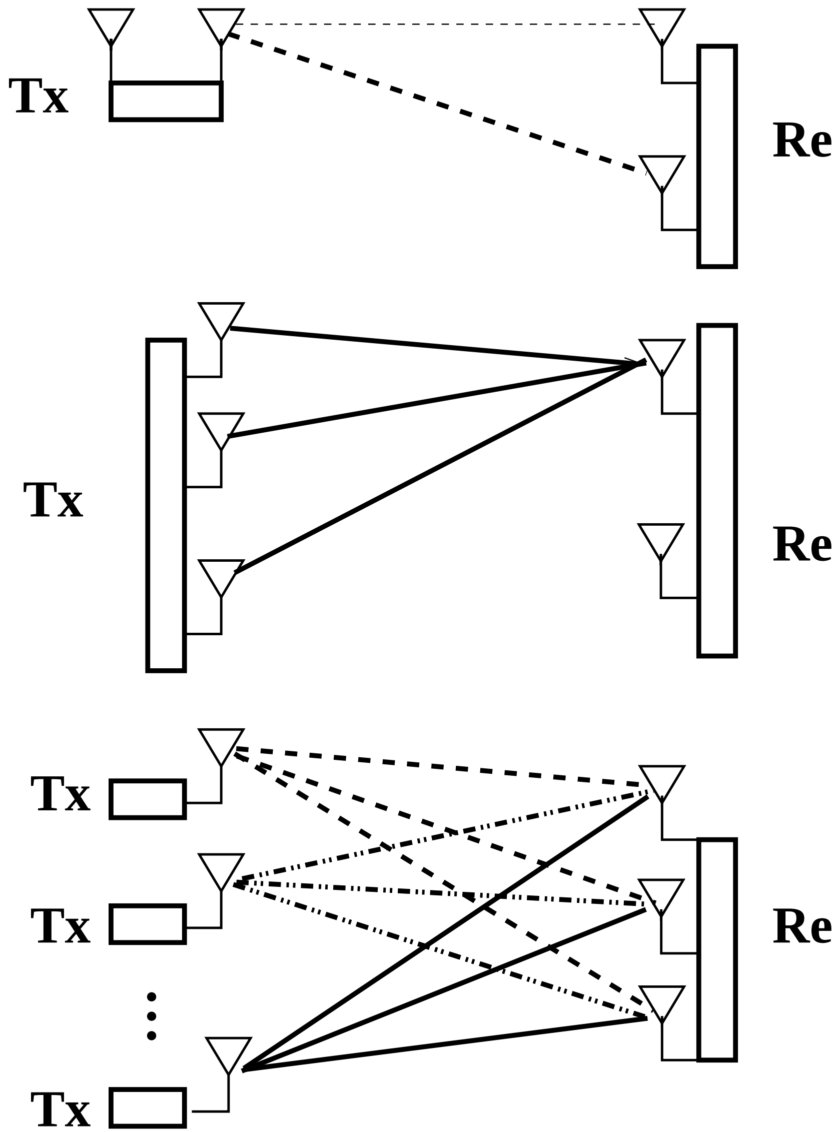
MIMO Introduction

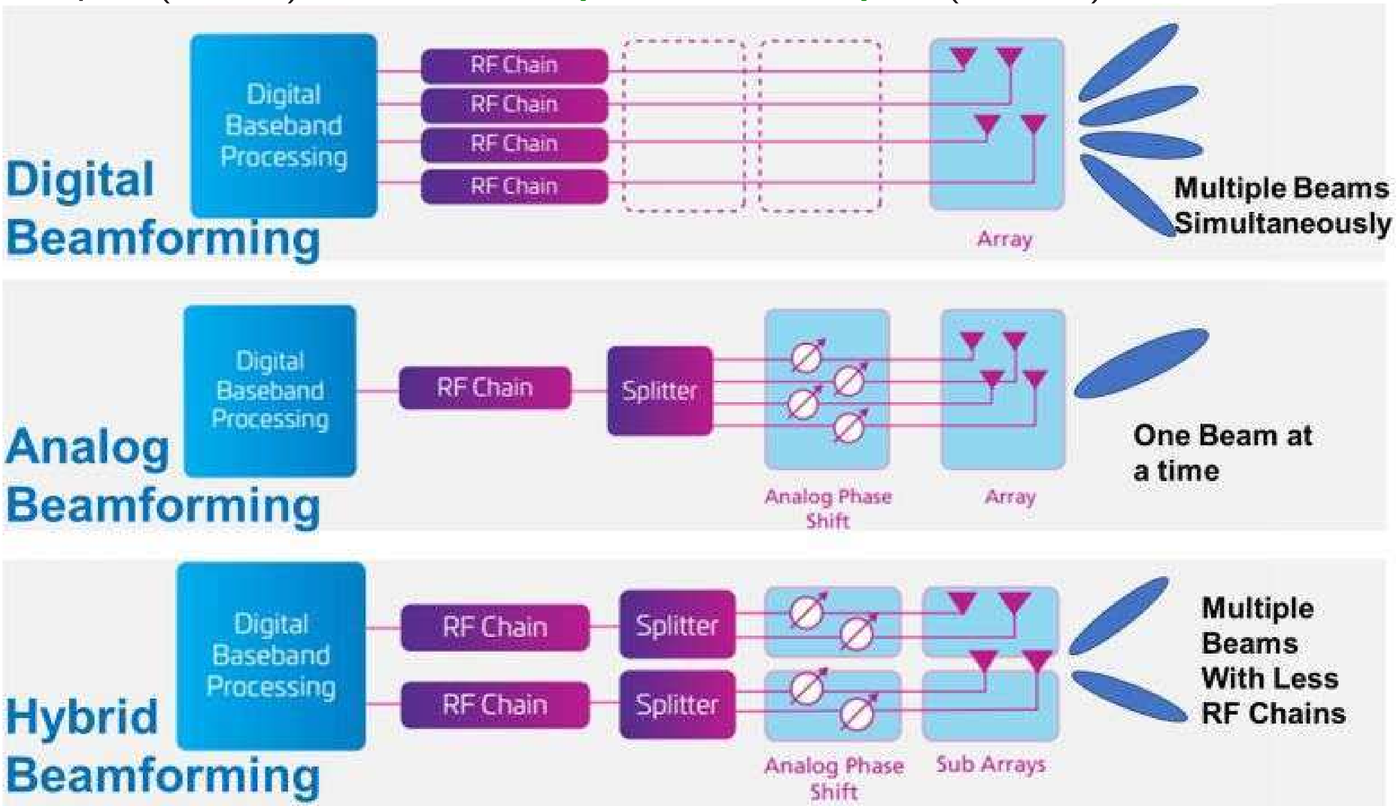
- MIMO are for a) **diversity** gain, and/or b) **multiplexing** gain

