Revision of Lecture Two

- Factors causing **power lost**: propagation path loss, slow (large-scale) fading, and fast (small-scale) fading
  - Power budget rule:
$$P_{\text{Tx}} = P_{\text{Rx}} + L_{\text{total}}$$
$$L_{\text{total}} = L_{\text{pathloss}} + L_{\text{slow}} + L_{\text{fast}}$$
  - Collaborative or relaying communication as seeing from simple model for receive signal power

- Two killer factors in mobile medium:
  - **Doppler spread**: time-varying nature of channel causes frequency dispersion, and a physical dimension/quantity – Doppler frequency f_D
  - **Multipath**: which causes time dispersion, and a physical dimension/quantity – excess delay τ

- We will have in-depth look into these two phenomenas

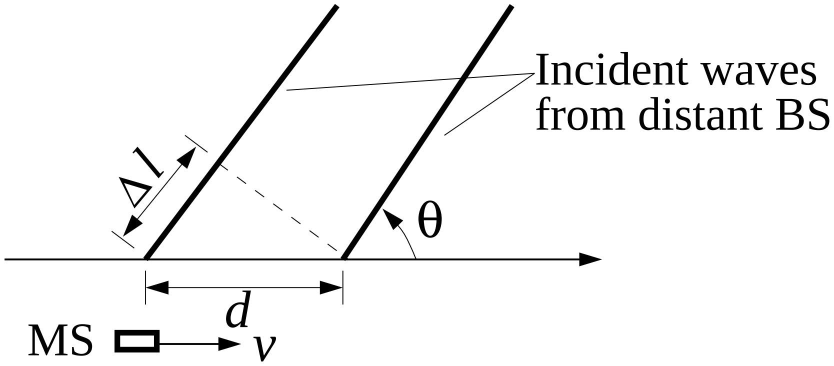
Doppler Spread: Physics

- Mobile medium is hostile environment for communication, and one reason is channel is nonstationary or **time-varying**
  - Famous Doppler effect: moving changes frequency
  - Time varying nature of channel hence **broadens** signal spectrum
  - We have a new physical dimension or quantity called Doppler frequency f_D
- Recall in Digital Coding and Transmission, we learnt that signal spectrum must be strictly shaped
  - But Doppler effect may seriously destroy this careful shaped signal spectrum
- To fully understand the effects of this physical phenomena, we need to know distribution of signal power in the Doppler-frequency domain
  - Power spectral density (PSD) in Doppler frequency, or **Doppler spectrum**, characterises this spectrum broadening caused by time-varying nature of channel

We consider **far-field**: EM waveform is planner (For near-field: EM waveform is spherical

Doppler Frequency: Derivation

- Consider mobile station (MS) moving at speed v:
  - Moving "changes" frequency $\longrightarrow$ **Doppler shift**
  - Assuming far-away base station (BS) and incident EM waves are "parallel"
- Difference in path lengths from BS to MS is $\Delta l = d\cos\theta = v\Delta t\cos\theta$



- Let λ be wavelength, then phase change in received signal due to difference in path lengths is:

$$\Delta\phi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi v\Delta t\cos\theta}{\lambda}$$

- **Doppler frequency** is defined as rate of phase change due to moving:

$$f_D = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta = f_m\cos\theta$$

  - f_m is the maximum Doppler frequency, unit in [Hz]
- The arrival angle θ can be viewed as uniformly distributed

$$\mathrm{PDF}(\theta) = \frac{1}{2\pi}, \quad 0 \le \theta \le 2\pi$$

  - **Doppler frequency** $f_D = f_m\cos\theta$ is then cosine distributed

Doppler Spectrum: Math Model

- Received power in $d\theta$ around θ is proportional to $\frac{|d\theta|}{2\pi}$ (absolute operation as power ≥ 0), and

