

Revision of Lecture Twenty-Five

- FFT / IFFT most widely found operations in communication systems
- Important to know what are going on inside a FFT / IFFT algorithm
- With the aid of FFT / IFFT, this lecture looks into OFDM system and other multicarrier systems

OFDM is most successful wireless transmission technology so far, and 4G is based on it. It is also used as a basic transmission method in 5G



OFDM Modem

- **OFDM basic concepts recall:** Let $\{S_k\}$ be the complex-valued symbol sequence (e.g. QAM symbols) transmitted at rate f_s or symbol period T_s
 - I have deliberately used the capital letter S_k to denote transmitted symbols (instead of the usual small letter s_k) and there is a good reason for it
 - Indeed transmitted symbols S_k are now viewed as “frequency”-domain quantities or samples
- At the OFDM **transmitter:** during the period $T = NT_s$, N symbols S_0, S_1, \dots, S_{N-1} are transmitted, and complex baseband OFDM signal during period T is therefore

$$s(t) = \sum_{k=0}^{N-1} S_k e^{j2\pi \frac{k}{T} t}$$

- At the OFDM **receiver:** the received signal is multiple by $e^{-j2\pi \frac{n}{T} t}$ and integrated over T to obtain S_n , $n = 0, 1, \dots, N - 1$

$$\frac{1}{T} \int_0^T s(t) e^{-j2\pi \frac{n}{T} t} dt = \frac{1}{T} \sum_{k=0}^{N-1} S_k \int_0^T e^{j2\pi \frac{k}{T} t} e^{-j2\pi \frac{n}{T} t} dt = S_n$$

- No one really implements OFDM Modem this way, as it needs N oscillators (N pairs of hardware modulators/demodulators)



DFT/FFT Implementation

- Sample complex-valued baseband signal $s(t)$ N times during a period T : i.e. $t = \frac{m}{N}T$

$$s_m = s\left(\frac{m}{N}T\right) = \sum_{k=0}^{N-1} S_k e^{j2\pi \frac{km}{N}}, \quad m = 0, 1, \dots, N-1$$

This is just IDFT formula multiplying by a factor N , thus one can view

- **Transmitted symbols** $\{S_n\}_{n=0}^{N-1}$ as a set of N “frequency” samples (hence capital S)
- **Baseband signal samples** $\{s_m\}_{m=0}^{N-1}$ as a set of “time” samples (hence small s)
- That is, from the set of N frequency samples to obtain the set of N time samples via IDFT:

$$s_m = N \cdot \text{IDFT} \left(\{S_k\}_{k=0}^N \right), \quad m = 0, 1, \dots, N-1$$

- At receiver, during a period T , the set of N transmitted symbols $\{S_n\}_{n=0}^{N-1}$ is recovered from the set of N time samples $\{s_m\}_{m=0}^{N-1}$ using DFT:

$$S_n = \frac{1}{N} \cdot \text{DFT} \left(\{s_m\}_{m=0}^N \right), \quad n = 0, 1, \dots, N-1$$

- IDFT/DFT are of course implemented by IFFT/FFT



OFDM Transceiver

- OFDM transmitter/receiver:

The carrier modulation and demodulation is standard

Also clock recovery, not shown, is standard

New components are **cyclic extension** add and remove

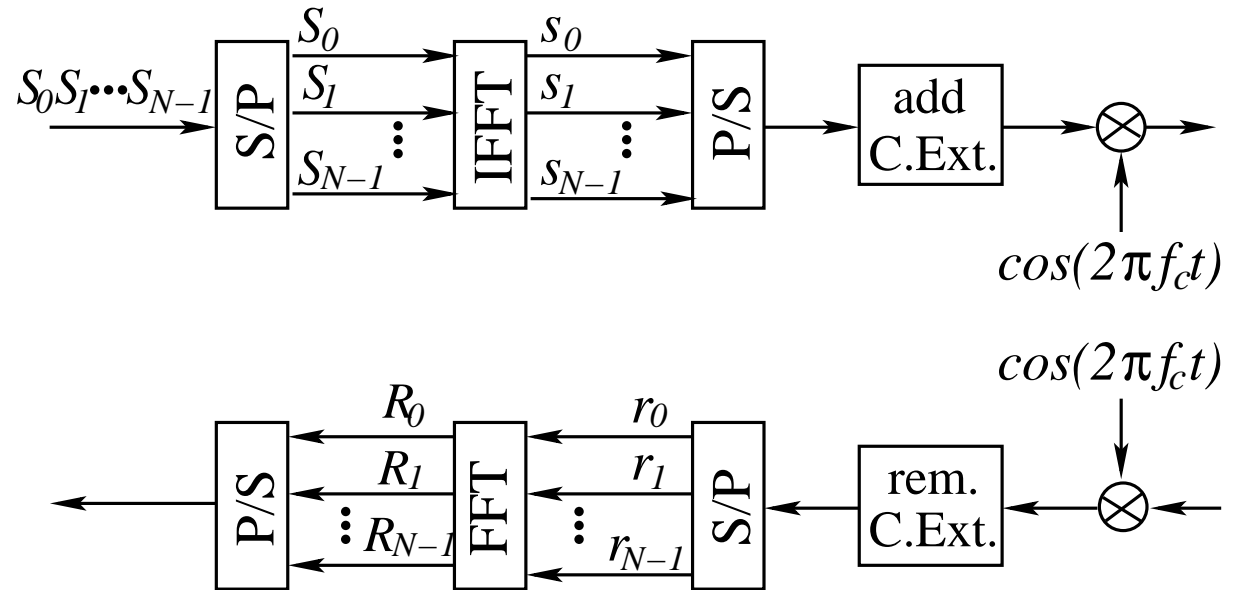
- Let transmitted frequency

frame be $[S_0 S_1 \cdots S_{N-1}]$ and transmitted time frame be $[s_0 s_1 \cdots s_{N-1}]$

- From discrete Fourier theory

$$\{s_m\}_{m=0}^{N-1} \leftrightarrow \{S_n\}_{n=0}^{N-1}$$

– Actually, from finite N frequency samples, time domain signal has infinite duration, but this time domain signal is periodic with a period of N samples



Cyclic Extension

- Why **cyclic extension**

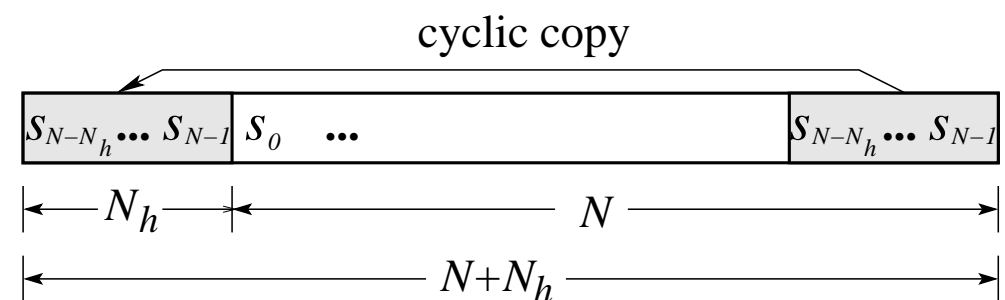
- Thus if the channel is ideal, at receiver, from N time samples $\{r_k\}_{k=0}^{N-1}$, N frequency samples $\{R_k\}_{k=0}^{N-1}$, i.e. the transmitted symbols can be recovered via FFT
- If the channel is dispersive, say, the CIR length is $N_h T_s$, then the transmitted length of N time symbols will spread to a length of $N_h + N$, and N frequency samples is insufficient

- A solution is to add some dummy symbols to make it $N + N_h$ frequency samples or cyclic extension

- An equivalent and more efficient alternative is to add cyclic extension in a transmitted time frame

- **Add cyclic extension at transmitter:**

The last N_h time samples is copied back to the beginning of the frame, and transmitted samples are $N + N_h$



Cyclic Extension (continue)

- **Remove cyclic extension at receiver:** number the current frame as i and the $N + N_h$ times samples as $-N_h, -N_h + 1, \dots, -1, 0, 1, \dots, N - 1$
 - Let the CIR be h_0, \dots, h_{N_h} and ignore noise for simplicity, we have:

$$r_{i,-N_h} = h_0 s_{i,N-N_h} + h_1 s_{i-1,N-1} + h_2 s_{i-1,N-2} + \dots + h_{N_h} s_{i-1,N-N_h}$$

$$r_{i,-N_h+1} = h_0 s_{i,N-N_h+1} + h_1 s_{i,N-N_h} + h_2 s_{i-1,N-1} + \dots + h_{N_h} s_{i-1,N-N_h+1}$$

$$\vdots$$

$$r_{i,-1} = h_0 s_{i,N-1} + h_1 s_{i,N-2} + h_2 s_{i,N-3} + \dots + h_{N_h} s_{i-1,N-1}$$

