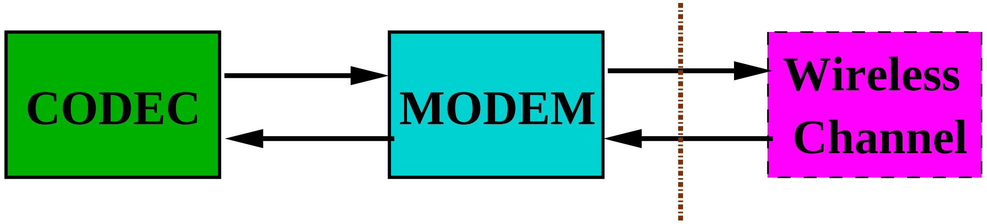
Revision of Wireless Channel

- Quick recap system block diagram

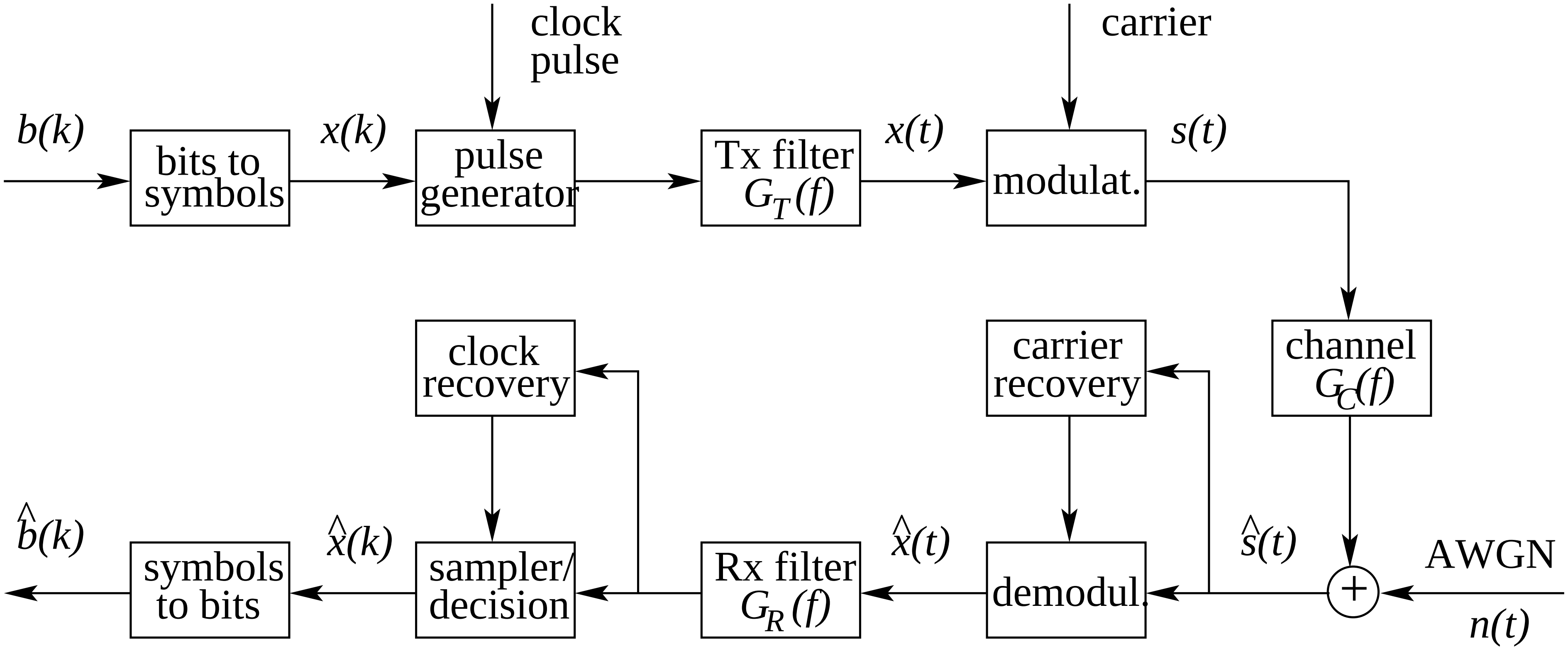


- Previous three lectures looked into **wireless mobile channels**
  - To understand mobile communication technologies, one needs a deep understand of **mobile communication media**
  - Two main sources of hostility in mobile media are **Doppler spread** and **multipath**
  - Many techniques developed are counter measures for **fading** and **frequency selective**
  - How spatial/angular dimension fits into wireless mobile landscape

