Revision of Lecture Twenty-Eight

- MIMO classification: roughly four classes – create diversity, increase throughput, support multi-users, beamforming

- Single-user fractional-spaced receiver

  Baseband continuous-time model, discrete-time multirate model, discrete-time multichannel model

- SDMA induced MIMOs

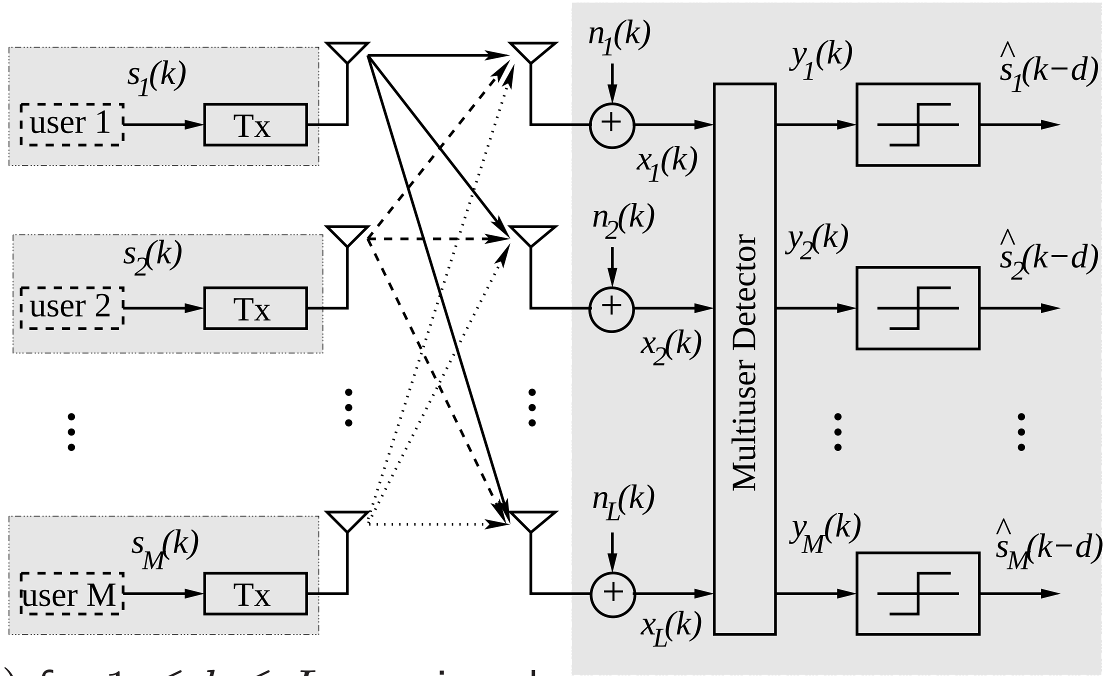
  digital beamforming assisted receiver, and digital transmit beamforming

- This lecture carries on MIMO A, B, C

SDMA Systems

- Previous lecture considers flat MIMO, we now considers frequency-selective MIMO, requiring space-time processing
- SDMA induced MIMO system:
  - Assume one transmit antenna and L receiver antennas supporting M users
  - No specific antenna array structure is assumed, so it is most generic
  - Channels are frequency selective, and CIR connecting user m and lth receiver antenna is

