

SC-FDE

Single-Carrier Block Transmission With Frequency-Domain Equalisation

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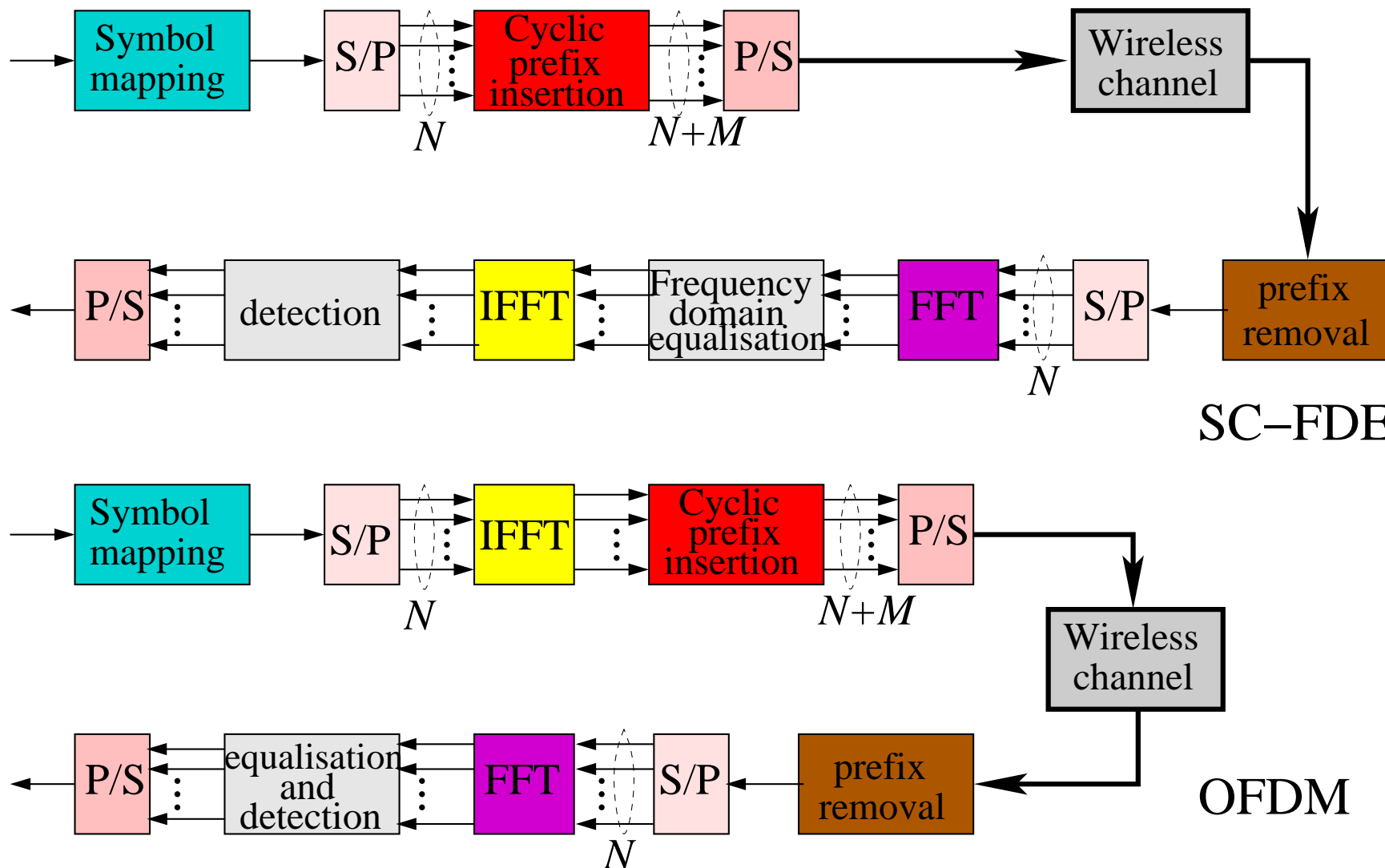
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Motivations