1. Create **diversity** for combating fading
   - With sufficient antenna spacing (10 wavelengths), each antenna experiences independent fading → When one signal is in its deep fade, others are unlikely the same
2. Increase **throughput**
   - Data stream is first S/P, each sub-sequence mapped to an antenna → This creates many "digital pipes" to support higher rate
3. When multiplexing gain is not used to increase throughput, it can support **multiple users**
   - With multiple receive antennas, each spatially separated user has a unique set of CIRs seen at receiver → This enables SDMA
4. **Beamforming**, antenna spacing half of wavelength
   - Improve signal quality or support SDMA



MIMO/Beamforming

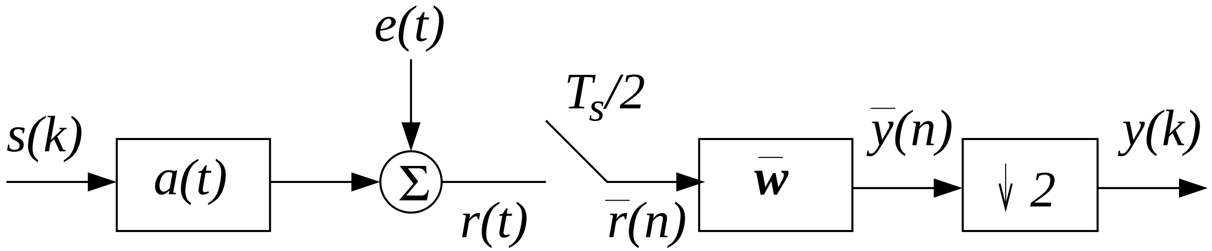
- MIMO classification: single-input single-output (SISO), single-input multi-output (SIMO), multi-input single-output (MISO), and **multi-input multi-output** (MIMO)



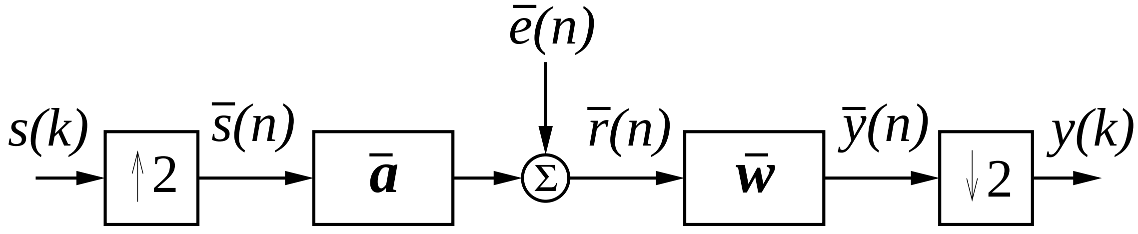
- Beamforming (transmit/receive) classification
  - **Digital** beamforming: baseband, one RF chain for each antenna, multi data streams/users
  - **Analog** beamforming: RF band, one RF chain for whole antenna array, one data stream
  - **Hybrid** beamforming, both base band/RF band, multi data streams/users

Fractional-Spaced Sampling

- Single-user (SISO) system with **fractional-space** sampling
  - At each symbol period take more than one sample $\Rightarrow$ multiple (symbol-rate) subchannel models
  - **Advantage**: robust to carrier/timing recovery errors, perfect reconstruction with finite equalizer
  - **Disadvantage**: noise sample no longer white
- **Baseband** model with $T_s/2$-spaced receiver:
  - Take two samples during each symbol period
  - $T_s/2$-spaced equaliser: $\bar{y}(n) = \bar{\mathbf{w}}^H \bar{\mathbf{r}}(n)$
    with $\bar{\mathbf{r}}(n) = [\bar{r}(n)\ \bar{r}(n-1) \cdots \bar{r}(n-2m+1)]^T$ and $\bar{\mathbf{w}} = [\bar{w}_0\ \bar{w}_1 \cdots \bar{w}_{2m-1}]^T$
  - $\bar{y}(n)$ is decimated by a factor of 2 to get T_s-spaced output $y(k)$



- **Multirate** model with $T_s/2$-spaced equaliser
  - k indicates T_s-spaced quantities and n indicates $T_s/2$-spaced quantities



Multirate/Multichannel Model

- $T_s/2$-spaced sequence $\{\bar{s}(n)\}$ is zero-filled transmitted symbol sequence $\{s(k)\}$ defined by

$$\bar{s}(n) = \begin{cases} s(n/2), & \text{for even } n \\ 0, & \text{for odd } n \end{cases}$$

