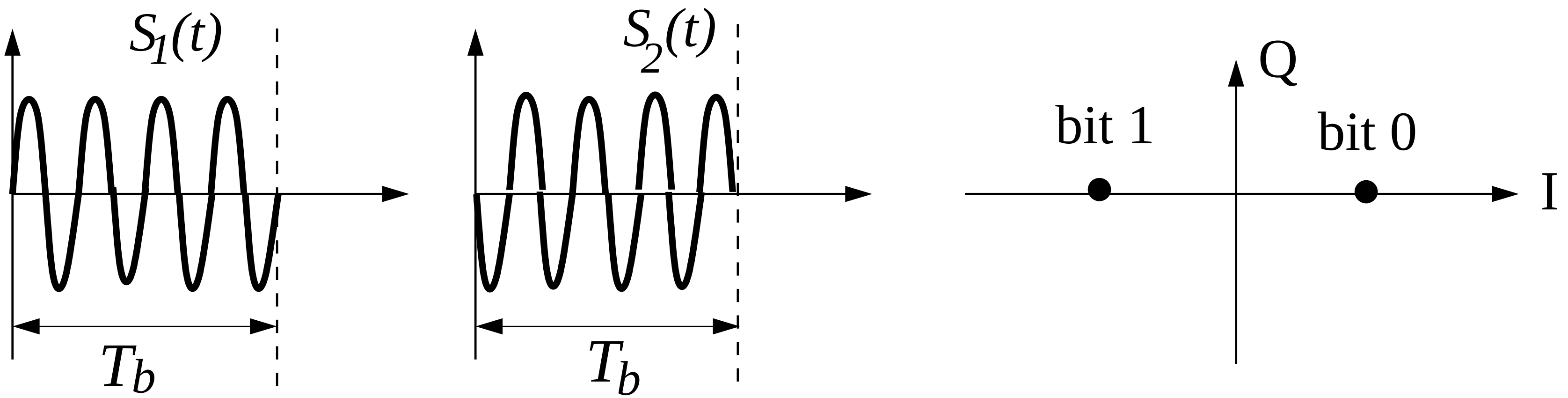
$$s(t) = A\cos(\omega_c t + \phi_i(t)) = \underbrace{A\cos(\phi_i(t))}_{\text{inphse symbol } x_{\mathrm{i}}(t)} \cdot \cos(\omega_c t) + \underbrace{(-A\sin(\phi_i(t)))}_{\text{quadrature symbol } x_{\mathrm{q}}(t)} \cdot \sin(\omega_c t)$$

- Recall previously in slide **32**, we say transmitted signal is

$$s(t) = x_{\mathrm{i}}(t)\cos(\omega_c t) + x_{\mathrm{q}}(t)\sin(\omega_c t)$$

Binary Phase Shift Keying (BPSK)

- One bit per symbol, note the mapping from bits to symbols in constellation diagram, where quadrature branch is not used



- Symbol rate equals to bit rate and symbol period equals bit period

- Modulation signal set $s_i(t) = A\cos(\omega_c t + \phi_i)$, $i = 1, 2$

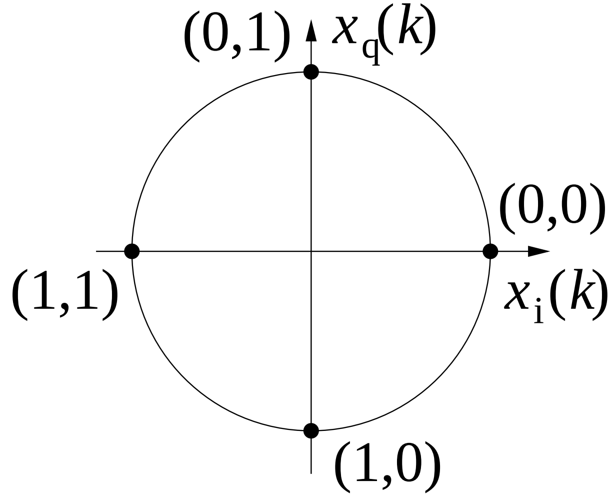
bit **0** or symbol 1 (**+1**): $\phi_1 = 0$
bit **1** or symbol 2 (-**1**): $\phi_2 = \pi$

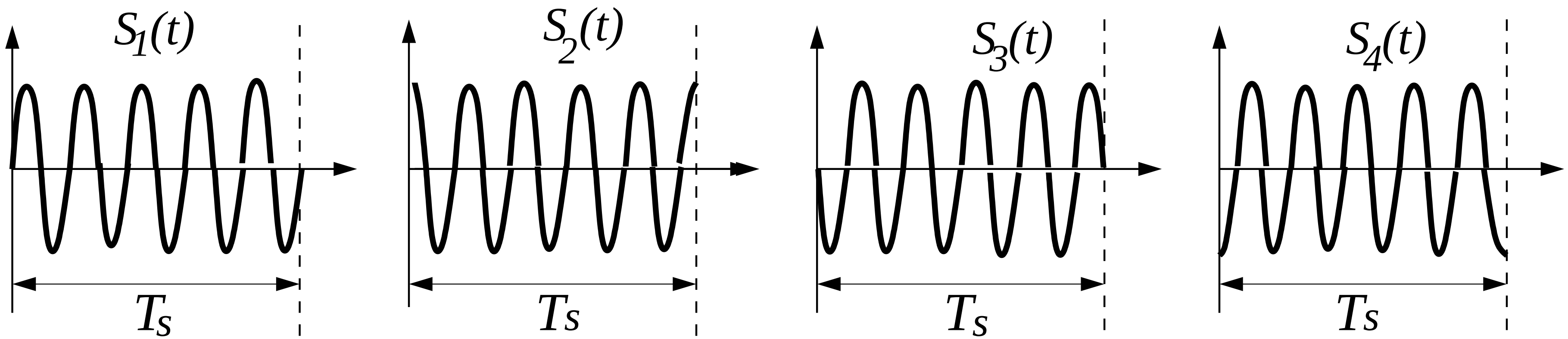
Phase separation: π

Quadrature Phase Shift Keying (QPSK)