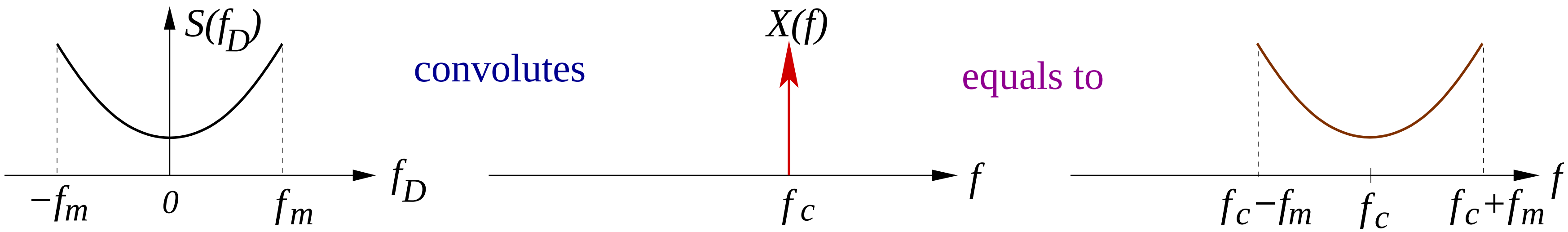
$$\frac{d\cos^{-1}x}{dx} = -\frac{1}{\sqrt{1-x^2}}, \quad -1 \leq x \leq 1$$

- **Doppler power spectrum density**: (absolute operation because power ≥ 0)

$$S(f_D) \propto \frac{1}{2\pi}\left|\frac{d\theta}{df_D}\right| = \frac{1}{2\pi}\left|\frac{d(\cos^{-1}(f_D/f_m))}{df_D}\right| \quad \text{or} \quad S(f_D) = \frac{C}{\sqrt{1-(f_D/f_m)^2}}$$

- Implications: **frequency dispersion**
  - Single frequency f_c broadened to a spectrum $(f_c - f_m, f_c + f_m)$

$$x(t) = a\cos\left(2\pi f_c + \varphi\right) \Rightarrow X(f) = \delta\left(f - f_c\right) \text{ and } X(f) \star S\left(f_D\right) = S\left(f - f_D\right)$$



  - Signal with bandwidth $B_p = 2B$ centred at f_c broadened to a bandwidth approximately $2B + 2f_m$

Doppler Spread

- **Doppler spread** B_D is defined as the "bandwidth" of Doppler spectrum. It is a measure of spectral broadening caused by the time varying nature of the channel

- **Coherence time** $T_C \propto \frac{1}{B_D}$ is used to characterise the time varying nature of the frequency dispersion of the channel in time domain

- **Fading** effects due to Doppler spread: determined by mobile speed and signal bandwidth. Let baseband signal bandwidth be B_S and symbol period T_S, then
  - "Slow fading" channel: $T_S \ll T_C$ or $B_S \gg B_D$, signal bandwidth is much greater than Doppler spread, and effects of Doppler spread are negligible
  - "Fast fading" channel: $T_S > T_C$ or $B_S < B_D$, channel changes rapidly in during one symbol period T_S

- Here slow and fast fading are used to describe relationship between **time rate of change** in the **channel** and in the transmitted **signal**
  - Do not confuse with slow (large-scale) and fast (small-scale) fadings in propagation pathloss model

Normalised Doppler Frequency

- Velocity of mobile and signal bandwidth determine whether a signal undergoes fast or slow fading
  - i.e. ratio of Doppler bandwidth over signal bandwidth determines fast or slow fading
- Fading rate describes the relationship between rate of change in channel and rate of change in signal
  - Rate of change in channel is specified by velocity of mobile v and carrier frequency f_c, as characterised in the (maximum) Doppler frequency

$$f_m = \frac{v}{\lambda} = \frac{v \cdot f_c}{c}, \qquad \lambda \text{ being wavelength}, \; c \text{ being speed of light}$$