- Received $T_s/2$-spaced signal sample is

$$\bar{r}(n) = \sum_{i=0}^{2N_c-1} \bar{a}_i \bar{s}(n-i) + \bar{e}(n)$$

- $T_s/2$-spaced complex-valued channel impulse response (CIR) is given by

$$\bar{\mathbf{a}} = [\bar{a}_0 \ \bar{a}_1 \ \bar{a}_2 \ \bar{a}_3 \cdots \bar{a}_{2N_c-1}]^T$$

- Sampling at twice of symbol rate $\Rightarrow$ Two symbol-spaced models: Odd sample model and even sample model, i.e., **multichannel** model
- Take even and odd samples of noise and Rx signal

$$e^{\text{e}}(k) = \bar{e}(2n), \ e^{\text{o}}(k) = \bar{e}(2n+1), \ r^{\text{e}}(k) = \bar{r}(2n), \ r^{\text{o}}(k) = \bar{r}(2n+1)$$

- Define even and odd channels and equalisers (all are symbol-rate sequences) as:

$$\begin{aligned} \bar{\mathbf{a}}^{\text{e}} &= [\bar{a}_0 \ \bar{a}_2 \cdots \bar{a}_{2N_c-2}]^T, \ \bar{\mathbf{a}}^{\text{o}} = [\bar{a}_1 \ \bar{a}_3 \cdots \bar{a}_{2N_c-1}]^T, \\ \bar{\mathbf{w}}^{\text{e}} &= [\bar{w}_0 \ \bar{w}_2 \cdots \bar{w}_{2m-2}]^T, \ \bar{\mathbf{w}}^{\text{o}} = [\bar{w}_1 \ \bar{w}_3 \cdots \bar{w}_{2m-1}]^T \end{aligned}$$

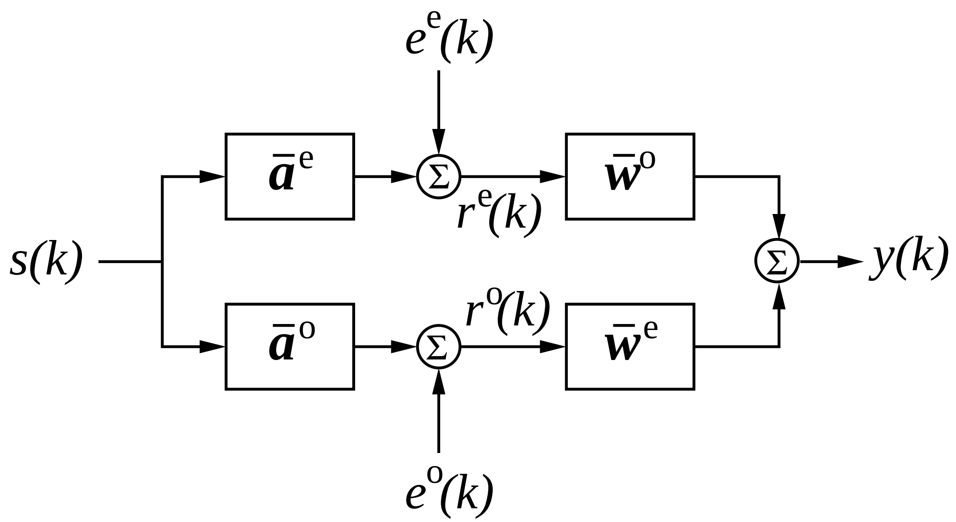
Multichannel Model (continue)

- Symbol-rate output

$$y(k) = \sum_{i=0}^{2m-1} w_i^* r(k-i) = \mathbf{w}^H \mathbf{r}(k)$$

with $\mathbf{w} = \left[(\bar{\mathbf{w}}^{\text{o}})^T \;\; (\bar{\mathbf{w}}^{\text{e}})^T \right]^T$ and

$\mathbf{r}(k) = \left[(\mathbf{r}^{\text{e}}(k))^T \;\; (\mathbf{r}^{\text{o}}(k))^T \right]^T$



- In general, T_s/K-spaced sampling will result in K channel models
- As we have symbol-rate model $y(k) = \mathbf{w}^H \mathbf{r}(k)$, all equalisation results apply
- **Blind equalisation example**: consists of a 22-tap $T_s/2$ channel and a 26-tap $T_s/2$ equaliser with 256-QAM and SNR$=60$ dB
- In simulation, an estimated MSE based on a separate block of data and the maximum distortion measure defined by

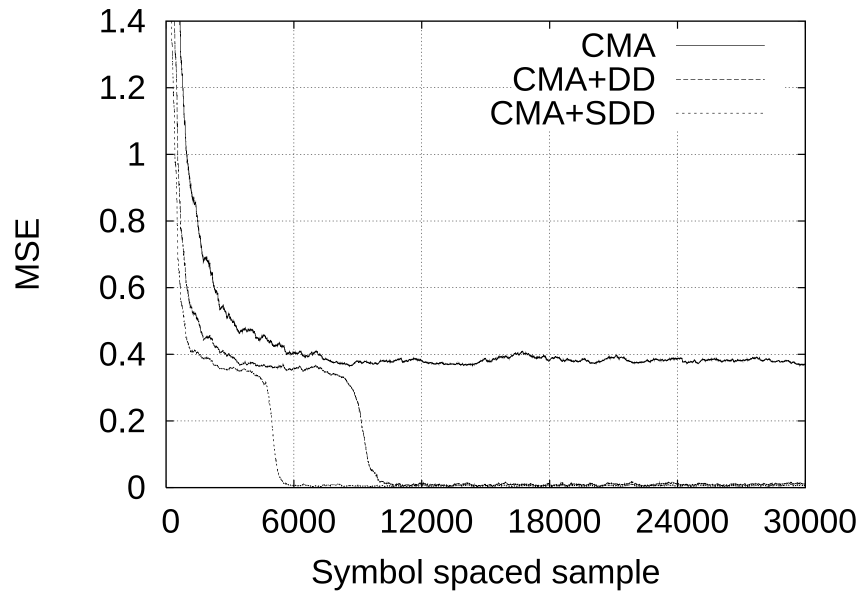
$$\text{MD} = \frac{\sum_{i=0}^{n_{\text{tot}}} |f_i| - |f_{i\max}|}{|f_{i\max}|}$$