- For 4G and B4G **high-speed broadband applications**, data rate of tens Mbps or higher over wireless channel of typical delay spread in microseconds
 - ISI spanning tens or even hundreds of symbols
 - Nightmare for time-domain equalisation: impractically long equaliser, excessively slow convergence \Rightarrow poor performance
- **Orthogonal frequency division multiplexing**, a multi-carrier technique, offers a viable low-complexity high-performance solution for ISI mitigation
 - High peak-to-average power ratio, intolerance to amplifier nonlinearity, and high sensitivity to carrier frequency offsets
- Alternative solution for ISI mitigation is **single-carrier** modulation with **frequency-domain equalisation**
 - Similar low-complexity and performance, but avoiding OFDM's drawbacks
 - Not as flexible as OFDM in managing bandwidth and energy resources

Quick Comparison

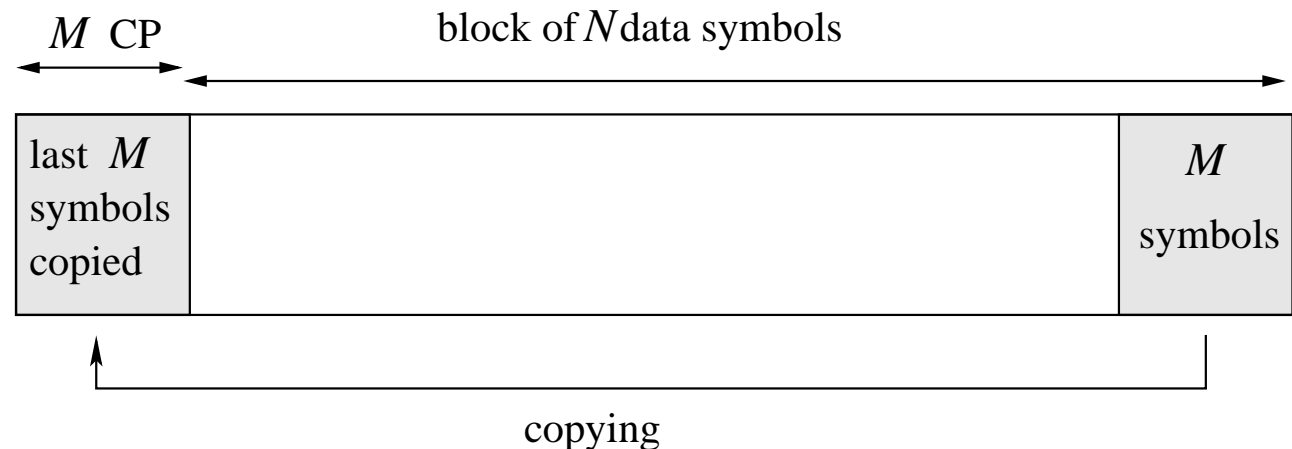


Both transceivers have similar implementation complexity, but SC-FDE transmitter is simpler and hence better for uplink handset

Block Transmission

- Data symbols $\{s_n\}$ are transmitted in **blocks** of N symbols with **cyclic prefix** of length $M \Rightarrow$ block length of $N + M$
 - M is chosen to be larger than channel impulse response length

- Cyclic prefix insertion



- Last M symbols may be **training** symbols, e.g. known PN sequence
- Spot “**difference**”
 - SC-FDE: data symbols are “**time-domain**” quantity and transmitted directly
 - OFDM: data symbols are “**frequency-domain**” quantity (and are IDFT into “time-domain” for transmission)

OFDM

- **Cyclic prefix** at beginning of each block has two main functions
 - Prevent contamination of a block by ISI from previous block, by simply dropping first M time-domain samples of received block of length $N + M$
 - Make received block **periodic with period** N , essential for DFT to lead single-tap equalisation in frequency domain
- Received block has **cyclicity property**

$$r_m = \sum_{k=0}^{N_h-1} h_k s_{m-(k \bmod N)}, \quad 0 \leq m \leq N - 1$$

where $N_h \leq M$ is length of CIR, and $h_k, 0 \leq k \leq N_h - 1$, CIR taps

- Process received block $\{r_m\}_{m=0}^{N-1}$ by DFT:

$$R_l = \sum_{m=0}^{N-1} r_m e^{-j\frac{2\pi lm}{N}} = H_l \cdot S_l + V_l, \quad 0 \leq l \leq N - 1$$