  - As signal bandwidth is much smaller than f_c, Doppler spread is approximately f_m
  - Rate of change in signal is specified by symbol rate or symbol period T_s
- Often **normalised Doppler frequency** is used to specify fading rate

$$\bar{f}_m = f_m \cdot T_s$$

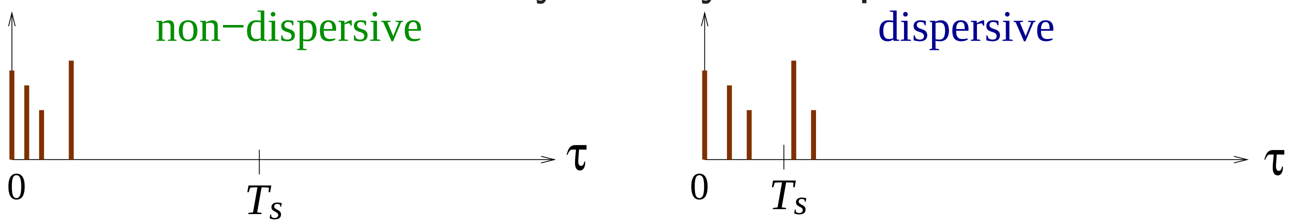
  - $\bar{f}_m = 10^{-6}$ is considered very slow fading, $\bar{f}_m = 10^{-4}$ quite fast
  - Example: Carrier frequency of 1 GHz $\longrightarrow$ wavelength $\lambda = c/f_c = 3 \cdot 10^8/10^9 = 0.3$ m

    User velocity of 10 m/s (36 km/h) and $\lambda = 0.3$ m $\longrightarrow$ Doppler frequency $f_m = v/\lambda \approx 33$ Hz

    At symbol rate of 3.3 Msymbols/s, the normalised Doppler frequency becomes $\bar{f}_m = f_m \cdot T_s = 33/(3.3 \cdot 10^6) = 10^{-5}$

Multipath: Physics

- Mobile medium is hostile environment for communication, and one reason is **multipath** distortion
  - EM wave propagation by reflection, diffraction and scattering, copies of signal arrives at receiver with different attenuations, phase shifts and delays
  - We have a new physical quantity or dimension called **excess delay** τ
- Depend on ratio of this excess delay and symbol period, channel may be dispersive



  - Recall in Digital Coding and Transmission, we learnt that pulse shaping achieves zero ISI, but multipath distortion may simply destroy it
- To fully understand the effects of this physical phenomena, we need to know distribution of channel/signal power in the excess-delay domain
  - PSD in excess delay, or **power delay profile**, characterises this time dispersion

Impulse Response of Multipath Channels

- Multipath causes time dispersion, as described by bandpass CIR $h(t,\tau)$
  - As channel can be time-varying, time t is needed, and τ is multipath delay
  - Generally, $h(t,\tau)$ is a function of two inputs t and τ

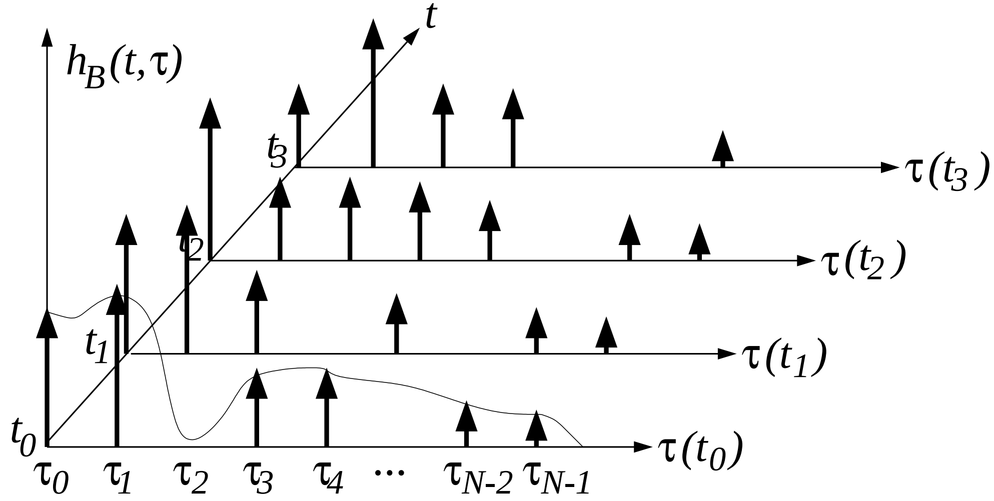
- Let equivalent baseband **complex-envelope** channel impulse response be $h_B(t,\tau)$