- Two bits per symbol with a minimum phase separation of $\frac{\pi}{2}$
- A QPSK constellation diagram:
  (A "different" one shown in slide **47**)
- Modulation signal set

$$s_i(t) = A\cos(\omega_c t + \phi_i), \quad 1 \le i \le 4$$

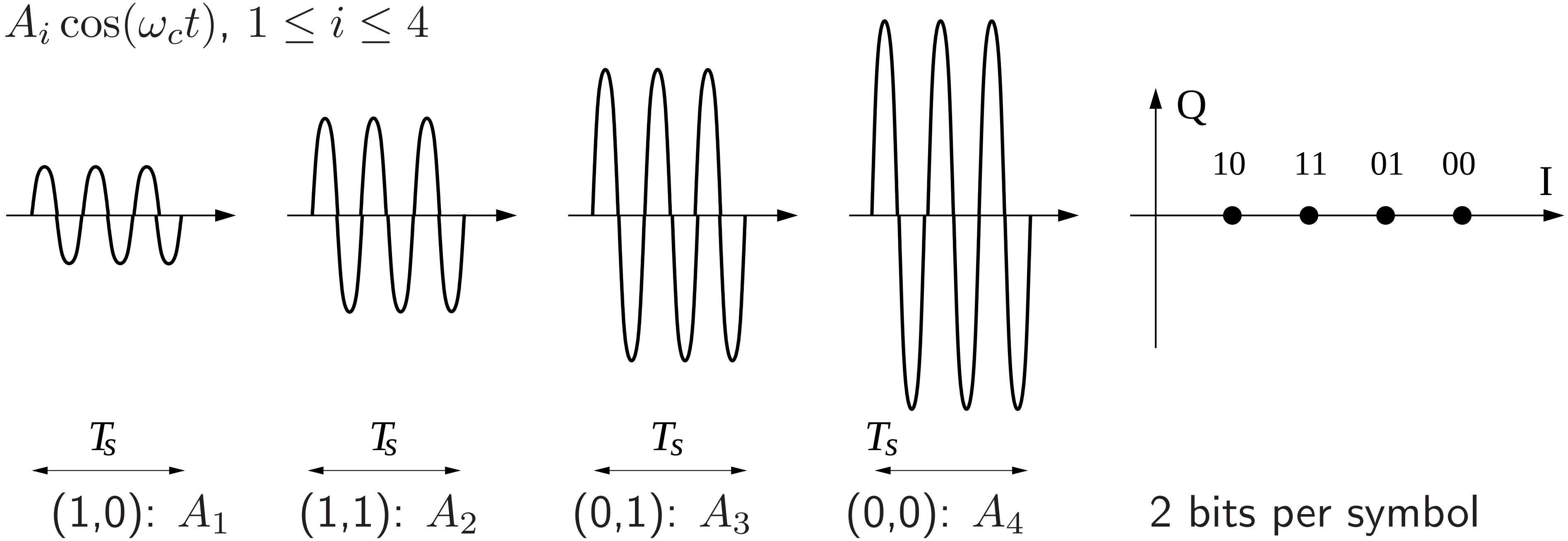




bit (0,0): $\phi_1 = 0$ bit (0,1): $\phi_2 = \frac{\pi}{2}$ bit (1,1): $\phi_3 = \pi$ bit (1,0): $\phi_4 = \frac{3\pi}{2}$

Amplitude Shift Keying (ASK)

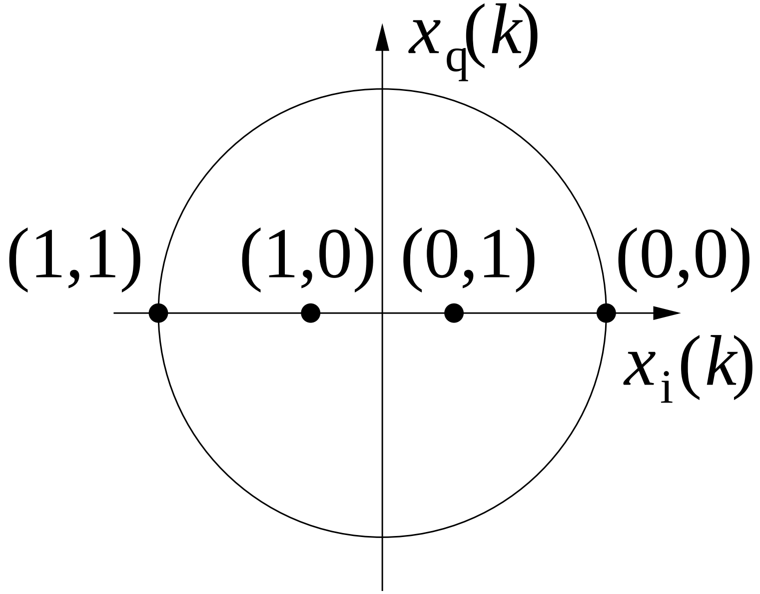
- Pure ASK: carrier amplitude is used to carry symbol information
- An example of 4-ASK with constellation diagram and modulation signal set $s_i(t) = A_i \cos(\omega_c t)$, $1 \le i \le 4$