$$\mathbf{c}_{l,m} = [c_{0,l,m}\ c_{1,l,m} \cdots c_{n_C-1,l,m}]^T$$



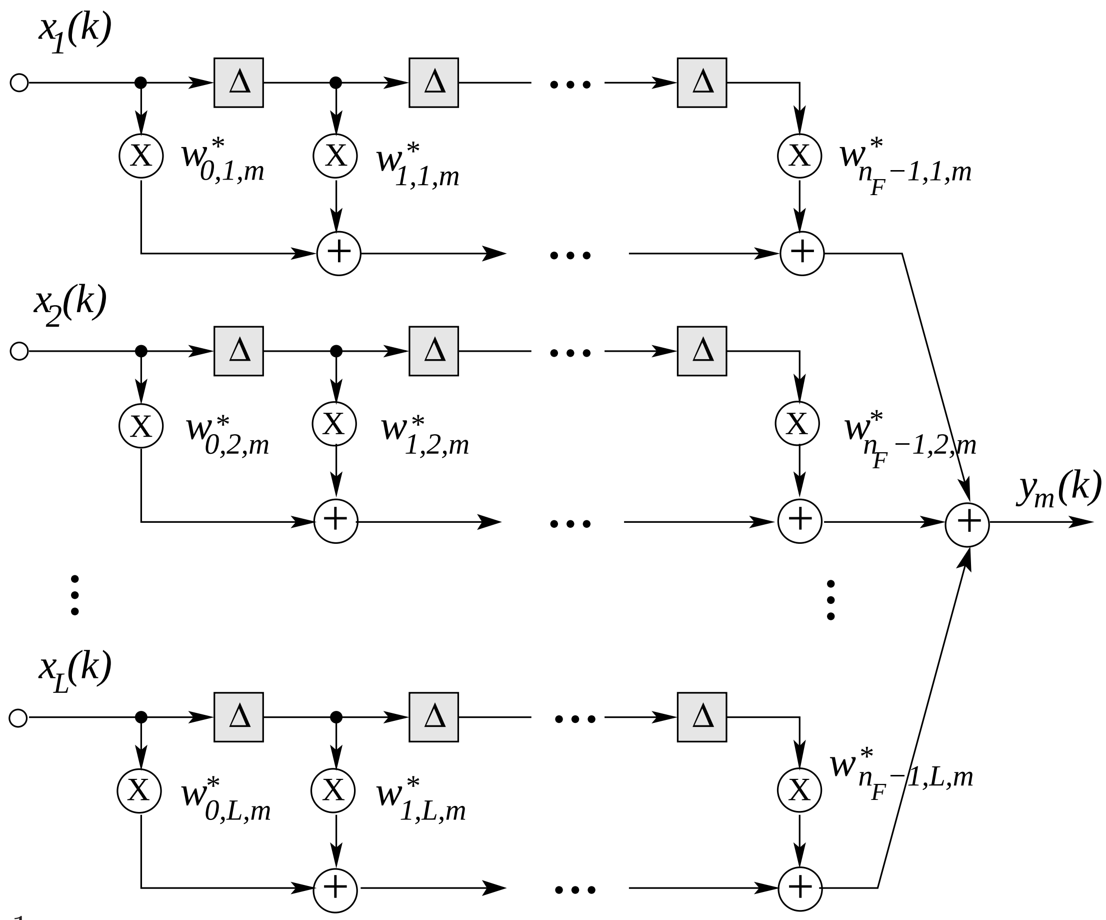
- Symbol-rate received signal samples $x_l(k)$ for $1 \le l \le L$ are given by

$$x_l(k) = \sum_{m=1}^{M} \sum_{i=0}^{n_C-1} c_{i,l,m} s_m(k-i) + n_l(k) = \bar{x}_l(k) + n_l(k)$$

$n_l(k)$ is complex-valued AWGN with $E[|n_l(k)|^2] = 2\sigma_n^2$, $\bar{x}_l(k)$ is noise-free part of lth receive antenna's output, $s_m(k)$ is kth transmitted symbol of user m (assuming BPSK for simplicity)

Multiuser Detection in SDMA Systems

- Multiuser supporting capability
  - CDMA: each user is separated by a unique user-specific spreading code
  - SDMA: each user is associated with a unique user-specific CIR encountered at receiver antennas
  - Unique user-specific CIR plays role of user-specific CDMA signature
  - Owing to non-orthogonal nature of CIRs, effective multiuser detection is required for separating users



- A bank of M space-time equalisers forms MUD, whose soft outputs are

$$y_m(k) = \sum_{l=1}^{L} \sum_{i=0}^{n_F-1} w^*_{i,l,m} x_l(k-i), \ 1 \le m \le M$$

$\mathbf{w}_{l,m} = [w_{0,l,m} \ w_{1,l,m} \cdots w_{n_F-1,l,m}]^T$ is mth user detector's equaliser weight vector associated with lth receive antenna, STE has order n_F and decision delay d

System Model

- Define $n_F \times (n_F + n_C - 1)$ CIR matrix associated with user m and lth receive antenna

$$\mathbf{C}_{l,m} = \begin{bmatrix} c_{0,l,m} & c_{1,l,m} & \cdots & c_{n_C-1,l,m} & 0 & \cdots & 0 \\ 0 & c_{0,l,m} & c_{1,l,m} & \cdots & c_{n_C-1,l,m} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & c_{0,l,m} & c_{1,l,m} & \cdots & c_{n_C-1,l,m} \end{bmatrix}$$

- Introduce overall system CIR convolution matrix

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{1,1} & \mathbf{C}_{1,2} & \cdots & \mathbf{C}_{1,M} \\ \mathbf{C}_{2,1} & \mathbf{C}_{2,2} & \cdots & \mathbf{C}_{2,M} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{C}_{L,1} & \mathbf{C}_{L,2} & \cdots & \mathbf{C}_{L,M} \end{bmatrix}$$

- Then received signal vector $\mathbf{x}(k) = [\mathbf{x}_1(k)\ \mathbf{x}_2(k) \cdots \mathbf{x}_L(k)]^T$ can be expressed by

$$\mathbf{x}(k) = \mathbf{C}\,\mathbf{s}(k) + \mathbf{n}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k)$$

where $\mathbf{x}_l(k) = [x_l(k)\ x_l(k-1) \cdots x_l(k-n_F+1)]^T$ for $1 \leq l \leq L$, $\mathbf{n}(k) = [\mathbf{n}_1(k)\ \mathbf{n}_2(k) \cdots \mathbf{n}_L(k)]^T$ with $\mathbf{n}_l(k) = [n_l(k)\ n_l(k-1) \cdots n_l(k-n_F+1)]^T$, and $\mathbf{s}(k) = [\mathbf{s}_1^T(k)\ \mathbf{s}_2^T(k) \cdots \mathbf{s}_M^T(k)]^T$ with $\mathbf{s}_m(k) = [s_m(k)\ s_m(k-1) \cdots s_m(k-n_F-n_C+2)]^T$