$$r_{i,0} = h_0 s_{i,0} + h_1 s_{i,N-1} + h_2 s_{i,N-2} + \dots + h_{N_h} s_{i,N-N_h}$$

$$\vdots$$

$$r_{i,N_h} = h_0 s_{i,N_h} + h_1 s_{i,N_h-1} + h_2 s_{i,N_h-2} + \dots + h_{N_h} s_{i,0}$$

$$\vdots$$

$$r_{i,N-1} = h_0 s_{i,N-1} + h_1 s_{i,N-2} + h_2 s_{i,N-3} + \dots + h_{N_h} s_{i,N-N_h-1}$$

- **Inter-frame interference:** transmitted samples from the previous $(i - 1)$ th frame spread into the first N_h time samples of i frame. Thus, the first N_h time samples are discarded



Cyclic Extension (continue)

- Remaining N samples are used to generate $R_{i,k}$, $0 \leq k \leq N - 1$
- Noting the cyclic extension: $s_{-1} = s_{N-1}, \dots, s_{-N_h} = s_{N-N_h}$, and the last N time samples in the current i frame can be written as:

$$r_{i,n} = \sum_{j=0}^{N_h} h_j s_{i,n-(j \bmod N)}, \quad 0 \leq n \leq N - 1$$

- The N frequency samples are obtained via FFT:

$$R_{i,k} = \sum_{n=0}^{N-1} r_{i,n} e^{-j2\pi \frac{n}{N}k}, \quad 0 \leq k \leq N - 1$$

- Using

$$e^{-j2\pi \frac{0}{N}k} = e^{-j2\pi \frac{N-1+1}{N}k} = e^{-j2\pi \frac{N-2+2}{N}k} = \dots, \dots$$

we have

$$R_{i,k} = \sum_{n=0}^{N_h} h_n e^{-j2\pi \frac{n}{N}k} \sum_{n=0}^{N-1} s_{i,n} e^{-j2\pi \frac{n}{N}k} = H_k S_{i,k}$$

where $\{H_k\}$ are the DFTs of the CIR $\{h_k\}$, call **frequency domain channel transfer functions (FDCTFs)**

Equalisation in OFDM

- Even though channel intersymbol interference occurs, as can be seen in received time samples $r_{i,k}$, but DFT removes this ISI in frequency-domain:

$$R_{i,k} = H_k S_{i,k} + N_{i,k}$$

with $N_{i,k}$ being a channel noise component

- The transmitted symbols are determined by passing $R_{i,k}$ through a decision device:

$$\hat{S}_{i,k} = \text{Detector}(W_k \cdot R_{i,k}), \quad 0 \leq k \leq N - 1$$

with frequency domain one-tap equaliser weight W_k

- This is a beauty of OFDM: equalisation in frequency domain becomes very simple, involving
 - Estimate the CIR taps $\{h_j\}_{j=0}^{N_h}$ or FDCTFs $\{H_i\}_{i=0}^{N-1}$
 - With estimated FDCTFs $\{\hat{H}_i\}_{i=0}^{N-1}$, compute frequency-domain equaliser weights $\{W_k\}_{k=0}^{N-1}$
- For example, zero-forcing equaliser

$$\hat{S}_{i,k} = \text{Detector}\left(\frac{R_{i,k}}{\hat{H}_k}\right), \quad 0 \leq k \leq N - 1$$