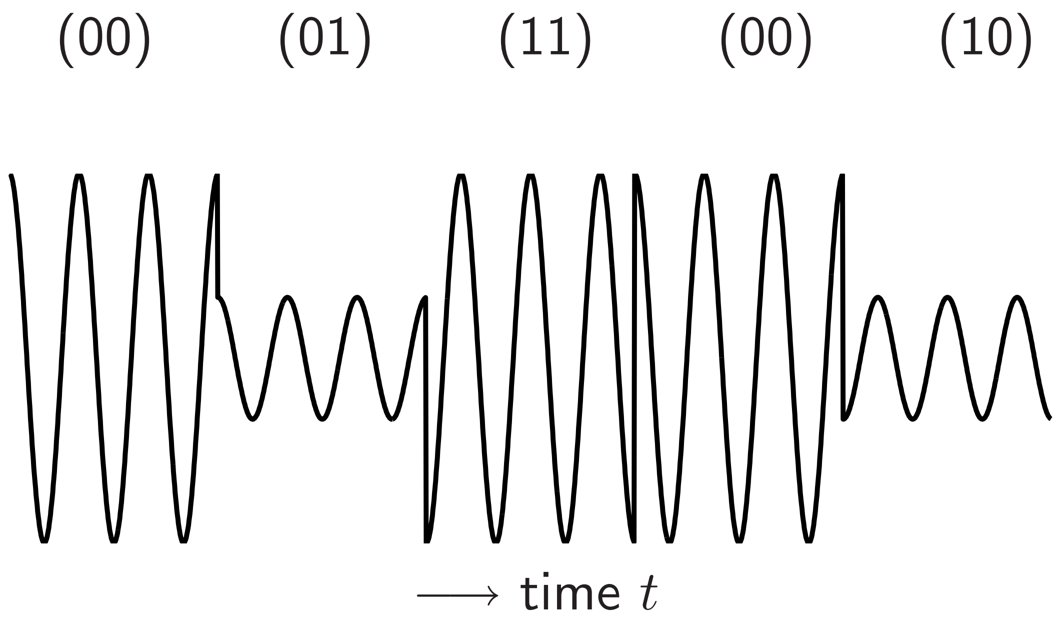


- Note quadrature branch is not used, pure ASK rarely used itself as amplitude can easily be distorted by channel
  - Channel AWGN can seriously distort pure ASK

Combined ASK / PSK

- PSK and ASK can be combined. Here is an example of 4-ary or 4-PAM (pulse amplitude modulation) with constellation pattern and transmitted signal $s(t)$:





- 2 amplitude levels and phase shift of π are combined to represent 4-ary symbols
  - 4-ary: 2 bits per symbol

- Note in $\sqrt{M}$-ary or $\sqrt{M}$-PAM, quadrature component is not used, a more generic scheme of combining PSK/ASK is QAM, which uses both I and Q branches

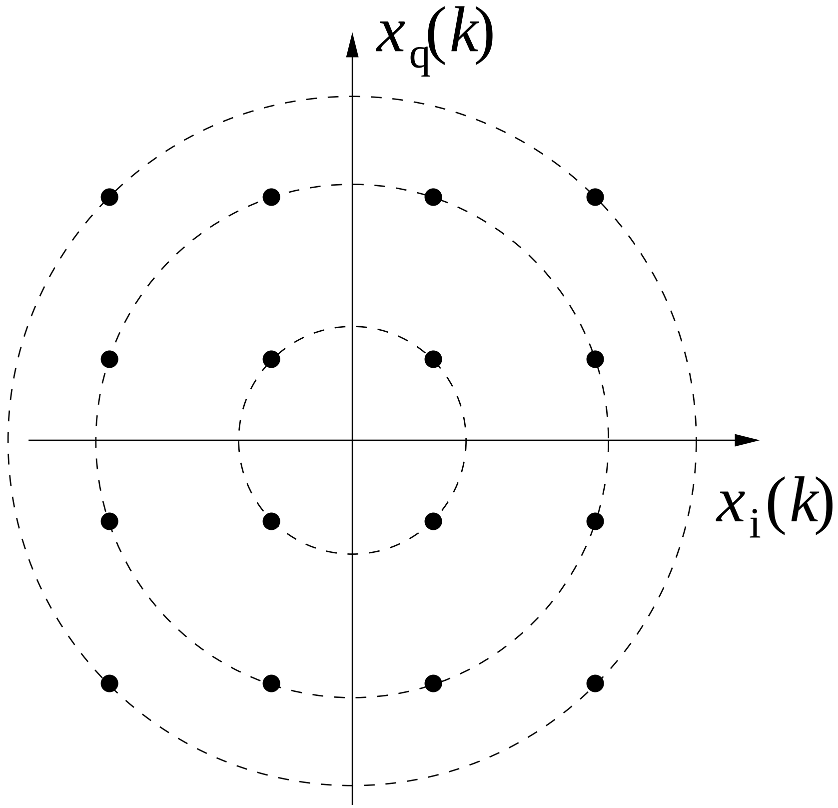
Quadrature Amplitude Modulation (QAM)

- QAM: combines features of PSK and ASK, uses both I and Q components, and is bandwidth very efficient