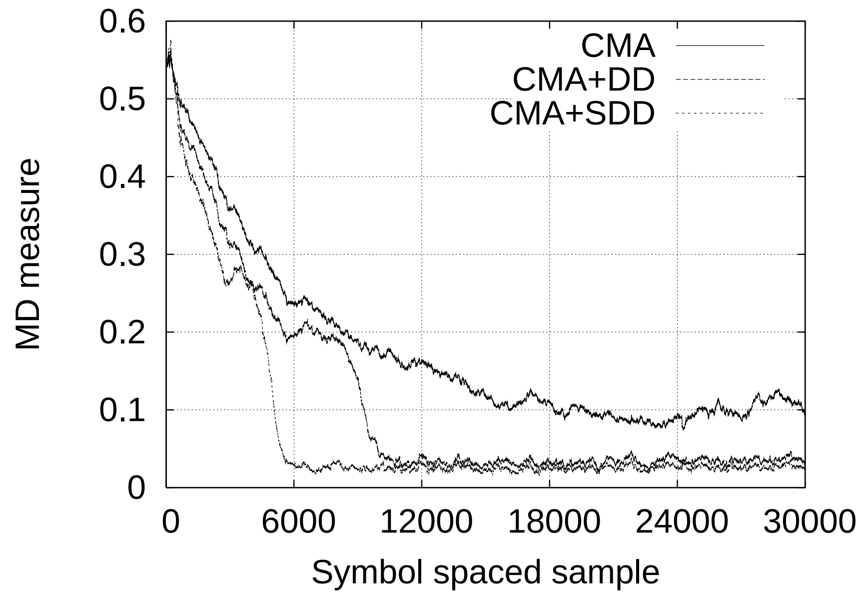
are used to assess convergence rate, where

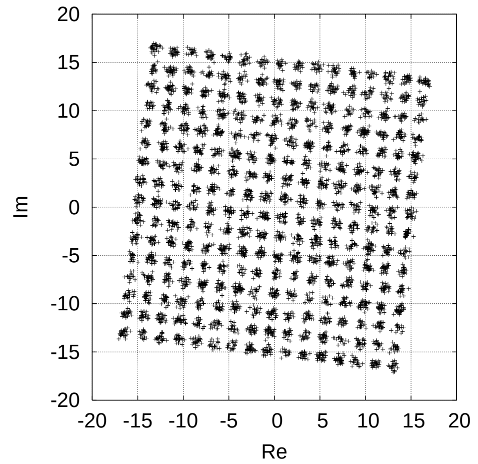
$$\mathbf{f} = [f_0 \; f_1 \cdots f_{n_{\text{tot}}}]^T = (\bar{\mathbf{w}}^{\text{o}})^* \star \bar{\mathbf{a}}^{\text{e}} + (\bar{\mathbf{w}}^{\text{e}})^* \star \bar{\mathbf{a}}^{\text{o}}$$

is the combined impulse response of the channel and equaliser, $f_{i\max} = \max\{f_i, \; 0 \le i \le n_{\text{tot}}\}$