$$h_B(t,\tau) = \sum_{i=0}^{N-1} a_i(t,\tau)\exp(-j\theta_i(t,\tau))\delta(\tau - \tau_i(t))$$

  - $h_B(t,\tau)$ a three-D surface with two inputs: time and excess delay

- Useful to discretize τ into delay bins, each bin represents a multipath component

- $a_i(t,\tau)$, $\theta_i(t,\tau)$ and $\tau_i(t)$ are amplitude, phase shift and excess delay of ith multipath component, respectively



Channel Impulse Response: Dispersion

- Interpretation of $h_B(t,\tau)$: there are N multipaths, i.e. there are N copies of transmitted signal arriving at the receiver

- At time t, each copy arrives at the receiver with a different amplitude $a_i(t,\tau)$, goes through a different phase shift $\theta_i(t,\tau)$ and has a different excess delay $\tau_i(t)$

- Excess delay τ is function of t, amplitude is function of t and τ, phase shift is function of t and τ, and they are stochastic processes

- A special case is the time invariant channel, where

$$h_B(\tau) = \sum_{i=0}^{N-1} a_i \exp(-j\theta_i)\delta(\tau - \tau_i)$$

  θ_i: uniformly distributed, a_i: Rayleigh distributed, τ_i: Poisson distributed

- **Multipath** causes **time dispersion** and results in **intersymbol interference**

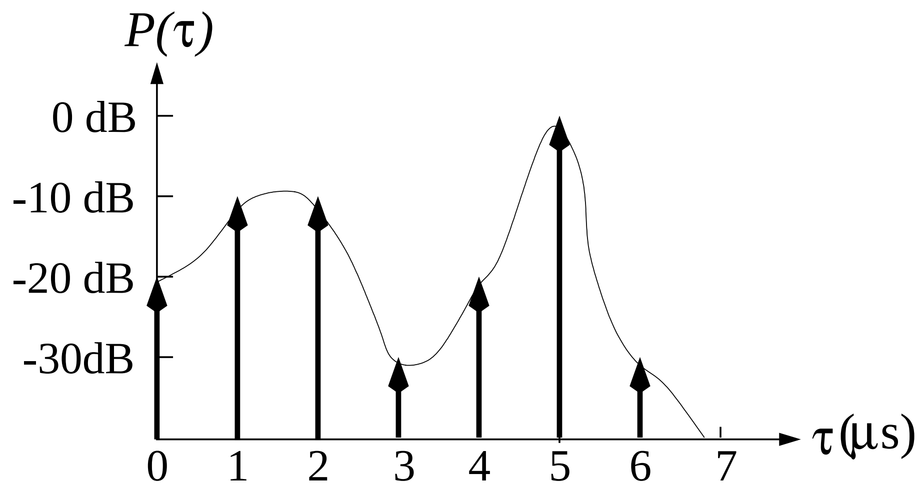
Power Delay Profile

- **Power delay profile** $P(\tau)$: channel power spectral density as a function of excess delay, i.e. how channel power is distributed along **dimension excess delay** τ

- Consider a local area around a spatial position, averaging $|h_B(t,\tau)|^2$ over time gives rise to $P(\tau)$

- Specifically, $P(\tau)$ is Fourier transform of autocorrelation function of $h_B(t,\tau)$

- Power delay profile: two-D curve over τ



- Power delay profile or power spectral density has "properties" of probability density function, so one can talks about moments of the underlying "stochastic process"
  - Again, it is useful to discretize excess delay τ into bins