- An example of (squared) 16-QAM:
  - 4 bits per symbol

- Note for squared M-QAM, I and Q branches are both $\sqrt{M}$-ary (of previous slide)

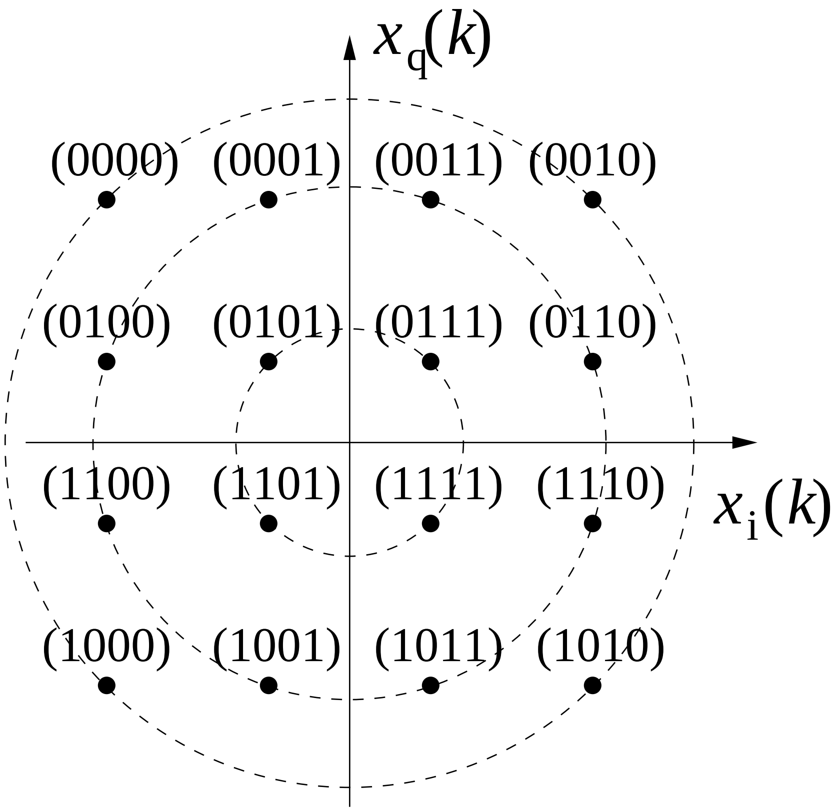
- Depending on the channel quality, 64-QAM or 256-QAM or higher order QAM are possible
  - 6 bits per symbol or 8 bits per symbol or higher number of bits per symbol

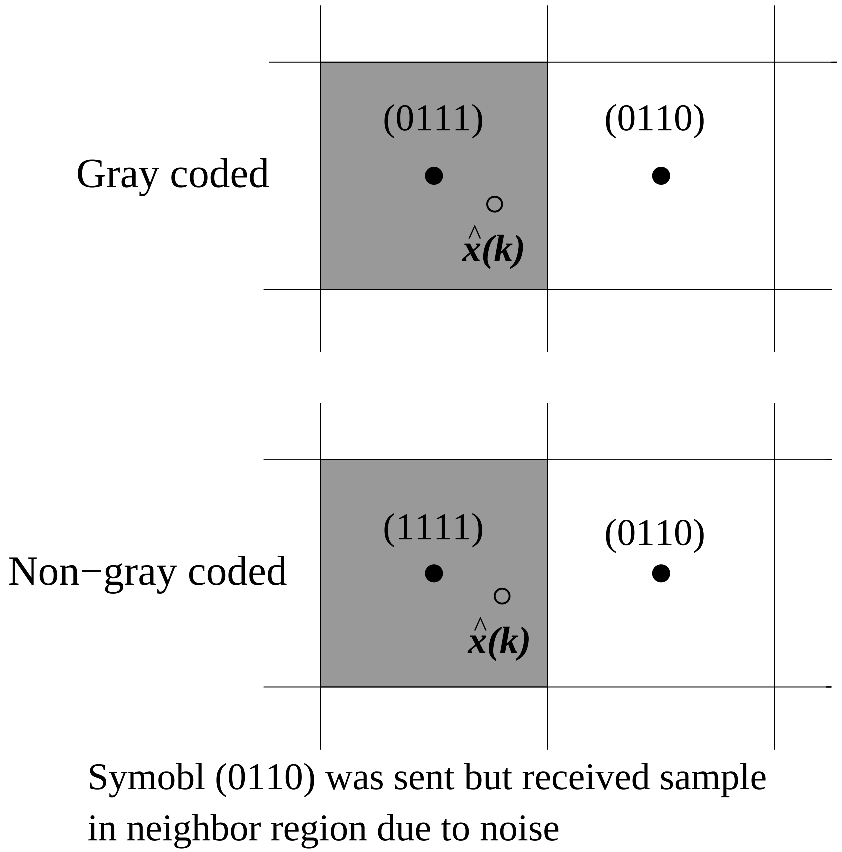


- Why high-order QAM particularly bandwidth efficient? and what is penalty paid?