Space-Time Equalisation

- Output of mth STE detector can be written as

$$y_m(k) = \sum_{l=1}^{L} \mathbf{w}_{l,m}^H \mathbf{x}_l(k) = \mathbf{w}_m^H \mathbf{x}(k)$$

where $\mathbf{w}_m = [\mathbf{w}_{1,m}^T \ \mathbf{w}_{2,m}^T \cdots \mathbf{w}_{L,m}^T]^T$

- With $y_{R_m}(k) = \mathrm{Re}[y_m(k)]$, M user detectors' decisions are defined by

$$\hat{s}_m(k-d) = \mathrm{sgn}\left(y_{R_m}(k)\right), \ 1 \le m \le M$$

- Minimum mean square error solution is defined by closed-form

$$\mathbf{w}_{(\mathrm{MMSE})m} = \left(\mathbf{C}\,\mathbf{C}^H + 2\sigma_n^2 \mathbf{I}\right)^{-1} \mathbf{C}_{|(m-1)(n_F+n_C-1)+(d+1)}$$

for $1 \le m \le M$, where $\mathbf{I}$ denotes $Ln_F \times Ln_F$ identity matrix and $\mathbf{C}_{|i}$ the ith column of $\mathbf{C}$

- Adaptive implementation using LMS algorithm

$$\mathbf{w}_m(k+1) = \mathbf{w}_m(k) + \mu \mathbf{x}(k) \epsilon^*(k)$$

where $\epsilon(k) = s_m(k-d) - y_m(k)$

Bit Error Rate of Space-Time Equaliser

- Note transmitted symbol sequence $\mathbf{s}(k) \in \{\mathbf{s}^{(q)}, 1 \le q \le N_s\}$, where $N_s = 2^{M(n_F+n_C-1)}$
- Let the element of $\mathbf{s}^{(q)}$ corresponding to desired symbol $s_m(k-d)$ be $s_{m,d}^{(q)}$
- Noise-free part of mth detector input signal $\bar{\mathbf{x}}(k)$ assumes values from signal set $\mathcal{X}_m = \{\bar{\mathbf{x}}^{(q)} = \mathbf{C}\,\mathbf{s}^{(q)}, 1 \le q \le N_s\}$
- $\mathcal{X}_m$ can be partitioned into two subsets, depending on the value of $s_m(k-d)$, as follows $\mathcal{X}_m^{(\pm)} = \{\bar{\mathbf{x}}^{(q,\pm)} \in \mathcal{X}_m : s_m(k-d) = \pm 1\}$
- Similarly, noise-free part of mth detector's output $\bar{y}_m(k)$ assumes values from the scalar set

$$\mathcal{Y}_m = \{\bar{y}_m^{(q)} = \mathbf{w}_m^H \bar{\mathbf{x}}^{(q)}, 1 \le q \le N_s\}$$

- Thus $\bar{y}_{Rm}(k) = \mathrm{Re}[\bar{y}_m(k)]$ can only take the values from the set

$$\mathcal{Y}_{Rm} = \{\bar{y}_{Rm}^{(q)} = \mathrm{Re}[\bar{y}_m^{(q)}], 1 \le q \le N_s\}$$

- $\mathcal{Y}_{Rm}$ can be divided into the two subsets conditioned on the value of $s_m(k-d)$

$$\mathcal{Y}_{Rm}^{(\pm)} = \{\bar{y}_{Rm}^{(q,\pm)} \in \mathcal{Y}_{Rm} : s_m(k-d) = \pm 1\}$$

Bit Error Rate of STE (continue)

- Conditional PDF of $y_{Rm}(k)$ given $s_m(k-d)=+1$ is a Gaussian mixture

$$p_m(y_R|+1) = \frac{1}{N_{sb}} \sum_{q=1}^{N_{sb}} \frac{1}{\sqrt{2\pi\sigma_n^2 \mathbf{w}_m^H \mathbf{w}_m}} e^{-\frac{\left(y_R - \bar{y}_{Rm}^{(q,+)}\right)^2}{2\sigma_n^2 \mathbf{w}_m^H \mathbf{w}_m}}$$

where $\bar{y}_{Rm}^{(q,+)} \in \mathcal{Y}_{Rm}^{(+)}$ and $N_{sb} = N_s/2$ is the number of points in $\mathcal{Y}_{Rm}^{(+)}$

- Thus BER of the mth detector associated with the detector's weight vector $\mathbf{w}_m$ is given by

$$P_E(\mathbf{w}_m) = \frac{1}{N_{sb}} \sum_{q=1}^{N_{sb}} Q\left(g^{(q,+)}(\mathbf{w}_m)\right)$$

where

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-\frac{v^2}{2}}\, d\, v \quad \text{and} \quad g^{(q,+)}(\mathbf{w}_m) = \frac{\mathrm{sgn}(s_{m,d}^{(q)})\bar{y}_{Rm}^{(q,+)}}{\sigma_n \sqrt{\mathbf{w}_m^H \mathbf{w}_m}}$$

- Note that BER is invariant to a positive scaling of $\mathbf{w}_m$
- Alternatively, the BER may be calculated based on the other subset $\mathcal{Y}_{Rm}^{(-)}$.