Power Delay Profile: Statistics

- **Mean excess delay** is defined as the first moment of power delay profile, and in general

$$\bar{\tau} = \frac{\int P(\tau)\, \tau\, d\tau}{\int P(\tau)\, d\tau}$$

but with discretized excess delay

$$\bar{\tau} = \frac{\sum_i P(\tau_i)\tau_i}{\sum_i P(\tau_i)}$$

- **Root mean square** (RMS) **delay spread** is defined as the square root of the second central moment of power delay profile:

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

where with discretized excess delay the second moment is given by

$$\overline{\tau^2} = \frac{\sum_i P(\tau_i)\tau_i^2}{\sum_i P(\tau_i)}$$

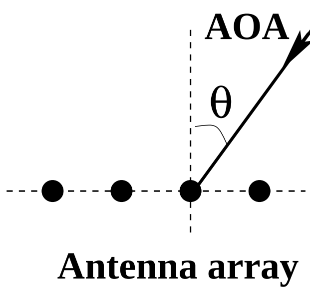
- **Coherence bandwidth** is a measure of the range of frequencies over which the channel is "flat" (i.e. passing spectral components with approximately equal gain and linear phase)
  - Coherence bandwidth $\propto \frac{1}{\sigma_\tau}$, and 50% coherence bandwidth is defined as:

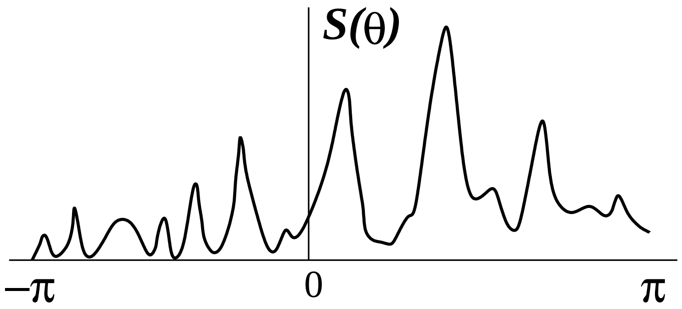
$$B_C \approx \frac{1}{5\sigma_\tau}$$

Angle Power Spectrum

- With antenna array sampling space, we may also consider power distribution in spatial or angular domain

- **Angle power spectrum** defines average power as a function of angle θ (angle-of-arrival for receive antenna and angle-of-departure for transmit antenna)





- Similar to delay power profile, we can define mean angle $\bar{\theta}$ and RMS angle spread σ_θ

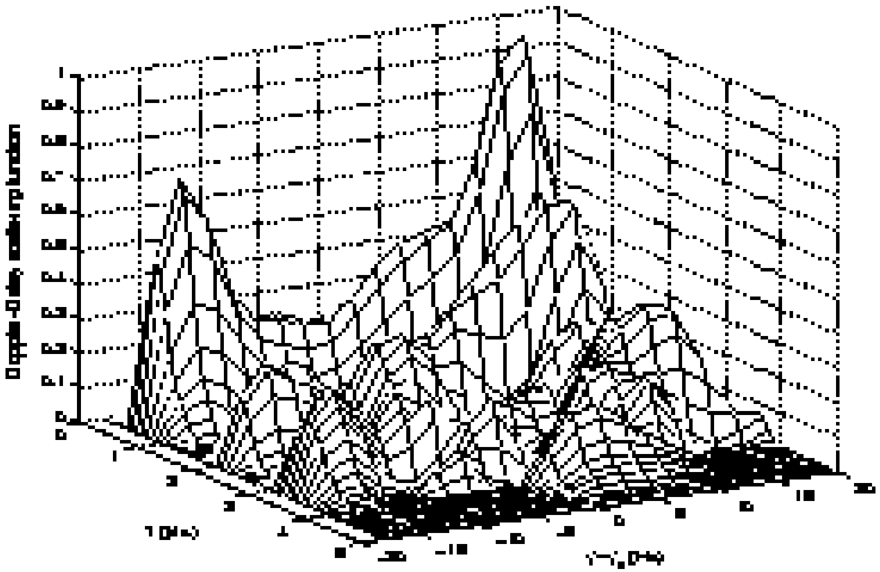
- **Angle spread** causes **space selective fading** $\rightarrow$ signal amplitude depends on spatial location of antenna/signals