Gray Mapping

- Gray coding: adjacent constellation points only differ in a single bit (minimum Hamming distance)





- If noise or distortions cause misclassification in the receiver, Gray coding can minimise the bit error rate

Frequency Shift Keying

- M-frequency shift keying: a **constant envelope** modulation with a set of frequencies $\{f_i, 1 \le i \le M = 2^q\}$ carrying symbol information

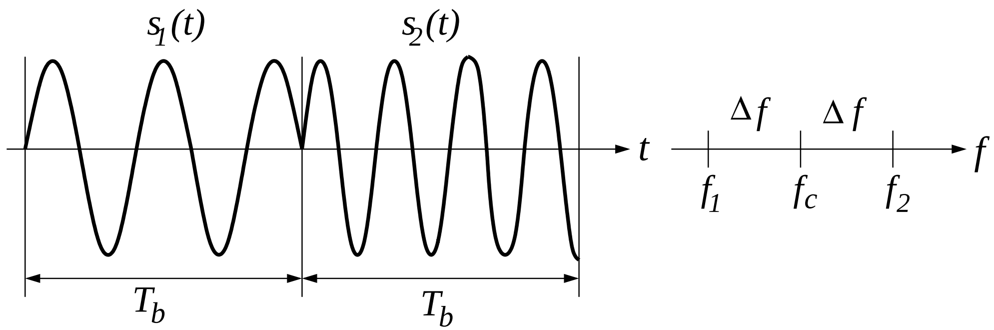
$$s_i(t) = A\cos(2\pi f_i t + \theta),\ 1 \le i \le M = 2^q,\ 0 \le t \le T_s$$

  – BFSK, QFSK, etc. 1 bit per symbol, 2 bits per symbol, etc.

- BFSK: $M = 2$, bit 0: $f_1 = f_c - \Delta f$, bit 1: $f_2 = f_c + \Delta f$

$$s_1(t) = A\cos(2\pi(f_c - \Delta f)t)$$

$$s_2(t) = A\cos(2\pi(f_c + \Delta f)t)$$

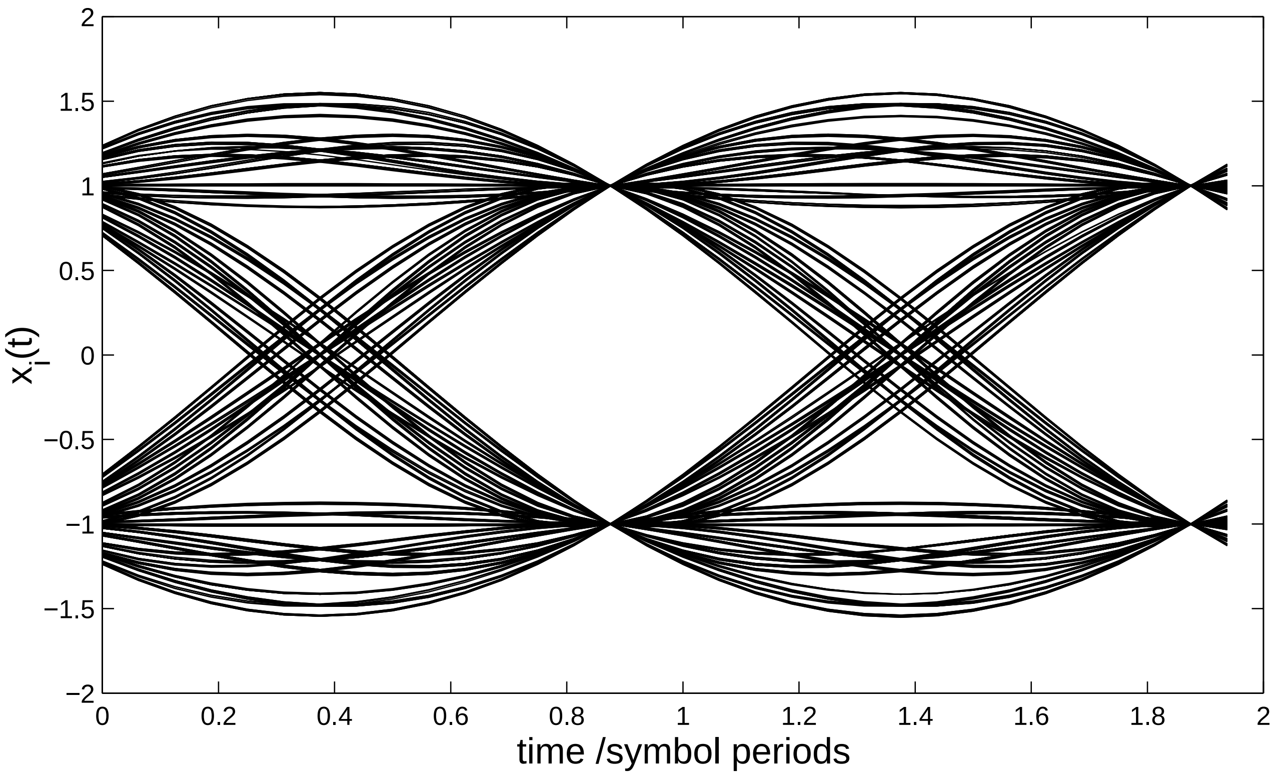


- With raised cosine pulse shaping, BFSK RF bandwidth is $B_p = 2\Delta f + 2B = 2\Delta f + (1+\gamma)R_b$, where B is the baseband signal bandwidth