Minimum Bit Error Rate Solution

- MBER solution for the mth STE detector is defined as

$$\mathbf{w}_{(\mathrm{MBER})m} = \arg\min_{\mathbf{w}_m} P_E(\mathbf{w}_m)$$

- No closed-form solution, but gradient of $P_E(\mathbf{w}_m)$ is

$$\nabla P_E(\mathbf{w}_m) = \frac{1}{2N_{sb}\sqrt{2\pi}\sigma_n\sqrt{\mathbf{w}_m^H\mathbf{w}_m}} \sum_{q=1}^{N_{sb}} e^{-\frac{\left(\bar{y}_{Rm}^{(q,+)}\right)^2}{2\sigma_n^2\mathbf{w}_m^H\mathbf{w}_m}} \mathrm{sgn}\left(s_{m,d}^{(q)}\right)\left(\frac{\bar{y}_{Rm}^{(q,+)}\mathbf{w}_m}{\mathbf{w}_m^H\mathbf{w}_m} - \bar{\mathbf{x}}^{(q,+)}\right)$$

Gradient optimisation can be applied to obtain a $\mathbf{w}_{(\mathrm{MBER})m}$

- Adaptive implementation using LBER algorithm

$$\mathbf{w}_m(k+1) = \mathbf{w}_m(k) + \mu\frac{\mathrm{sgn}(s_m(k-d))}{2\sqrt{2\pi}\rho_n} e^{-\frac{y_{Rm}^2(k)}{2\rho_n^2}}\mathbf{x}(k)$$

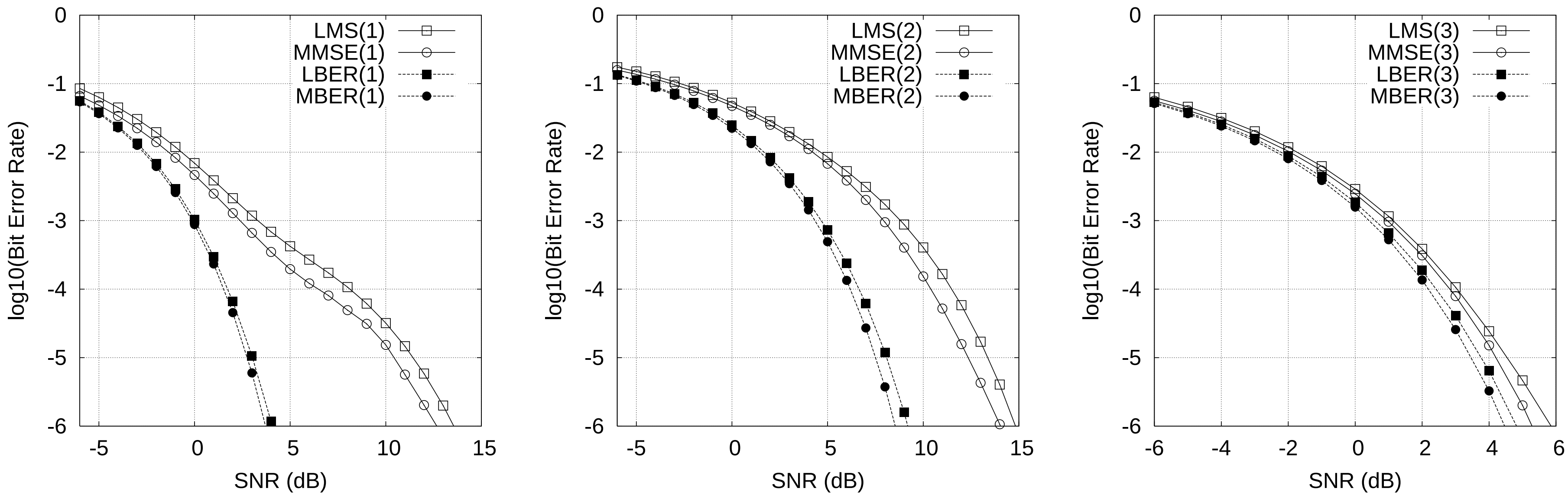
where μ is adaptive gain, and ρ_n kernel width