- Front end of transceiver is Modem, which faces hostile (fading and frequency selective) communication media
  - Next eight lectures we will have a close look into Modem

Digital Modulation Overview

- In Digital Coding and Transmission, we learn schematic of MODEM (modulation and demodulation) with its basic components:



- The purpose of MODEM: transfer the bit stream at required rate over the communication medium reliably
  - Given system bandwidth and power resource

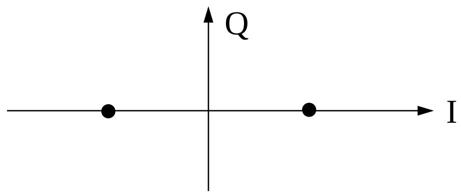
Constellation Diagram

- Digital **modulation signal** has finite states. This manifests in **symbol** (message) set: $\mathcal{M} = \{m_1, m_2, \cdots, m_M\}$, where each symbol contains $\log_2 M$ bits

- Or in modulation signal set: $\mathcal{S} = \{s_1(t), s_2(t), \cdots s_M(t)\}$. There is one-to-one relationship between two sets:

$$\mathcal{M} \overset{\text{modulation scheme}}{\longleftrightarrow} \mathcal{S}$$

- Example: BPSK, $M = 2$. Constellation diagram:



Modulation signal set:

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta) \qquad s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta)$$

- **Methods of modulation**: utilise amplitude, phase or frequency of carrier

Performance Measures

- Two key performance measures of a modulation scheme are power efficiency and bandwidth efficiency

- **Power efficiency** expressed as the ratio of the signal energy per bit (E_b) to the noise PSD (N_0) required to achieve a given probability of error (say 10^{-4}):

$$\eta_P = \frac{E_b}{N_0}$$ Small η_P is preferred

- **Bandwidth efficiency** defined as the ratio of the data bit rate R to the required RF bandwidth B_p:

$$\eta_B = \frac{R}{B_p} \text{ (bps/Hz)}$$ Large η_B is preferred

- **Channel capacity** gives an upper bound of achievable bandwidth efficiency:

$$\eta_{B_{\max}} = \frac{C}{B_p} = \log_2(1 + \text{SNR})$$

  – Capacity of ideal channel with Gaussian signal used here as limit for digital modulated signal

Modulation Schemes Classification

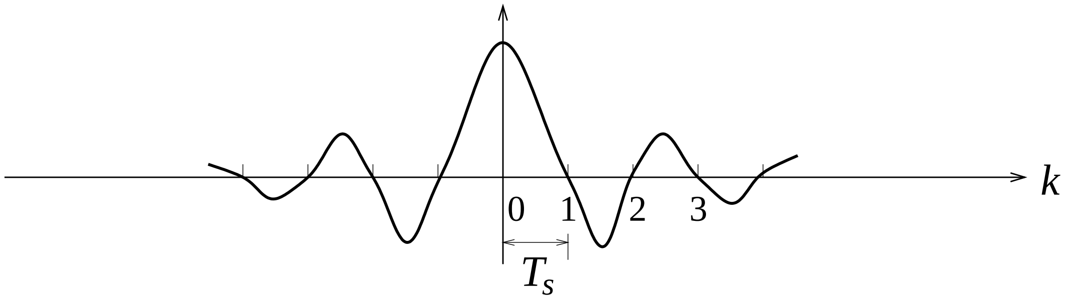
- According to pulse shaping techniques adopted
  - **Nyquist** pulse shaping: absolution bandwidth is finite, does not induce ISI
  - Nyquist modulation schemes are bandwidth more efficient but power less efficient, requiring expensive linear RF amplifier
  - **Non-Nyquist** pulse shaping: absolution bandwidth is infinite and can only be defined by e.g. 3 dB bandwidth, will induce certain level of ISI
  - Non-Nyquist modulation schemes are bandwidth less efficient but power more efficient, only requiring inexpensive nonlinear RF amplifier