CFR $\{H_l\}_{l=0}^{N-1}$ is N -point DFT of CIR $\{h_k\}_{k=0}^{N_h-1}$, V_l is noise term

- One-tap equalisation

$$Y_l = W_l \cdot R_l, \quad 0 \leq l \leq N - 1$$

$\{Y_l\}_{l=0}^{N-1}$ provides sufficient statistics for **detection** of transmitted data symbols $\{S_l\}_{l=0}^{N-1}$



Cyclic Prefix and Cyclicity

- Transmitted time-domain OFDM signal: $\mathbf{s}_{N+M} = [s_{N-1} s_{N-2} \cdots s_{N-M} \cdots s_1 s_0 | s_{-1} s_{-2} \cdots s_{-M}]^T$
 - N data symbols $\mathbf{s}_N = [s_{N-1} \cdots s_1 s_0]^T$, and
 - M -length **cyclic prefix** $[s_{-1} s_{-2} \cdots s_{-M}]^T = [s_{N-1} s_{N-2} \cdots s_{N-M}]^T$
 - $M \geq$ CIR length, and for convenience, let CIR be: $[h_0 h_1 \cdots h_M]^T$
- Received block of length $N + M$: $\mathbf{r}_{N+M} = [r_{N-1} \cdots r_1 r_0 | r_{-1} r_{-2} \cdots r_{-M}]^T$
 - Dropping $[r_{-1} r_{-2} \cdots r_{-M}]^T$ removes ISI from **previous block**
 - N -length received block $\mathbf{r}_N = [r_{N-1} \cdots r_1 r_0]^T$ has **cyclicity property**
- **Linear convolution** $\mathbf{r}_N = \mathbf{H}_L \mathbf{s}_{N+M}$, $\mathbf{H}_L : N \times (N + M)$

$$\begin{aligned}
 r_{N-1} &= h_0 s_{N-1} + h_1 s_{N-2} + \cdots + h_M s_{N-M+1} \\
 &\vdots \\
 r_M &= h_0 s_M + h_1 s_{M-1} + \cdots + h_M s_0 \\
 r_{M-1} &= h_0 s_{M-1} + h_1 s_{M-2} + \cdots + h_{M-1} s_0 + s_M s_{-1} \\
 &\vdots \\
 r_1 &= h_0 s_1 + h_1 s_0 + h_2 s_{-1} + \cdots + h_M s_{-M+1} \\
 r_0 &= h_0 s_0 + s_1 s_{-1} + \cdots + h_M s_{-M}
 \end{aligned}$$

Block Processing in SC-FDE

- **Cyclic prefix** at beginning of each block has two main functions
 - Prevent contamination of a block by intersymbol interference from previous block, by simply dropping first M samples of received block of length $N + M$
 - Make received block **periodic with period** N , essential for DFT to lead single-tap equalisation in frequency domain
- Received block has **cyclicity property**

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CFR $\{H_l\}_{l=0}^{N-1}$ is N -point DFT of CIR $\{h_k\}_{k=0}^{N_h-1}$, V_l is noise term

One-Tap Equalisation

- In **frequency-domain**, equalisation can be achieved by **one-tap** linear equaliser

$$Y_l = W_l \cdot R_l, \quad 0 \leq l \leq N - 1$$

- **Zero-forcing:**

$$W_l = \frac{H_l^*}{|H_l|^2}, \quad 0 \leq l \leq N - 1$$

- **Minimum mean square error:**

$$W_l = \frac{H_l^*}{|H_l|^2 + \frac{\sigma_v^2}{\sigma_s^2}}, \quad 0 \leq l \leq N - 1$$