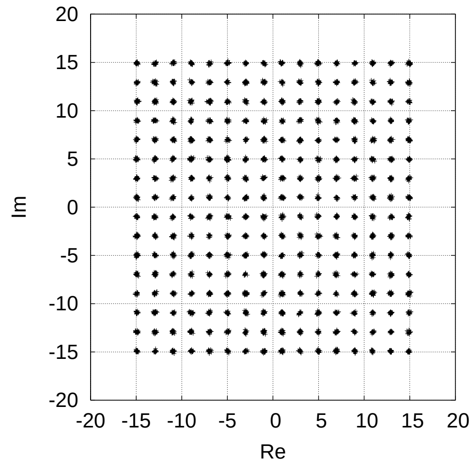
Simulation Results



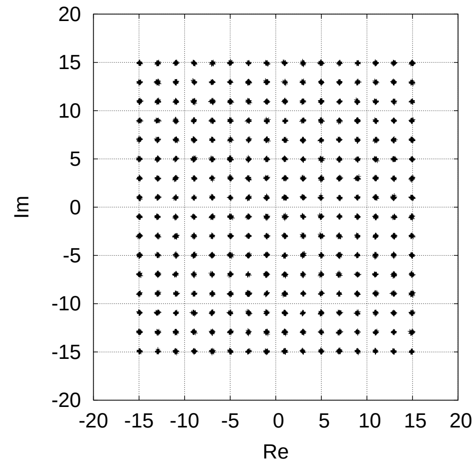




(a) CMA



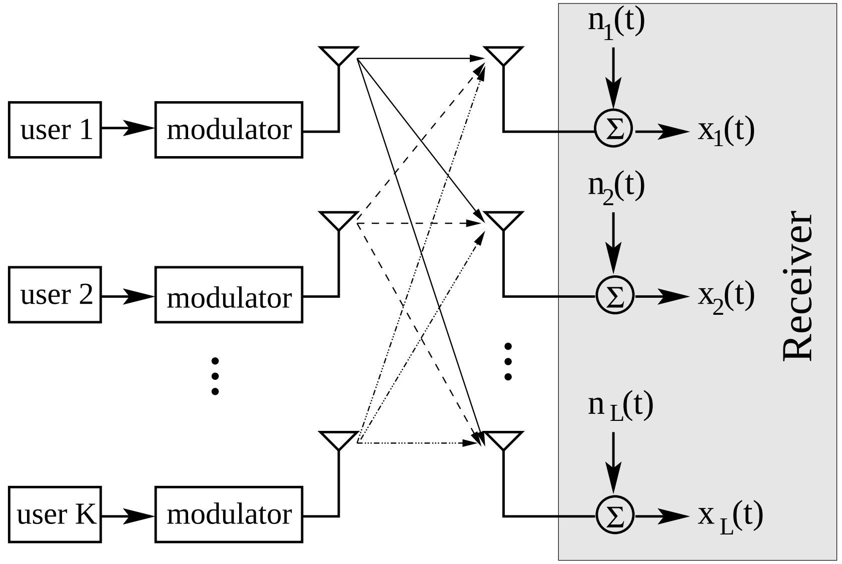
(b) CMA+DD



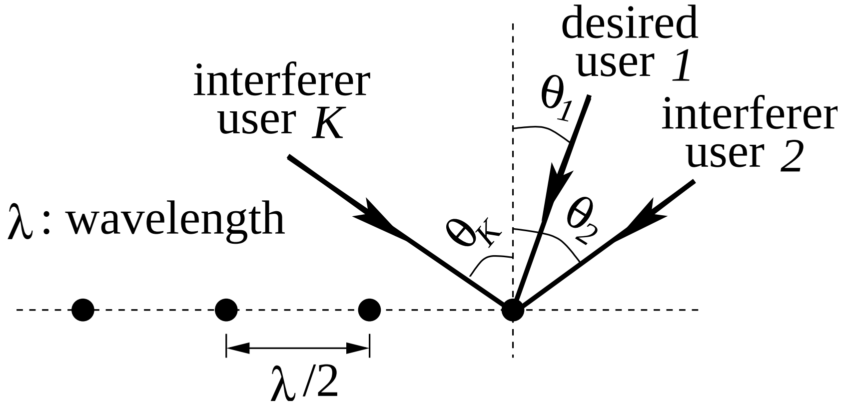
(c) CMA+SDD

Digital Beamforming Assisted Receiver

- MIMO based **SDMA** system:
  - Uplink, BS **digital beamforming** receiver to implement **multiuser detection**
  - Assume L receiver antennas supporting K single-antenna users
  - Narrowband channels with $m_i(k) = A_i b_i(k)$, A_i: channel coefficient for user i and $b_i(k)$: kth symbol of user i
  - Symbol-rate sampling, BS detects user i data for $1 \leq i \leq K$



- Uniformly spaced linear antenna array:
  - $t_l(\theta_i)$ be relative time delay at array element l for user i
  - θ_i angle of arrival for user i and carrier $\omega = 2\pi f_c$



$$x_l(k) = \sum_{i=1}^{K} m_i(k) \exp\left(j\omega t_l(\theta_i)\right) + n_l(k) = \bar{x}_l(k) + n_l(k), \;\; 1 \leq l \leq L$$

Beamforming Assisted Receiver (continue)

- System model: Antenna array output $\mathbf{x}(k) = [x_1(k)\ x_2(k) \cdots x_L(k)]^T$ is expressed as

$$\mathbf{x}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k) = \mathbf{H}\mathbf{b}(k) + \mathbf{n}(k)$$

  where $\mathbf{n}(k) = [n_1(k)\ n_2(k) \cdots n_L(k)]^T$ has a covariance matrix of $E[\mathbf{n}(k)\mathbf{n}^H(k)] = 2\sigma_n^2\mathbf{I}_L$ with $\mathbf{I}_L$ representing the $L \times L$ identity matrix, uplink channel matrix $\mathbf{H}$ is given by

$$\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2 \cdots \mathbf{h}_K] = [A_1\mathbf{s}_1\ A_2\mathbf{s}_2 \cdots A_K\mathbf{s}_K]$$

  steering vector for source i is

$$\mathbf{s}_i = [\exp(j\omega t_1(\theta_i))\ \ \exp(j\omega t_2(\theta_i)) \cdots \exp(j\omega t_L(\theta_i))]^T$$

  and transmitted symbol vector $\mathbf{b}(k) = [b_1(k)\ b_2(k) \cdots b_K(k)]^T$

- Given channel matrix $\mathbf{H}$, **maximum likelihood** MUD

$$\widehat{\mathbf{b}}_{\mathrm{ML}}(k) = \arg\min_{\mathbf{b}\in\mathcal{B}} \|\mathbf{x}(k) - \mathbf{H}\mathbf{b}\|$$

  Feasible solution set $\mathcal{B} = \{\mathbf{b}_1, \cdots, \mathbf{b}_{N_f}\}$, with $N_f = K^M$ for M-QAM

- **Linear** beanforming/MUD: BS beamformer output vector $\mathbf{y}(k) = [y_1(k) \cdots y_K(k)]^T$

$$\widehat{\mathbf{b}}(k) = \mathbf{y}(k) = \mathbf{W}^H\mathbf{x}(k)$$

  Beamformer complex-valued weight matrix $\mathbf{W} = [\mathbf{w}_1 \cdots \mathbf{w}_K]$ with $\mathbf{w}_i = [w_{1,i} \cdots w_{L,i}]^T$

Linear Beamforming/MUD

- **Zero-forcing** (ZF) beamforming/MUD

$$\mathbf{W}_{\text{ZF}}^H = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H \text{ or } \mathbf{W}_{\text{ZF}} = \mathbf{H}\left(\mathbf{H}^H\mathbf{H}\right)^{-1}$$