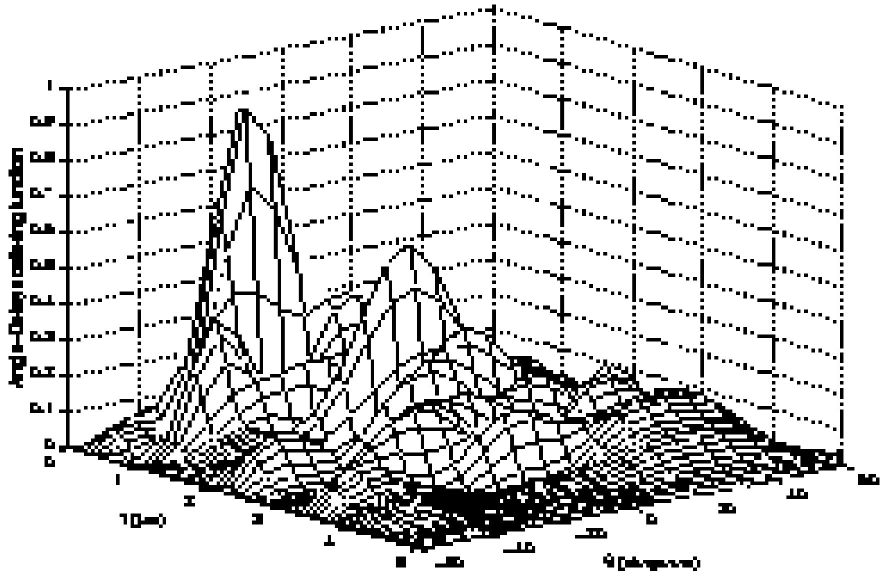
- **Coherence distance** D_C is spatial separation for which autocorrelation coefficient of spatial fading drops to 0.7

$$D_c \propto \frac{1}{\sigma_\theta}$$

Scattering Functions

- Complete channel statistics are captured in a triple scattering function: **Doppler-angle-delay scattering function** $S(f_D, \theta, \tau)$, four-D surface of three inputs
- Doppler-delay and angle-delay **scattering functions** $S(f_D, \tau)$ and $S(\theta, \tau)$ are two most widely used three-D **marginal spectra**





$$S(f_D, \tau) = \int S(f_D, \theta, \tau)\, d\theta, \quad S(\theta, \tau) = \int S(f_D, \theta, \tau)\, df_D$$

- Doppler, delay and angle power spectra $S(f_D)$, $P(\tau)$ and $S(\theta)$ are three two-D **marginal spectra** of single input, related to scattering function $S(f_D, \theta, \tau)$, e.g.

$$S(f_D) = \int\int S(f_D, \theta, \tau)\, d\theta d\tau, \quad P(\tau) = \int\int S(f_D, \theta, \tau)\, df_D d\theta$$

Summary

- Mobile channels are hostile due to:
  - Doppler spread which causes frequency dispersion
  - Multipath which causes time dispersion
- Doppler spectrum: speed broadens signal spectrum
  - Doppler PSD (spectrum): Doppler spread, and coherence time
  - What are **slow** and **fast** fading channels, and normalised Doppler frequency
- Multipath: excess delays of different copies of signal arrived
  - Power delay profile: mean excess delay, RMS delay spread, coherence bandwidth
- Complete characterisation of channel: Doppler-angle-delay scattering function
  - Doppler spectrum, power delay profile and angle power spectrum are its marginal spectra
  - Angle power spectrum: RMS angle spread, and coherence distance