where σ_v^2 is noise power and $\sigma_s^2 = E[|s_n|^2]$ signal power

- SC-FDE with decision feedback equaliser

- $\{y_n\}_{n=0}^{N-1}$, IDFT of $\{Y_l\}_{l=0}^{N-1}$

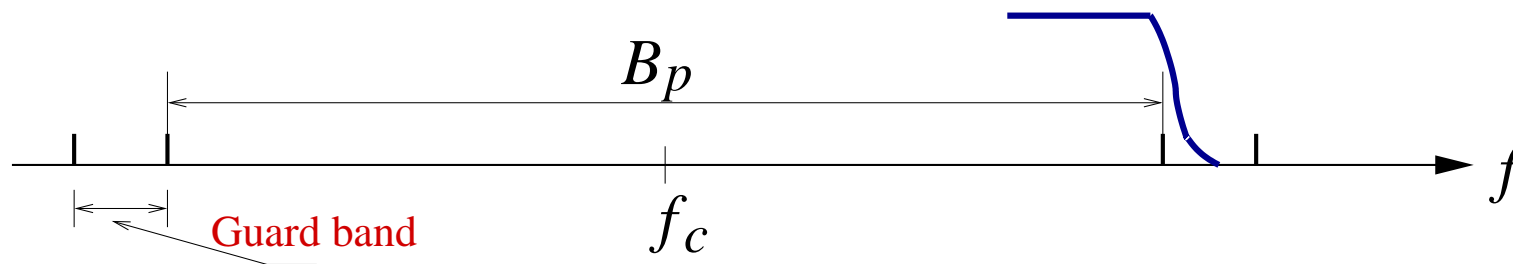
$$y_n = \frac{1}{N} \sum_{l=0}^{N-1} Y_l e^{j\frac{2\pi ln}{N}}, \quad 0 \leq n \leq N - 1$$

provides sufficient statistics for **detection** of transmitted data symbols $\{s_n\}_{n=0}^{N-1}$

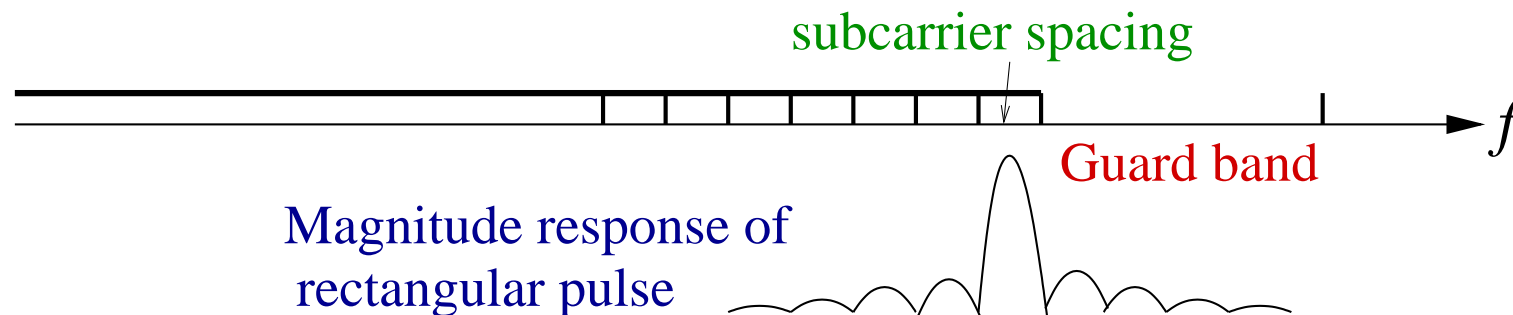


Quick Comparison Again

- OFDM and SC-FDE have similar low-complexity and performance
 - OFDM has high peak to average power ratio, and sensitive to carrier phase noise
 - SC-FDE requires much tighter pulse shaping with associated higher complexity
- For single-carrier systems with wideband signals, pulse shaping must be very tight, say $\gamma \leq 0.1$