- Modulation schemes can be classified as linear or nonlinear
  - **Linear** modulation: RF signal amplitude varies linearly with modulating digital signal, e.g. QAM
  - **Nonlinear** modulation: RF signal amplitude does not vary linearly with modulating digital signal, e.g. constant envelope modulation
  - Linear modulation is bandwidth more efficient but power less efficient, nonlinear modulation is reverse

Nyquist Pulse Shaping

- In Digital Coding and Transmission, we learn **Nyquist criterion for zero ISI**: The impulse response of the baseband system $h_{eff}(t)$ must satisfy

$$h_{eff}(kT_s) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$



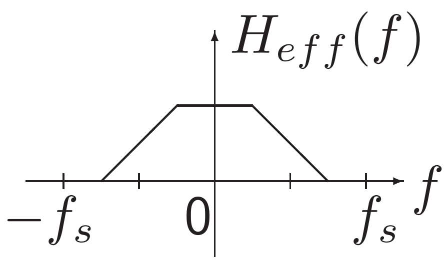
- Equivalently, the transfer function $H_{eff}(f)$ must satisfy

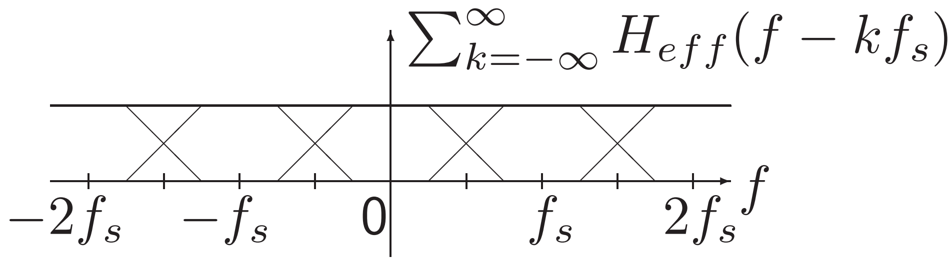
$$\sum_{k=-\infty}^{\infty} H_{eff}(f - kf_s) = \text{constant}, \text{ for } |f| < \frac{f_s}{2}$$

  – where T_s is the symbol period and $f_s = \frac{1}{T_s}$ the symbol rate

Nyquist Pulse Shaping (Implication)

- Illustration of condition for zero ISI, seeing from frequency domain:





- Note that $H_{eff}(f) = H_T(f)H_{Ch}(f)H_R(f)$. Assuming $H_{Ch}(f) = 1$, the transmit and receive filter pair provides the desired spectrum shape:

$$H_T(f)H_R(f) = H_{eff}(f)$$

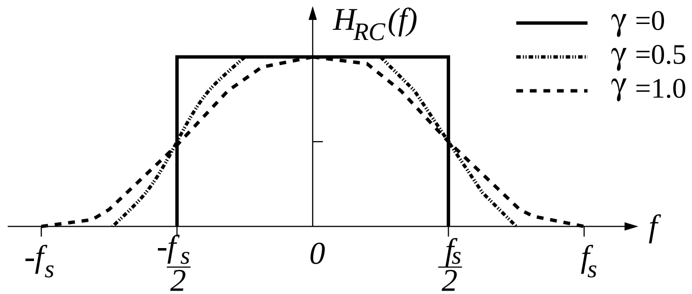
- The **minimum** required baseband **bandwidth** for zero ISI is $B_{\min} = \frac{f_s}{2}$, and this corresponds to the sinc pulse shaping

- Recall that given the baseband signal bandwidth B, the required RF bandwidth is $B_p = 2B$