- **Minimum mean square error** (MMSE) beamforming/MUD

$$\mathbf{W}_{\text{MMSE}}^H = \left(\mathbf{H}^H\mathbf{H} + \frac{2\sigma_n^2}{\sigma_s^2}\mathbf{I}_K\right)^{-1}\mathbf{H}^H \text{ or } \mathbf{W}_{\text{MMSE}} = \mathbf{H}\left(\mathbf{H}^H\mathbf{H} + \frac{2\sigma_n^2}{\sigma_s^2}\mathbf{I}_K\right)^{-1}$$

  with $E[|b_i(k)|^2] = \sigma_s^2$. For user i,

$$\mathbf{w}_{i,\text{MMSE}} = \left(\mathbf{H}\mathbf{H}^H + \frac{2\sigma_n^2}{\sigma_s^2}\mathbf{I}_L\right)^{-1}\mathbf{h}_i$$

  MMSE solution can be implemented adaptively using the LMS or RLS algorithm

- Further consider for user i, beamformer's output is given by

$$y_i(k) = \mathbf{w}_i^H\mathbf{x}(k) = \mathbf{w}_i^H\bar{\mathbf{x}}(k) + \mathbf{w}_i^H\mathbf{n}(k) = \bar{y}_i(k) + e_i(k)$$

  where $e_i(k)$ is Gaussian distributed having a zero mean and $E[|e_i(k)|^2] = 2\sigma_n^2\mathbf{w}_i^H\mathbf{w}_i$

- Assume M-QAM modulation and define combined impulse response of beamformer and system as

$$\mathbf{w}_i^H\mathbf{H} = \mathbf{w}_i^H[\mathbf{h}_1 \ \mathbf{h}_2 \cdots \mathbf{h}_K] = [c_{i,1} \ c_{i,2} \cdots c_{i,K}]$$

Linear Beamforming/MUD (continue)

- **Minimum symbol error rate**: beamformer's output can also be expressed as

$$y_i(k) = c_{i,i} b_i(k) + \sum_{1 \le l \le K, l \ne i} c_{i,l} b_l(k) + e_i(k)$$

- Define decision variable as $d_i(k) = d_{i_R}(k) + \mathrm{j} d_{i_I}(k) = \frac{y_i(k)}{c_{i,i}}$, then symbol decision $\widehat{b}_i(k) = \widehat{b}_{i_R}(k) + \mathrm{j}\widehat{b}_{i_I}(k)$ is given by

$$\widehat{b}_{i_R}(k) = \begin{cases} u_1, & \text{if } d_{i_R}(k) \le u_1 + 1 \\ u_l, & \text{if } u_l - 1 < d_{i_R}(k) \le u_l + 1 \text{ for } 2 \le l \le \sqrt{M} - 1 \\ u_{\sqrt{M}}, & \text{if } d_{i_R}(k) > u_{\sqrt{M}} - 1 \end{cases}$$

$$\widehat{b}_{i_I}(k) = \begin{cases} u_1, & \text{if } d_{i_I}(k) \le u_1 + 1 \\ u_q, & \text{if } u_q - 1 < d_{i_I}(k) \le u_q + 1 \text{ for } 2 \le q \le \sqrt{M} - 1 \\ u_{\sqrt{M}}, & \text{if } d_{i_I}(k) > u_{\sqrt{M}} - 1 \end{cases}$$

  where M-QAM symbol set is defined as $\{u_l + \mathrm{j} u_q, \ 1 \le l, q \le \sqrt{M}\}$

- Given $\mathbf{H}$, we have set of $\bar{y}_i$, which is function of $\mathbf{w}_i$, and symbol error rate is **mixture** of Q functions

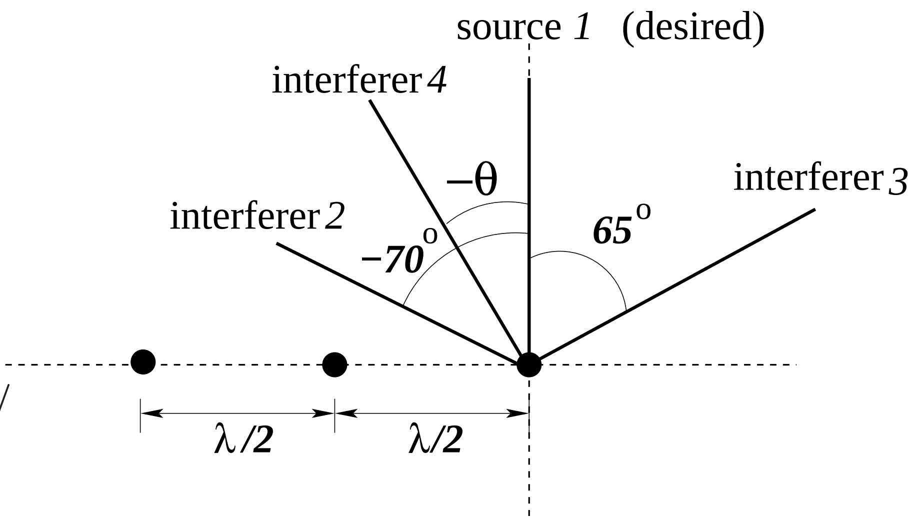
- Minimize this symbol error rate leads to **MSER** solution $\mathbf{w}_{i\text{MSER}}$

Adaptive MBER Beamforming Solution

- We can derive minimum symbol error rate solution $\mathbf{w}_{i,\mathrm{MSER}}$ for general QAM
  - Unlike MMSE solution, there is no closed-form solution for $\mathbf{w}_{i,\mathrm{MSER}}$
  - Gradient optimisation must be used
- MSER solution can be implemented adaptively using the LSER algorithm
- For details see:

  S. Chen, H.-Q. Du and L. Hanzo, "Adaptive minimum symbol error rate beamforming assisted receiver for quadrature amplitude modulation systems," in *VTC2006-Spring* (Melbourne, Australia), May 7-10, 2006

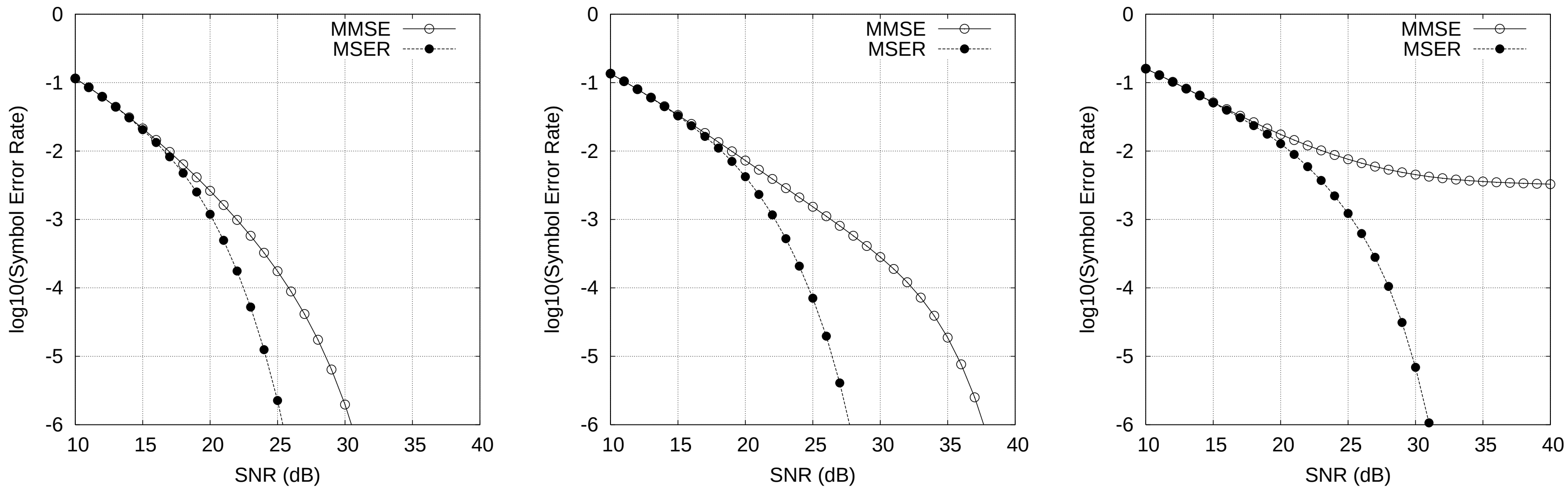
  PDF copy can be download from: https://www.southampton.ac.uk/∼sqc/ELEC6214/



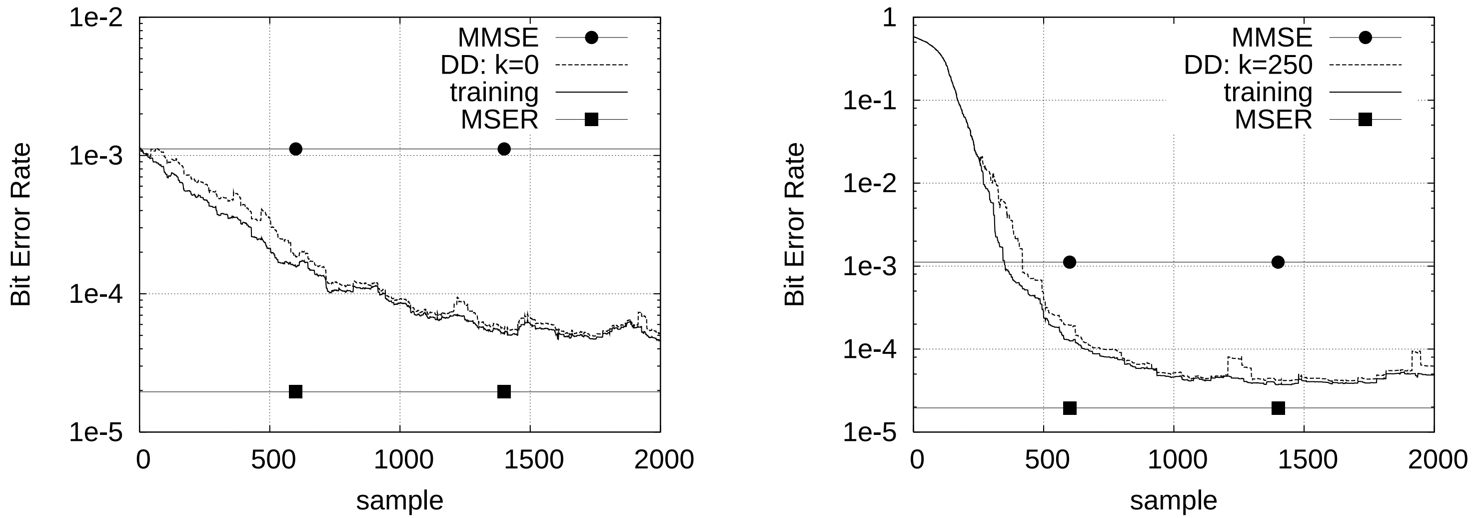
- Example: 16QAM, 4 users, 3-element antenna array