Similarly, one may use MMSE equaliser

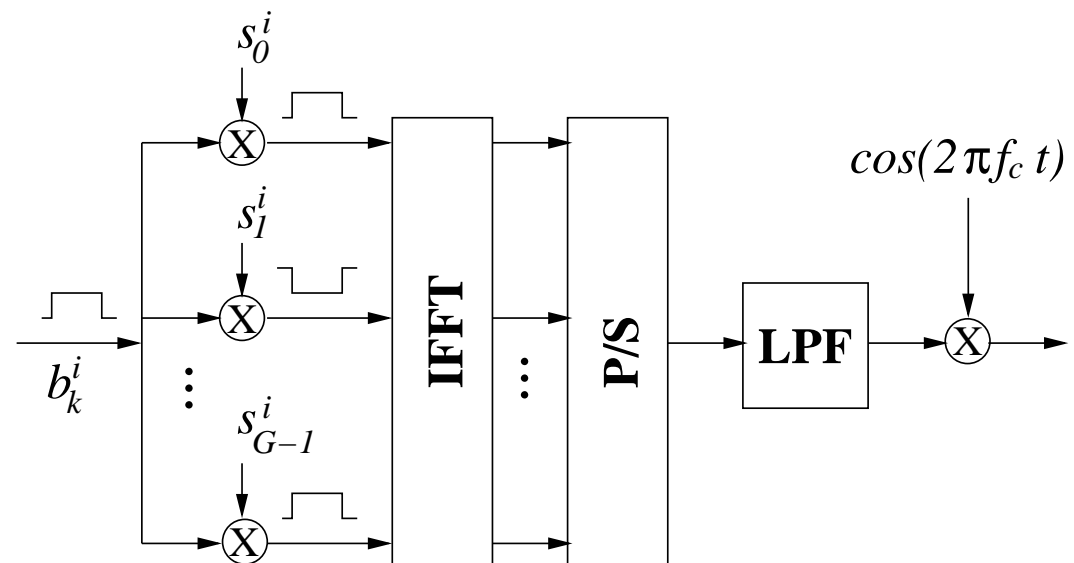
Multi-Carrier CDMA

- Map a different chip of a spreading sequence to an individual OFDM subcarrier
- Each OFDM subcarrier has a data rate identical to original input data rate
- Multicarrier absorbs increased rate due to spreading in a wider frequency band
- MC-CDMA transmitter:

b_k^i : i th user's k th bit

$\mathbf{s}^i = [s_0^i \ s_1^i \ \dots \ s_{G-1}^i]$: i th user's spreading code

Processing gain is G (subcarrier number is also G)



Multi-Carrier CDMA (continue)

- MC-CDMA receiver: Let number of users be I , and $i \in \{0, 1, \dots, I - 1\}$; k th received symbol (sample) for subcarrier l is

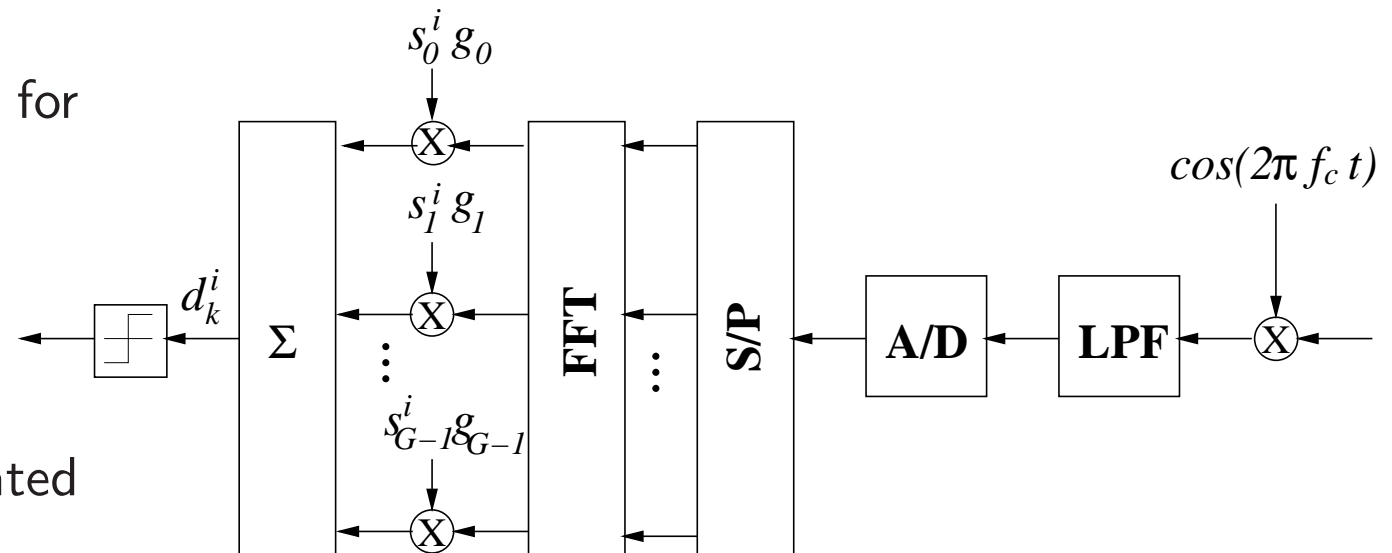
$$r_{k,l} = \sum_{i=1}^{I-1} H_l b_k^i s_l^i + n_{k,l}$$