Raised Cosine Pulse Shaping Filter

- The required baseband bandwidth $f_s/2 \leq B \leq f_s$, and the spectrum:

$$H_{RC}(f) = \begin{cases} 1 & |f| \leq \frac{f_s}{2} - \beta \\ \cos^2\left(\frac{\pi}{4\beta}|f| - \frac{f_s}{2} + \beta\right) & \frac{f_s}{2} - \beta < |f| \leq \frac{f_s}{2} + \beta \\ 0 & |f| > \frac{f_s}{2} + \beta \end{cases}$$



- where β is the extra bandwidth over the minimum $f_s/2$, and roll-off factor γ:

$$\gamma = \frac{\beta}{f_s/2} = \frac{B - f_s}{f_s} \quad \text{or} \quad B = \frac{f_s}{2}(1+\gamma)$$

- Widely used raised cosine filter achieves zero ISI, but requires power less inefficient linear amplifier

Example

• In GSM, the RF channel bandwidth is 200 kHz and the data rate is 270.833 kbps. The bandwidth efficiency is:

$$\eta_B = \frac{270.833}{200} \approx 1.4 \text{ bits}$$

For SNR=10 dB=10, such a channel has

$$\eta_{B\text{max}} = \log_2(1 + 10) \approx 3.5 \text{ bits}$$

The bandwidth efficiency of GSM under the SNR=10 dB is only 40% of the limit:

$$\frac{\eta_B}{\eta_{B\text{max}}} \approx 40\%$$

(Note: GSM uses 2 bits per symbol digital modulation scheme, so its channel capacity is smaller than ideal Gaussian signal. Thus actual GSM bandwidth efficiency is more than 40%)

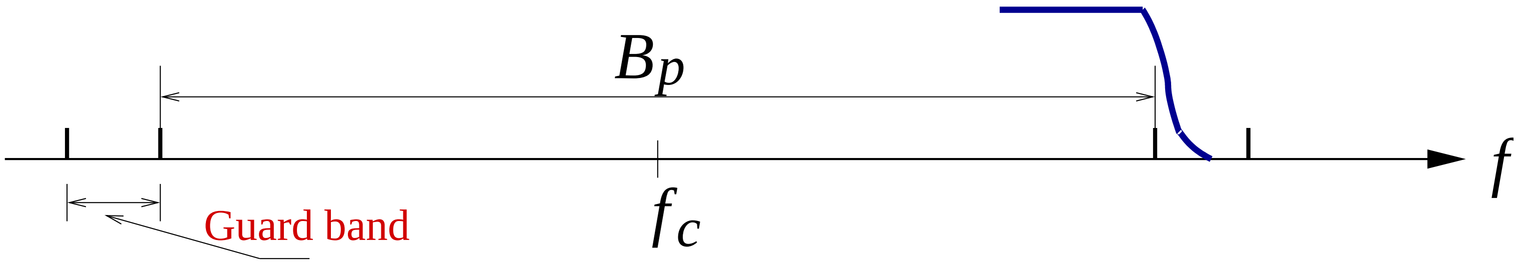
• The symbol period is $T_s = 41.06\ \mu$s and the raised cosine filter has a roll-off factor $\gamma = 0.35$. The filter (absolute) baseband bandwidth is