Simulation Results: Stationary System

- CIRs of 3-user 4-antenna stationary system

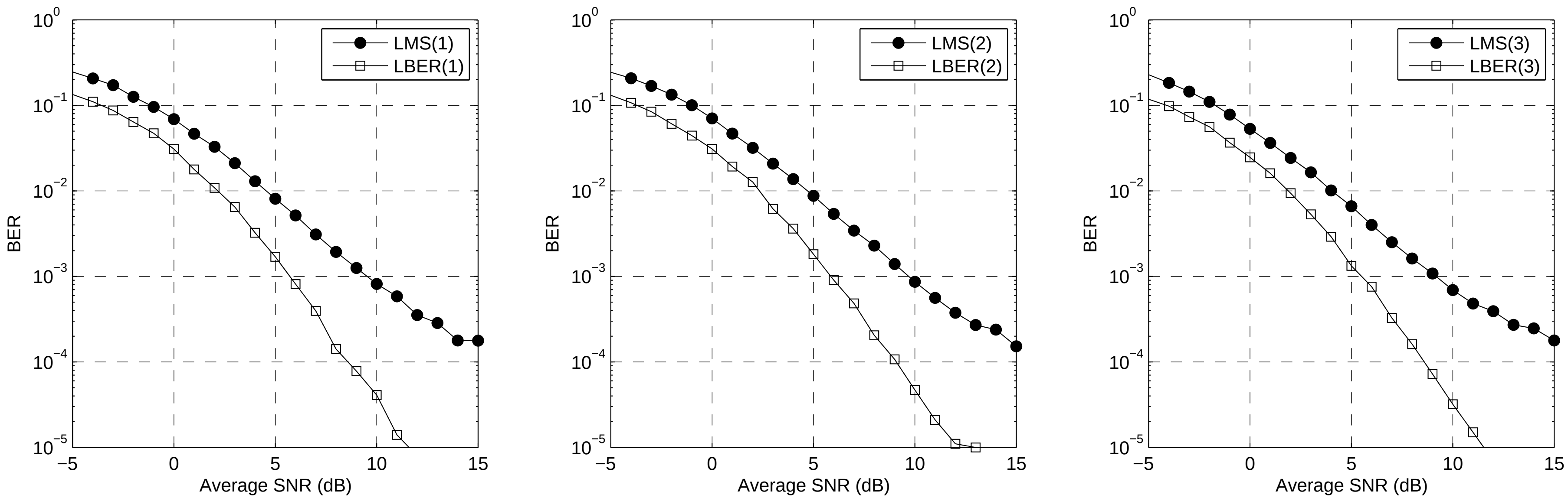
$C_{l,m}(z)$	$m=1$	$m=2$	$m=3$
$l=1$	$(-0.5+j0.4)+(0.7+j0.6)z^{-1}$	$(-0.1-j0.2)+(0.7+j0.6)z^{-1}$	$(-0.7+j0.9)+(0.6+j0.4)z^{-1}$
$l=2$	$(0.5-j0.4)+(-0.8-j0.3)z^{-1}$	$(-0.3+j0.5)+(-0.7-j0.9)z^{-1}$	$(-0.6+j0.8)+(-0.6-j0.7)z^{-1}$
$l=3$	$(0.4-j0.4)+(-0.7-j0.8)z^{-1}$	$(-0.1-j0.2)+(0.7+j0.6)z^{-1}$	$(0.3-j0.5)+(0.9+j0.1)z^{-1}$
$l=4$	$(0.5+j0.5)+(0.6-j0.9)z^{-1}$	$(-0.6-j0.4)+(0.9-j0.4)z^{-1}$	$(-0.6-j0.6)+(0.8+j0.0)z^{-1}$

- CIR order $n_C=2$, STE order $n_F=3$ and decision delay $d=1$
- BER comparison of MMSE/MBER and LMS/LBER for three users



Simulation Results: Fading System

- 3 users, 4 receive antennas, and Rayleigh fading channels with each of 12 CIRs having $n_C = 3$ taps
- Each channel tap has root mean power of $\sqrt{0.5} + j\sqrt{0.5}$
- Normalised Doppler frequency for simulated system was 10^{-5}, which for a carrier of 900 MHz and a symbol rate of 3 Msymbols/s corresponded to a user velocity of 10 m/s (36 km/h)
- STE order $n_F = 5$ and decision delay $d = 2$
- Frame structure: 50 training symbols followed by 450 data symbols
- BER comparison of LMS/LBER for three users



Multiuser MIMO OFDM Uplink

- M single-antenna MUs transmit to BS equipped with L receiving antennas using same resource block. Each OFDM transmission block of MU m has N data symbols

$$\boldsymbol{S}_m = \left[S_{m,0} \; S_{m,1} \cdots S_{m,N-1} \right]^{\mathrm{T}}$$

with $\mathrm{E}\{|S_{m,n}|^2\} = \sigma_s^2$

- Converting $\boldsymbol{S}_m$ by N-point IFFT yields TD signal block $\boldsymbol{s}_m = \left[s_{m,0} \; s_{m,1} \cdots s_{m,N-1} \right]^{\mathrm{T}}$
- Adding CP of length N_{cp} to $\boldsymbol{s}_m$ yields $\bar{\boldsymbol{s}}_m = \left[s_{m,-N_{\mathrm{cp}}} \; s_{m,-N_{\mathrm{cp}}+1} \cdots s_{m,-1} \middle| \boldsymbol{s}_m^{\mathrm{T}} \right]^{\mathrm{T}}$, in which $s_{m,-k} = s_{m,N-k}$, $1 \le k \le N_{\mathrm{cp}}$, and $N_{\mathrm{cp}} \ge n_H$
- Channel is frequency selective, and CIR of link connecting mth mobile to lth antenna of BS is given by $\boldsymbol{h}_{l,m} = \left[h_{0,l,m} \; h_{1,l,m} \cdots h_{n_H-1,l,m} \right]^{\mathrm{T}}$ for $1 \le l \le L$ and $1 \le m \le M$
- At BS, after CP removal, received signal blocks $\boldsymbol{x}_l = \left[x_{l,0} \; x_{l,1} \cdots x_{l,N-1} \right]^{\mathrm{T}}$, $1 \le l \le L$ are passing through N-point FFT to yield (compare this with **single-user** OFDM)

$$X_{l,n} = \sum_{m=1}^{M} H_{n,l,m} S_{m,n} + \Xi_{l,n}, \quad 0 \le n \le N-1$$