  – Compared with BPSK's RF bandwidth $B_p = (1+\gamma)R_b$

Eye Diagram — Perfect Channel

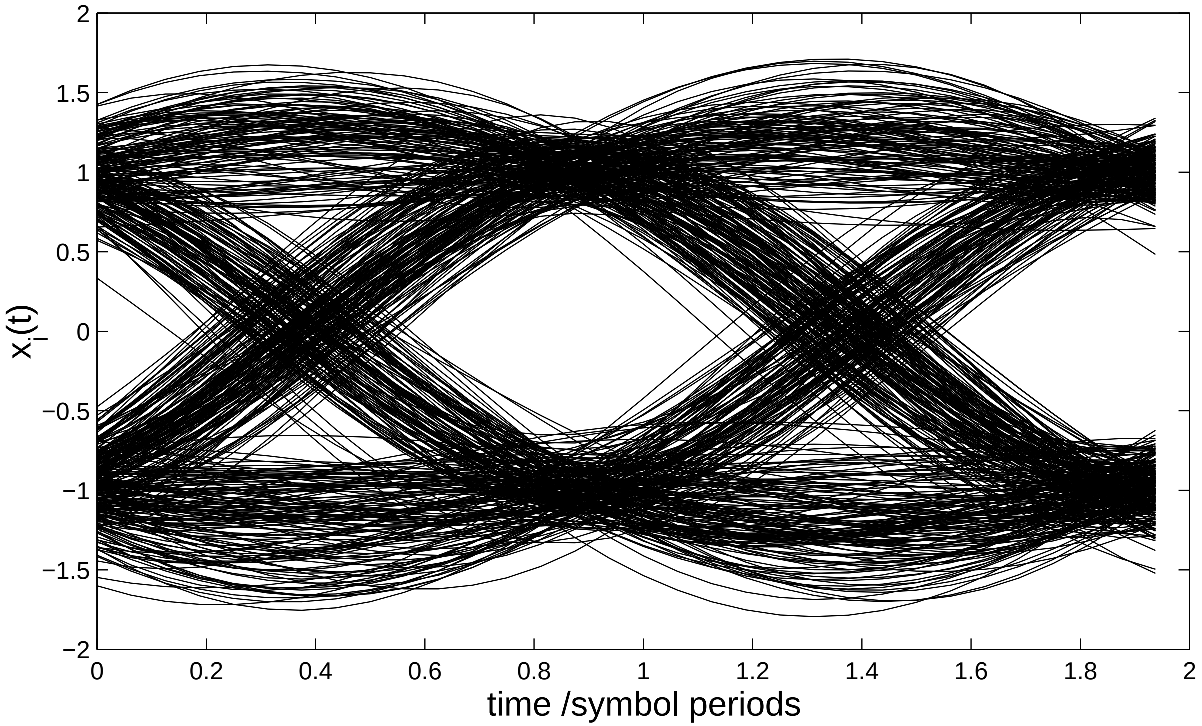
- We have discussed all components of MODEM for AWGN channel, and we will turn to "channel" again
- We have designed all components correctly, and in absence of noise, stacked 2 symbol period intervals of the demodulated signal $\hat{x}_{\rm i}(t)$ in QPSK scheme ($\hat{x}_{\rm i}(t)$ is BPSK) looks like:



- This is called an eye diagram; ideal sampling of $\hat{x}_{\rm i}(k)$ will sample the crossing points $\hat{x}_{\rm i}(t) = \pm 1$ $\longrightarrow$ clock/timing recovery ($\tau \approx 0.85T_s$ or $t_k = kT_s + 0.85T_s$)

Eye Diagram — Noisy Channel

- With channel noise at 10 dB SNR, the eye diagram looks different:

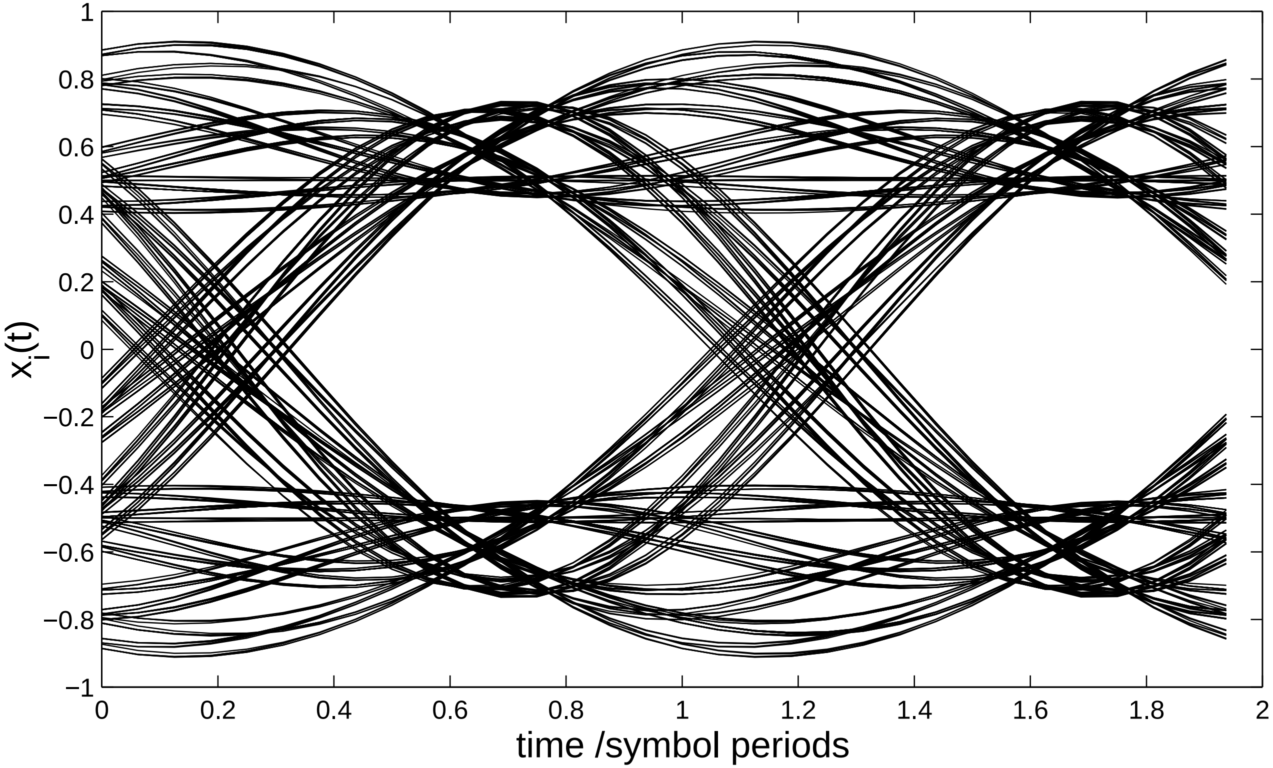


- As long as the sampling points can be clearly determined and the eye is "open", $\hat{x}_{\rm i}(k)$ will correctly resemble $x_{\rm i}(k)$

- At higher noise levels, misclassification can occur if the eye is "closed"

Eye Diagram — Distorting Channel

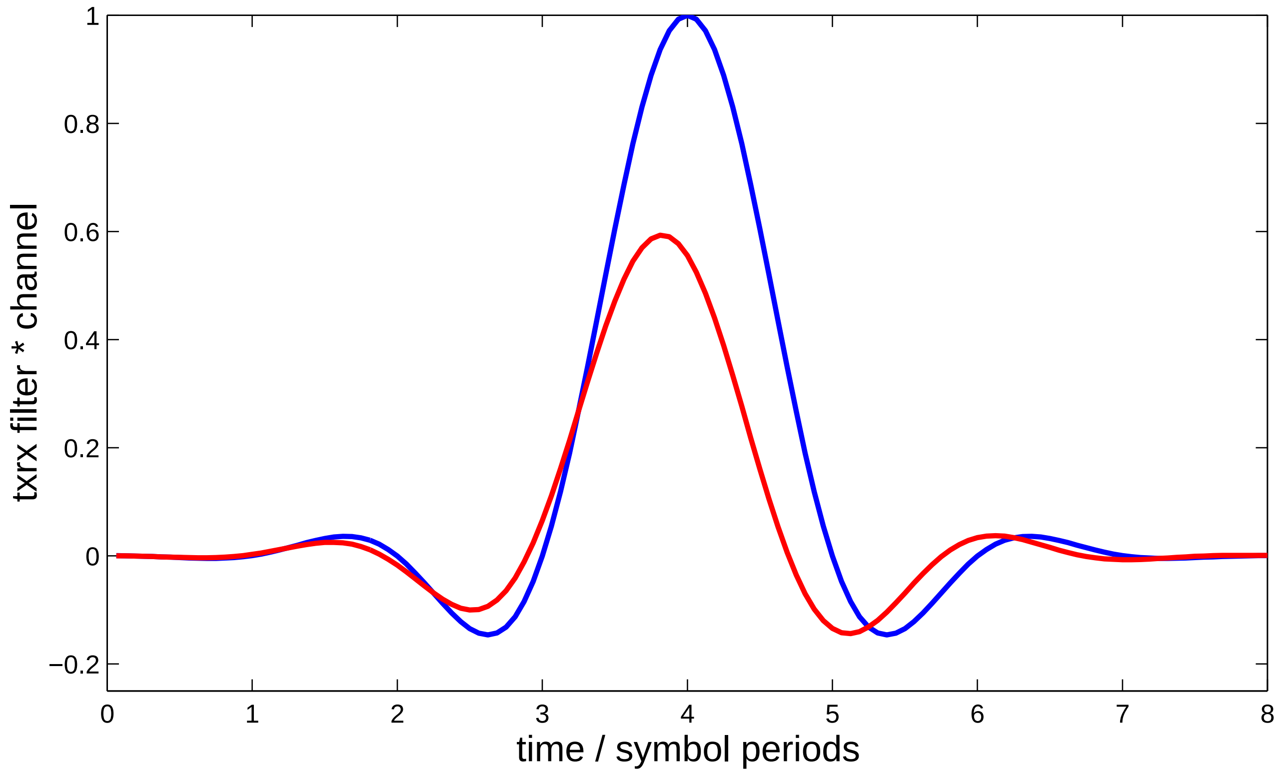
- Although we design all components correctly for AWGN channel, channel may actually be Non-ideal, and what happens?
- Let us consider a very **mild** non-ideal channel with impulse response $c(t) = \delta(t) - \frac{1}{2} \cdot \delta(t - T_s/4)$ in absence of noise, where T_s is the symbol period:



- Even with such a mild non-ideal channel, the eye diagram is distorted, this intersymbol interference together with noise effect will make the eye completely closed, leading to misclassification

Intersymbol Interference (ISI)