$$B = \frac{(1 + \gamma)}{2T_s} = 16.44 \text{ kHz}$$

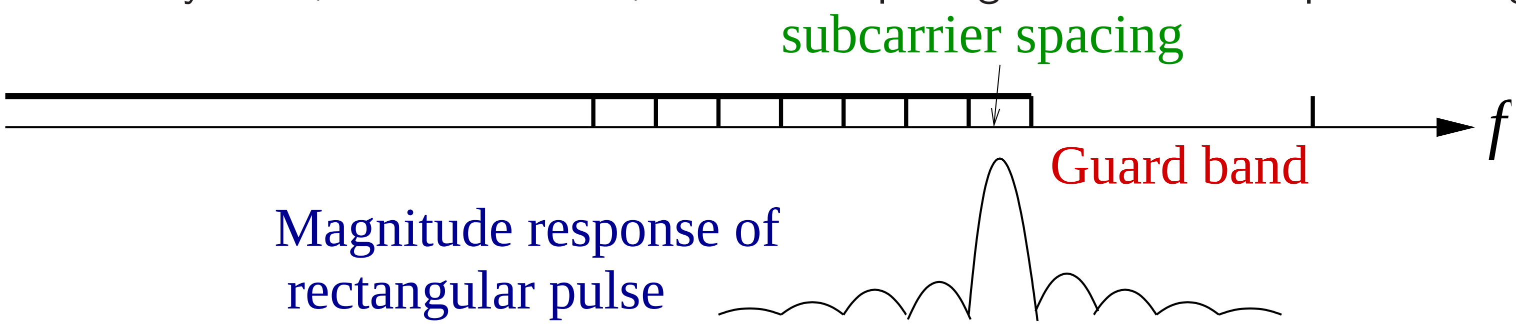
The require RF channel bandwidth is $B_p = 2B = 32.44$ kHz

Bandwidth Efficiency / Complexity

- Roll off factor γ of Nyquist pulse shaping determines bandwidth efficiency and implementation complexity
  - Larger roll off factor is bandwidth less efficient but less complex in implementation, and vice versu
- For single-carrier systems with wideband signals, such as DTV, pulse shaping must be very tight, say $\gamma \leq 0.1$



  - Channel bandwidth B_p is very large, 0.1 of which may be significant compared with guard band
- For multi-carrier systems, such as OFDM, subcarrier spacing is small in comparison to guard band



  - In fact, often no pulse shaping is required for multi-carrier OFDM systems

Gaussian Pulse-Shaping Filter

- This is a non-Nyquist pulse shaping filter with the transfer function:

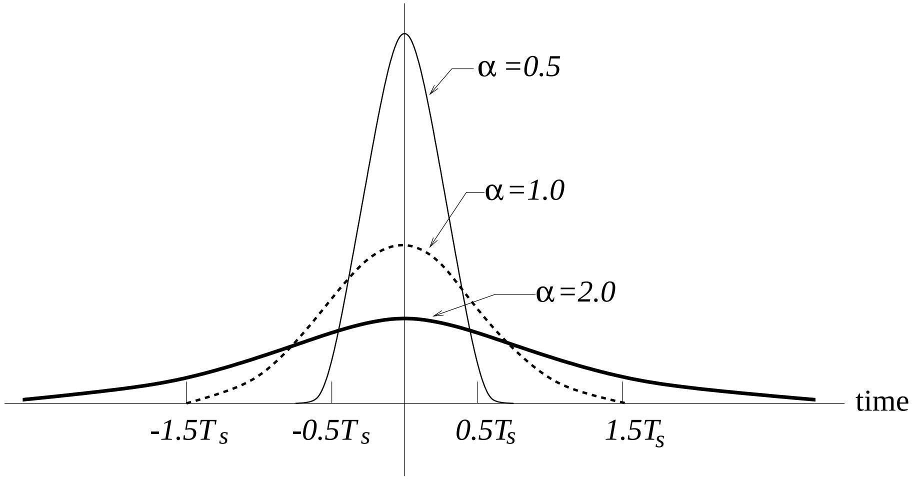
$$H_G(f) = \exp(-\alpha^2 f^2)$$

absolute bandwidth is infinity, α and 3-dB baseband bandwidth B satisfy

$$\alpha B = \frac{\sqrt{\ln 2}}{\sqrt{2}} = 0.5887$$

- Impulse response is:

$$h_G(t) = \frac{\sqrt{\pi}}{\alpha} \exp\left(-\frac{\pi^2}{\alpha^2} t^2 \right)$$



- This pulse shaping filter introduces ISI but only requires power efficient nonlinear amplifier: trade off is required between reducing RF bandwidth and increasing ISI
  - As α increases, required (3-dB) bandwidth decreases (Gaussian spectrum narrower) but ISI level increases (Gaussian time pulse wider)