$\Xi_{l,n}$: FD AWGN at lth receive antenna with $\mathrm{E}\{|\Xi_{l,n}|^2\} = 2\sigma_\xi^2$, and FDCTFC vector $\mathbf{H}_{l,m} = \left[H_{0,l,m} \; H_{1,l,m} \cdots H_{N-1,l,m} \right]^{\mathrm{T}}$ is N-point FFT of $\boldsymbol{h}_{l,m}$, $1 \le l \le L$, $1 \le m \le M$

MU MIMO OFDM (continue)

- For $0 \leq n \leq N-1$, define $\underline{\mathbf{X}}_n = \left[X_{1,n}\ X_{2,n} \cdots X_{L,n}\right]^{\mathrm{T}}$, $\underline{\mathbf{S}}_n = \left[S_{1,n}\ S_{2,n} \cdots S_{M,n}\right]^{\mathrm{T}}$, and $\underline{\mathbf{\Xi}}_n = \left[\Xi_{1,n}\ \Xi_{2,n} \cdots \Xi_{L,n}\right]^{\mathrm{T}}$ as well as nth subcarrier FD channel matrix

$$\underline{\mathbf{H}}_n = \begin{bmatrix} H_{n,1,1} & H_{n,1,2} & \cdots & H_{n,1,M} \\ H_{n,2,1} & H_{n,2,2} & \cdots & H_{n,2,M} \\ \vdots & \vdots & \cdots & \vdots \\ H_{n,L,1} & H_{n,L,2} & \cdots & H_{n,L,M} \end{bmatrix}$$

  Then

$$\underline{\mathbf{X}}_n = \underline{\mathbf{H}}_n \underline{\mathbf{S}}_n + \underline{\mathbf{\Xi}}_n, \ 0 \leq n \leq N-1$$

- **MUD** in FD is done on **subcarrier basis**, detection of $\underline{\mathbf{S}}_n$ can be obtained as

$$\widehat{\underline{\mathbf{S}}}_n = \underline{\mathbf{W}}_n \underline{\mathbf{X}}_n, \ 0 \leq n \leq N-1$$

  where $\underline{\mathbf{W}}_n$ is weight matrix of nth subcarrier MUD

- Given nth subcarrier FD channel matrix $\underline{\mathbf{H}}_n$, **MMSE** solution for $\underline{\mathbf{W}}_n$ is

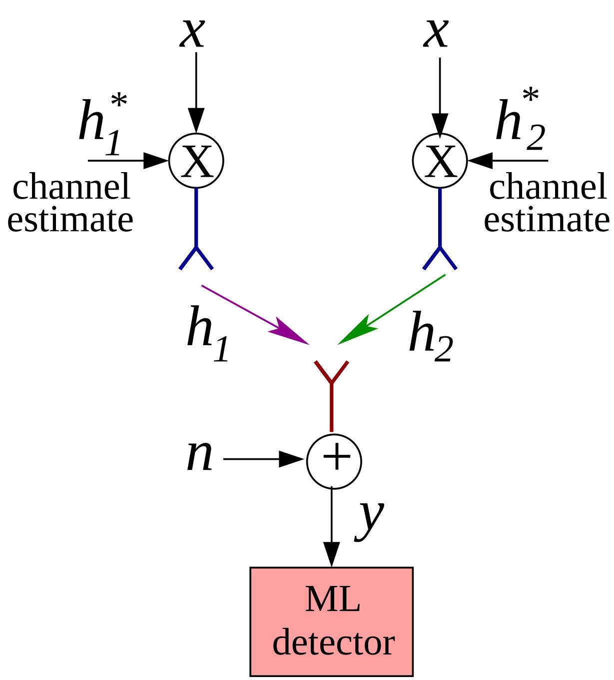
$$\widehat{\underline{\mathbf{W}}}_n = \left(\underline{\mathbf{H}}_n^{\mathrm{H}} \underline{\mathbf{H}}_n + \frac{2\sigma_\xi^2}{\sigma_s^2} \boldsymbol{I}_M\right)^{-1} \underline{\mathbf{H}}_n^{\mathrm{H}}, \ 0 \leq n \leq N-1$$