Beamforming Results

(a) $\theta = 32^\circ$ (b) $\theta = 30^\circ$ (c) $\theta = 28^\circ$

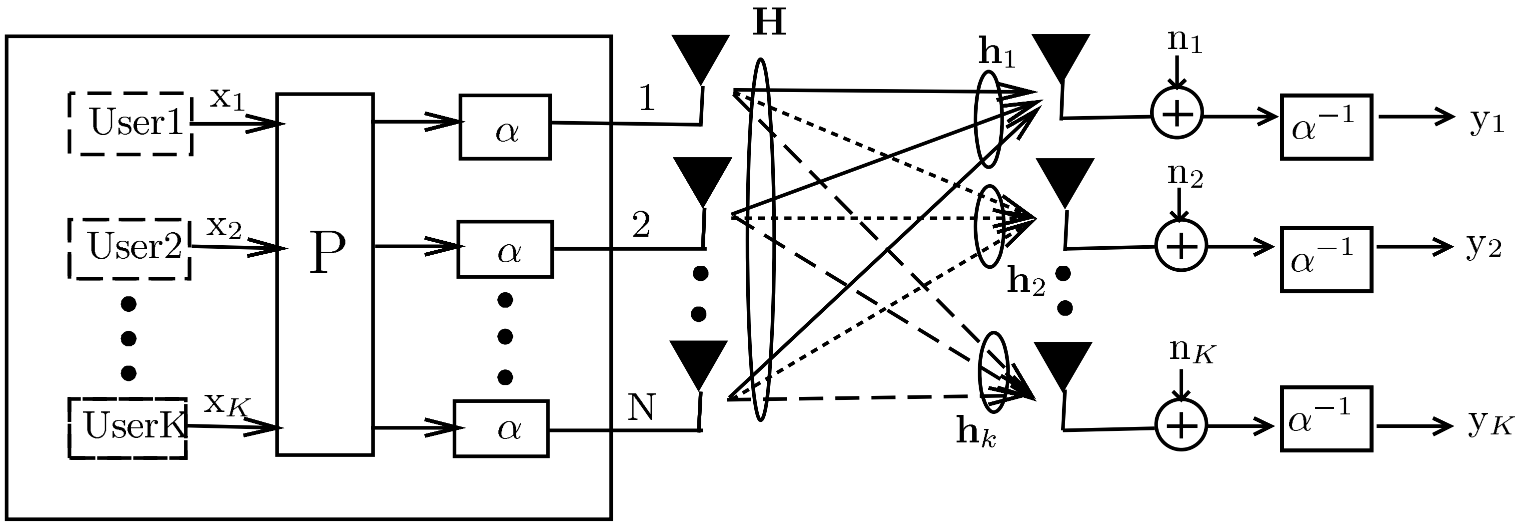


$\theta = 30^\circ$ and SNR$= 26$ dB, LSER



Transmit Beamforming/Precoding

- BS employs N transmit antennas to communicate with K single-receive-antenna MSs, i.e. downlink



- MSs unable to perform multiuser detection or receive beamforming
  - BS does **multiuser transmission**/**transmit beamforming**/**precoding** to combat MUI
- Received signal vector of K MSs, $\mathbf{y}(k) = [y_1(k)\ y_2(k) \cdots y_K(k)]^T$, is given by

$$\mathbf{y}(k) = \mathbf{H}^T \mathbf{P} \mathbf{x}(k) + \alpha^{-1} \mathbf{n}(k)$$

  - Transmit vector $\mathbf{x}(k) = [x_1(k)\ x_2(k) \cdots x_K(k)]^T$
  - Downlink channel matrix $\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2 \cdots \mathbf{h}_K] \in \mathbb{C}^{N \times K}$
  - Precoding matrix $\mathbf{P} = [\mathbf{p}_1\ \mathbf{p}_2 \cdots \mathbf{p}_K] \in \mathbb{C}^{N \times K}$
  - $\mathbf{n}(k)$ is AWGN vector, and α a transmit power normalization factor

Digital Precoding Design

- The task is to design the precoding matrix $\mathbf{P}$ so that
  - ith MS's receive signal $y_i(k)$ is **sufficient** statistic to detect $x_i(k)$
  - Zero-forcing precoding: $\mathbf{P}_{\mathrm{ZF}} = \mathbf{H}^* \left(\mathbf{H}^T \mathbf{H}^*\right)^{-1}$
  - Similarly one can have MMSE precoding, and even MBER precoding
- BS cannot estimate downlink channel matrix, and MSs have to feed back corresponding downlink channel estimates to BS
- For TDD, uplink channel and downlink channel are reciprocal
  - BS can estimate uplink channel, which is needed in uplink MUD, and it can use estimated uplink channel as downlink channel in precoding design
- Fundamentally, for TDD, uplink receive beamforming or MUD is exactly equivalent to downlink transmit beamforming or MUT

1. L.-L. Yang, "Design of linear multiuser transmitters from linear multiuser receivers," in *Proc. ICC 2007* (Glasgow, Scotland), June 24-28, 2007, pp. 5258–5263
2. S. Chen and L.-L. Yang, "Downlink MBER beamforming design based on uplink MBER receive beamforming for TDD-SDMA induced MIMO systems," *Communications and Networks*, vol.2, pp. 145–151, Aug. 2010

Summary

- MIMO system introduction: diversity gain and multiplexing gain
- Single-user fractional-spaced receiver

  Baseband continuous-time model, discrete-time multirate model, discrete-time multichannel model
- Digital beamforming assisted receiver for QAM modulation
  - Also known as multiuser detection
- Digital transmit beamforming or precoding
  - Also known as multiuser transmission
- For TDD, equivalency of uplink receive beamforming or MUD and downlink transmit beamforming or MUT