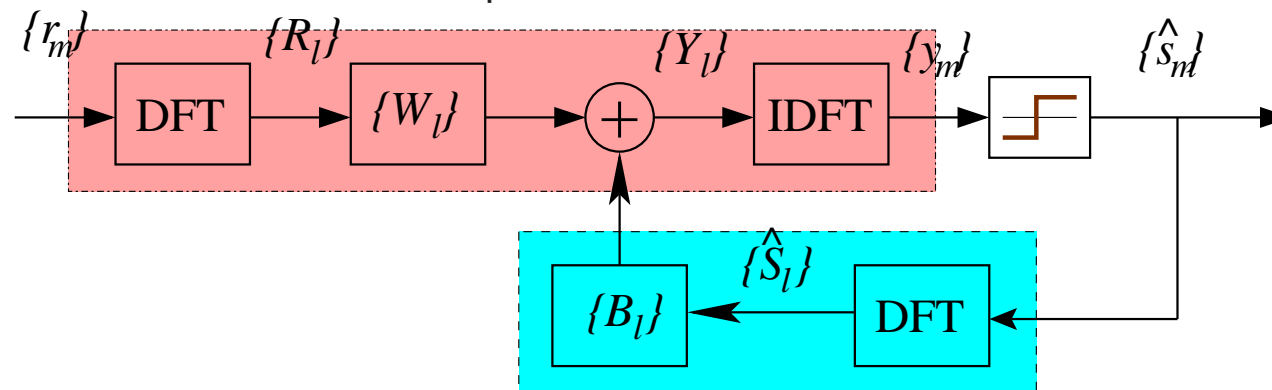
- Channel bandwidth B_p is very large, 0.1 of which may be significant compared with guard band
- For OFDM, subcarrier spacing is small in comparison to guard band



- In fact, often no pulse shaping is required for multi-carrier OFDM systems

FD Decision Feedback Equalizer

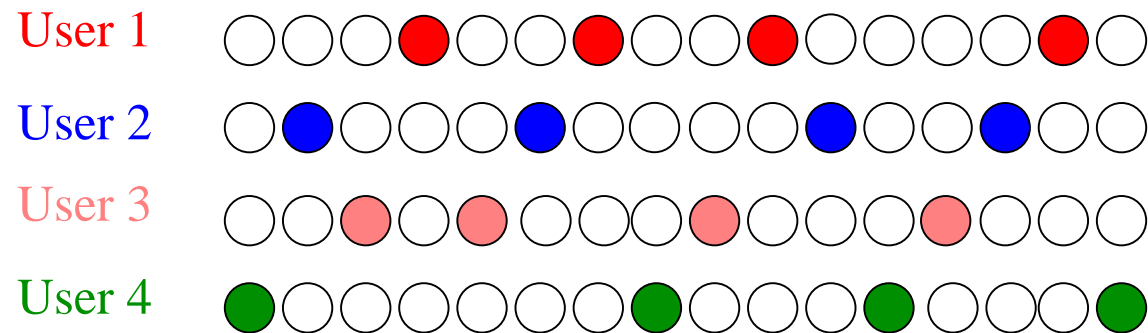
- SC-FDE and OFDM are basically linear equalisers
 - Suffer from usual noise enhancement
 - Performance degradation over deep frequency-selective fading channels
- Frequency-domain decision feedback equaliser: an iterative structure



- At k th iteration, feedforward equaliser $\{W_l^{(k)}\}$, and feedback equaliser $\{B_l^{(k)}\}$
 - First iteration is usual FD linear equaliser
 - Many ways of updating $\{W_l^{(k)}\}$ and $\{B_l^{(k)}\}$
1. N. Benvenuto and S. Tomasin, "Iterative design and detection of a DFE in the frequency domain," *IEEE Trans. Communications*, vol.53, no.11, pp.1867–1875, Nov. 2005
 2. C. Zhang, Z. Wang, C. Pan, S. Chen and L. Hanzo, "Low-complexity iterative frequency domain decision feedback equalization," *IEEE Trans. Vehicular Technology*, vol.60, no.3, pp.1295–1301, 2011

OFDMA / SC-FDMA

- Multi-carrier system to support multi users: orthogonal frequency division multiple access **OFDMA**
 - N subcarriers to support K users
 - Carrier assignment scheme **CAS** assigns N/K subcarriers to each user



4 users 16 subcarriers

- Each user assigns its data symbols to subcarriers it occupies, and assigns zero to unoccupied subcarriers
- Single-carrier system to support multi users: frequency division multiple access **SC-FDMA**



Total bandwidth

- For MC systems, such as OFDMA, flexible for resource allocation, such as **power allocation** to subcarriers

Summary

- Single-carrier block transmission with frequency-domain equalisation
 - Alternative to OFDM, similarity with OFDM
 - Advantages/disadvantage comparison with OFDM
- Cyclic prefix and Cyclicity: remove inter-block interference and make received block periodic
 - Frequency-domain one-tap equalisation
- Frequency-domain decision feedback equalisation
- Support multi users: OFDMA and SC-FDMA