  ZF solution for $\underline{\mathbf{W}}_n$ is

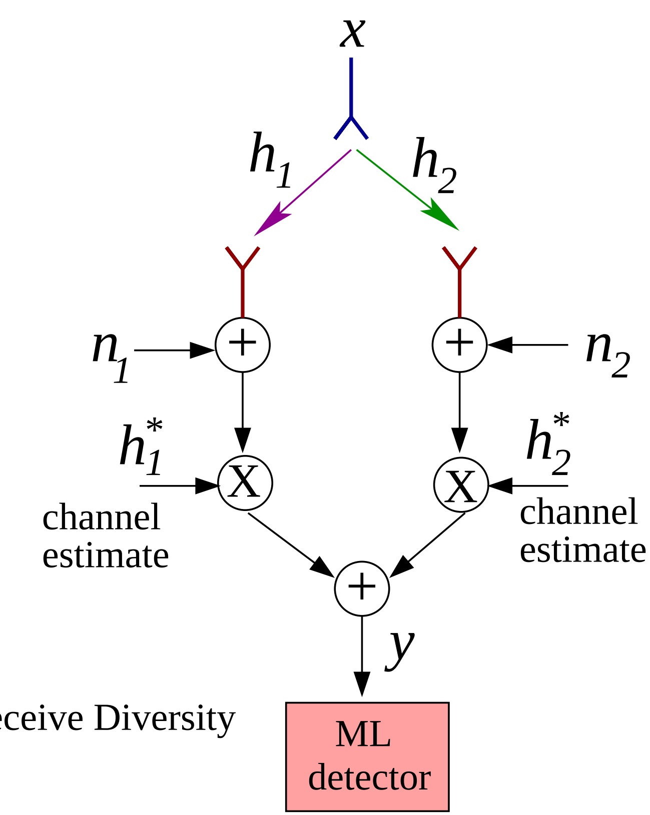
$$\widetilde{\underline{\mathbf{W}}}_n = \left(\underline{\mathbf{H}}_n^{\mathrm{H}} \underline{\mathbf{H}}_n\right)^{-1} \underline{\mathbf{H}}_n^{\mathrm{H}}, \ 0 \leq n \leq N-1$$

Diversity

- We now consider diversity gain aspect of MIMO
- Transmit diversity: assume
  - Two transmit antennas, which are sufficiently apart
  - One receive antenna
  - Two channel estimates are available at transmitter
- Receive diversity: assume
  - One transmit antenna
  - Two receive antennas, which are sufficiently apart
  - Two channel estimates are available at receiver



Transmit Diversity



Receive Diversity

- Transmit diversity order of two: two transmit signals are $h_1^* x$ and $h_2^* x$, and receive signal is

$$y = h_1 h_1^* x + h_2 h_2^* x + n = \left(|h_1|^2 + |h_2|^2\right) x + n$$

- Receive diversity order of two: optimal combined signal of two receive signals is

$$y = h_1^* (h_1 x + n_1) + h_2^* (h_2 x + n_2) = \left(|h_1|^2 + |h_2|^2\right) x + n$$

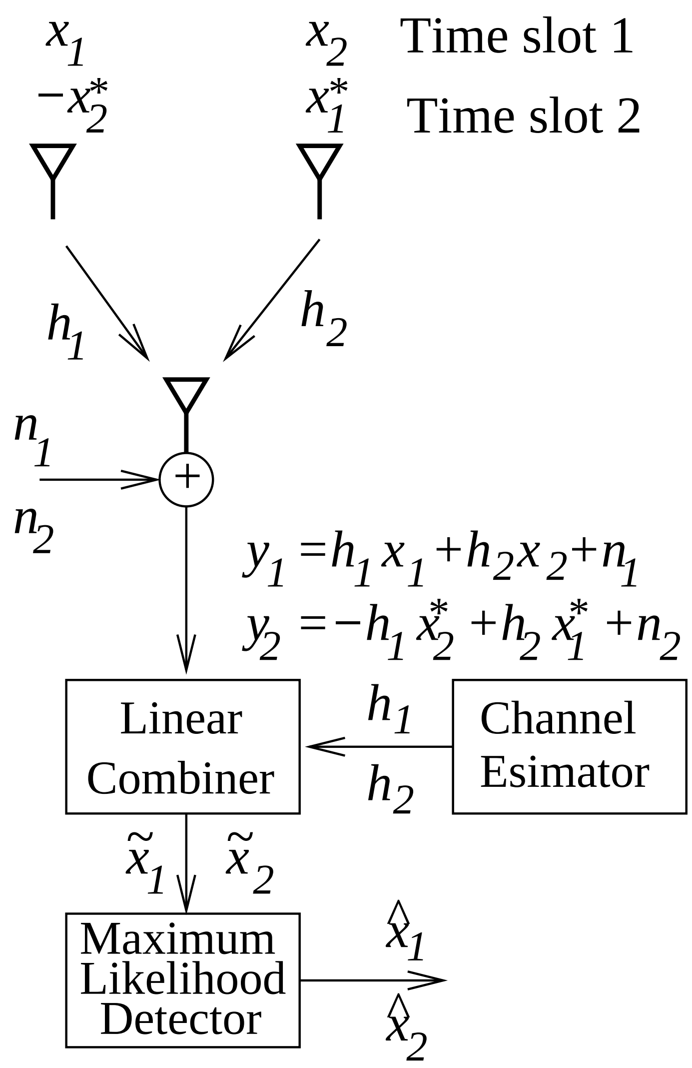
G_2 Space-Time Block Code

- Alamouti's G_2 space-time block code uses two transmitter antennas and one receiver antenna
  - In time slot 1 (one symbol period), two symbols (x_1, x_2) are transmitted
  - While in time slot 2, transformed (x_1, x_2), i.e. $(-x_2^*, x_1^*)$, are transmitted