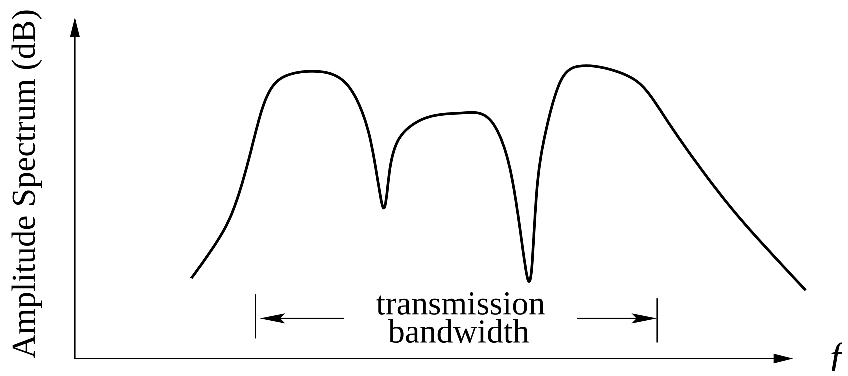
- Combined impulse response of an ideal pulse shaping filter of regular zero crossings with ideal channel $g_c(t) = \delta(t)$ and non-ideal channel $g_c(t) = \delta(t) - \frac{1}{2}\delta(t - t_s/4)$:



- For non-ideal channel, the combined Tx-filter – channel – Rx filter has lost the property of a Nyquist system, no longer has regular zero crossings at symbol spacing

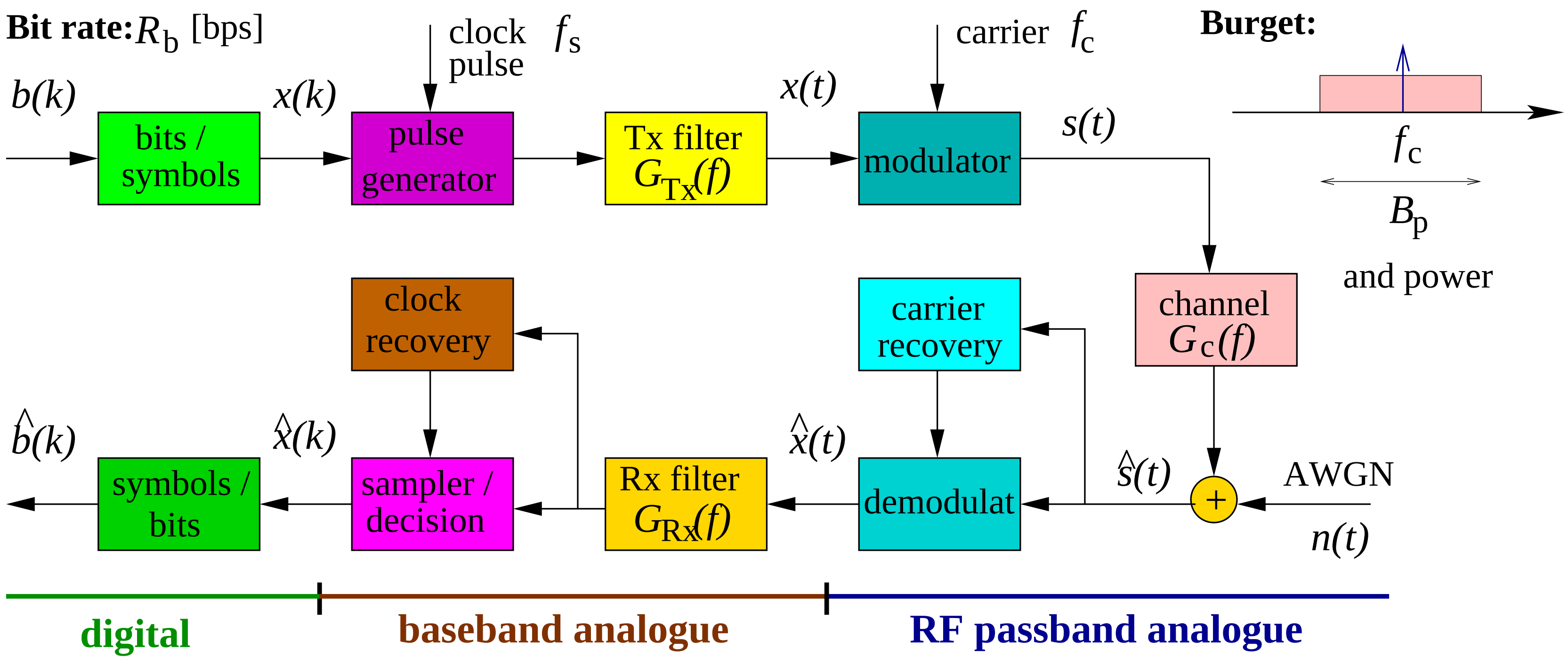
Dispersive Channel

- Recall that zero ISI is achieved if combined Tx and Rx filters is a Nyquist system
- But this is only true if the channel is ideal $\Rightarrow G_{Tx}(f)G_c(f)G_{Rx}(f) = G_{Tx}G_{Rx}(f)$
- If $G_c(f)$ is non-ideal, $G_{Tx}(f)G_c(f)G_{Rx}(f)$ will not be a Nyquist system; example of a distorting channel:



- Dispersive channel is caused by: (i) a restricted bandwidth (channel bandwidth is insufficient for the required transmission rate); or (ii) multipath distorting
- Equalisation is needed for overcoming this channel distortion (next lecture)

MODEM Summary



- Given bit rate R_b [bps] and resource of channel bandwidth B_p and power budget
  - Select a modulation scheme (bits to symbols map) so that symbol rate can fit into required baseband bandwidth of $B = B_p/2$ and signal power can met power budget
  - Pulse shaping ensures bandwidth constraint is met and maximizes receive SNR
  - At transmitter, baseband signal modulates carrier so transmitted signal is in required channel
  - At receiver, incoming carrier phase must be recovered to demodulate it, and timing must be recovered to correctly sampling demodulated signal
- Discussions are based on ideal AWGN channel, i.e. channel is non-dispersive (no memory)