H_l : frequency response of l th subcarrier (subchannel), $n_{k,l}$: noise sample

- Decision variable d_k^i for estimating b_k^i is

$$d_k^i = \sum_{l=0}^{G-1} s_l^i g_l r_{k,l}$$

g_l : reciprocal of estimated H_l

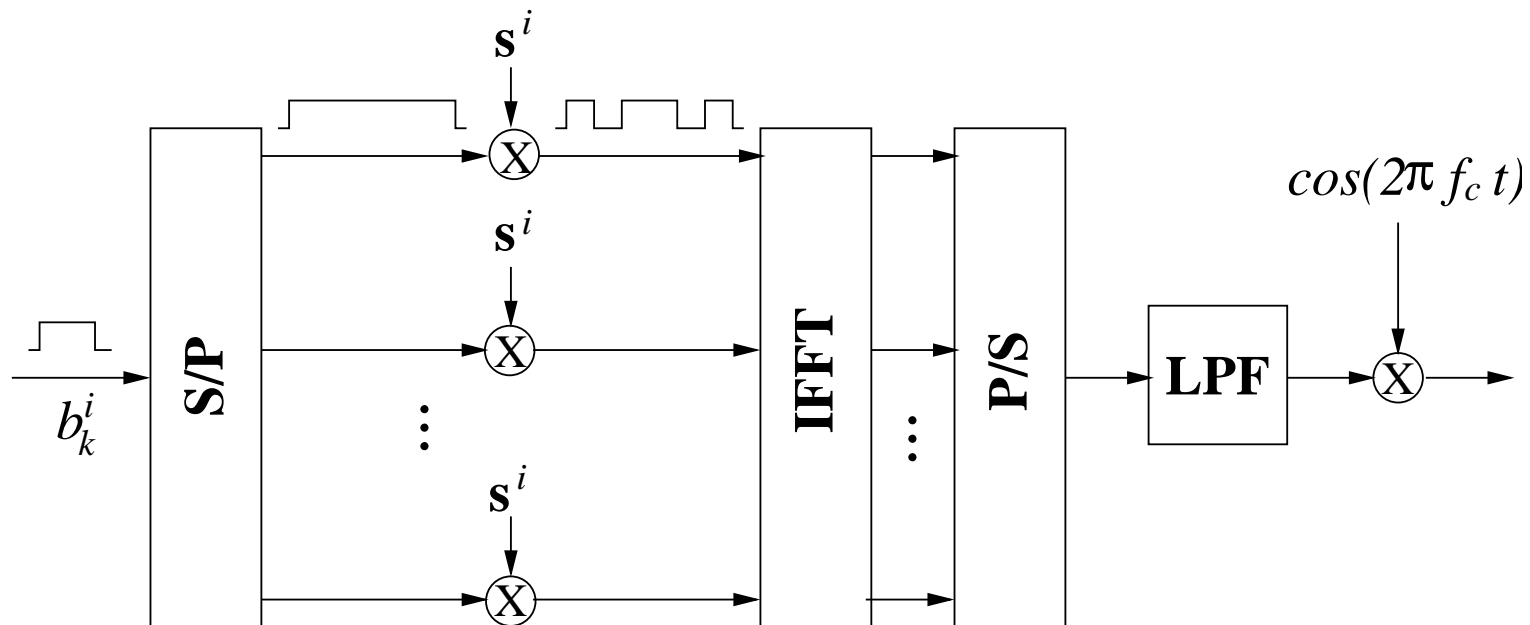


MC-DS-CDMA

- Parallel transmission of DS-CDMA signals using OFDM structure

R_b : input bit rate, N : number of subcarriers, G : processing gain

- MC-DS-CDMA transmitter for user i : information data rate is R_b bps, after S/P rate is $\frac{R_b}{N}$ bps, after spreading rate is $\frac{R_b}{N}G$ bps



Summary

- OFDM implementation with FFT: transmitted complex symbols S_k are frequency samples, and transmitted time signal samples s_m are the IDFT of S_k
- Cyclic extension: the channel with CIR length N_h will spread the transmitted frame from length N to $N + N_h$
 - By employing cyclic extension, the inter-frame interference can be removed by simply discard the first N_h received time samples
 - This also turns linear convolution with channel into circular convolution, essential for DFT to “remove” ISI in the received time-domain signal samples
- Equalisation in OFDM becomes “automatic” in frequency domain: one-tap equalisation, all required are estimating frequency domain channel transfer functions
- MC-CDMA and MC-DS-CDMA