- Assume narrowband channels with channel 1, $h_1 = |h_1|e^{j\alpha_1}$ and channel 2, $h_2 = |h_2|e^{j\alpha_2}$

- Antenna spacing is sufficiently large, e.g. 10 wavelengths, so two channels are independently faded

- Fading is sufficiently slow so during two time slots channels h_1, h_2 are unchanged



G_2 STBC (continue)

- Received signals at two time slots are respectively

$$\begin{aligned} y_1 &= h_1 x_1 + h_2 x_2 + n_1 \\ y_2 &= -h_1 x_2^* + h_2 x_1^* + n_2 \end{aligned}$$

- Assume perfect channel estimate h_1, h_2, linear combiner's outputs are

$$\tilde{x}_1 = h_1^* y_1 + h_2 y_2^* = (|h_1|^2 + |h_2|^2) x_1 + h_1^* n_1 + h_2 n_2^*$$
$$\tilde{x}_2 = h_2^* y_1 - h_1 y_2^* = (|h_1|^2 + |h_2|^2) x_2 + h_2^* n_1 + h_1 n_2^*$$

- Maximum likelihood decoding involves minimising decision metric

$$|\tilde{x}_1 - (|h_1|^2 + |h_2|^2) x_1|^2$$

for decoding x_1 and minimising decision metric

$$|\tilde{x}_2 - (|h_1|^2 + |h_2|^2) x_2|^2$$

for decoding x_2

Space-Time Block Codes

- Encoding: generic STBC is defined by $n \times p$ transmission matrix

$$G = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1p} \\ g_{21} & g_{22} & \cdots & g_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{np} \end{bmatrix} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,p} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,p} \\ \vdots & \vdots & \cdots & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,p} \end{bmatrix}$$

  Each entry $g_{ij} = x_{i,j}$ is a linear combination of k input symbols $x_1, x_2, \cdots x_k$ and their conjugates

- Number of rows n is equal to number of time slots, and number of columns is equal to number of transmit antennas
- During time slot i, encoded symbols $x_{i,1}, x_{i,2}, \cdots, x_{i,p}$ are transmitted simultaneously from transmit antennas $1, 2, \cdots, p$, respectively
- Code rate is obviously $R = k/n$
- Assume L receiver antennas, and channel connecting jth transmit antenna and lth receiver antenna is $h_{j,l}$, then received signal arriving at receiver l during time slot i is

$$y_{i,l} = \sum_{j=1}^{p} h_{j,l} x_{i,j} + n_{j,l}$$

  where $n_{j,l}$ is AWGN for j, l-th channel
  - ML detector or suboptimal low-complexity detector can be employed

Space-Time Block Codes (continue)

- Decoding: assuming perfect channel estimate, maximum likelihood decoding decides in favour of specific entry $x_{i,j}$, $1 \leq i \leq n$, $1 \leq j \leq p$, that minimises the decision metric

$$\sum_{i=1}^{n} \sum_{l=1}^{L} \left| y_{i,l} - \sum_{j=1}^{p} h_{j,l} x_{i,j} \right|^2$$

- An alternative is maximum a posteriori probability decoding, for details see relevant reference
- STBC examples (transmit antennas $p = 2, 3, 4$)

$$G_2 = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}, \quad G_3 = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \\ x_1^* & x_2^* & x_3^* \\ -x_2^* & x_1^* & -x_4^* \\ -x_3^* & x_4^* & x_1^* \\ -x_4^* & -x_3^* & x_2^* \end{bmatrix}, \quad G_4 = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{bmatrix}$$

G_2 has time slots $n = 2$, G_3 and G_4 have time slots $n = 8$

STBC Examples (continue)

- STBC examples (transmit antennas $p = 3, 4$)

$$H_3 = \begin{bmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{-x_1-x_1^*+x_2-x_2^*}{2} \\ \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \frac{x_2+x_2^*+x_1-x_1^*}{2} \end{bmatrix}, H_4 = \begin{bmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{-x_1-x_1^*+x_2-x_2^*}{2} & \frac{-x_2-x_2^*+x_1-x_1^*}{2} \\ \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \frac{x_2+x_2^*+x_1-x_1^*}{2} & \frac{-x_1-x_1^*-x_2+x_2^*}{2} \end{bmatrix}$$

H_3 and H_4 have time slots $n = 4$

- Parameters of space-time block codes

space-time block code	code rate R	number of transmitters p	number of input symbol k	number of time slots n
G_2	1	2	2	2
G_3	1/2	3	4	8
G_4	1/2	4	4	8
H_3	3/4	3	3	4
H_4	3/4	4	3	4

Summary

- Multiuser capacity of SDMA systems
- Space-time equalisation assisted multiuser detection for SDMA systems

  MMSE design and MBER design, adaptive implementation
- Multiuser MIMO OFDM
- Diversity order, and space-time block codes