Non-Nyquist Pulse Shaping: Practical Considerations

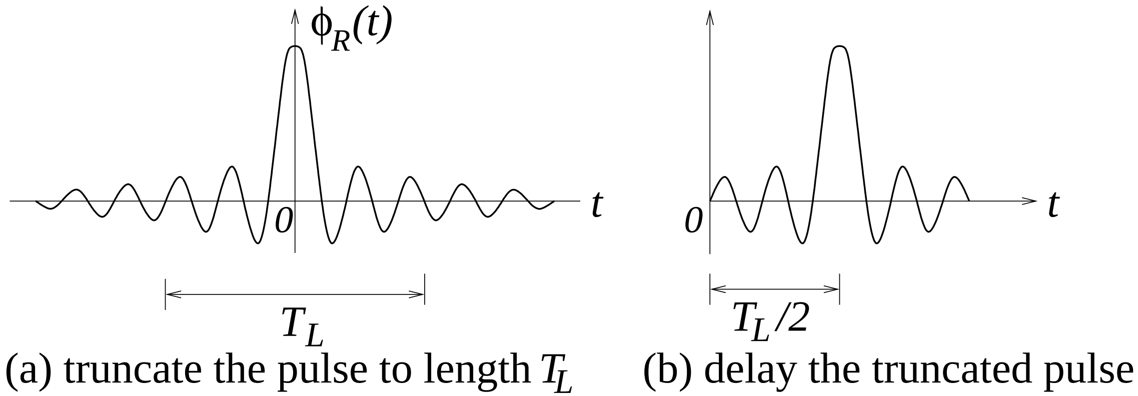
- Advantage and disadvantage of non-Nyquist pulse shaping
  - Power more efficient as inexpensive and power efficient nonlinear amplifier can be used, but bandwidth less efficient and introduce intersymbol interference

- Design trade off is between reducing RF bandwidth and increasing ISI
  - Modulation schemes with non-Nyquist pulse shaping introduced controlled ISI
  - System CIR is limited in length and known to both transmitter and receiver
  - Even introduced ISI may be very limited, it must be alleviated
  - **Pre-coding** or **pre-equalisation** can be implemented to eliminate ISI

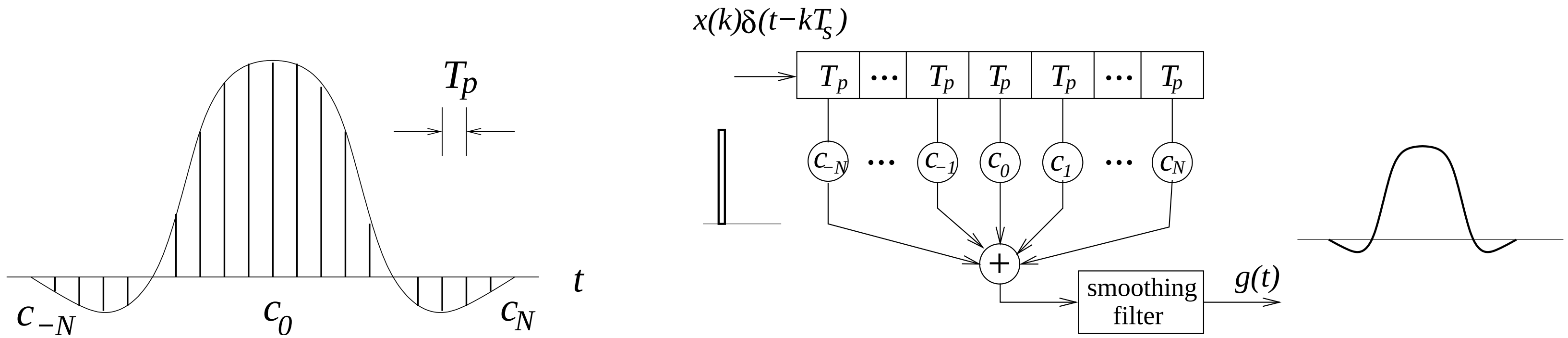
- Since transmitter knows CIR, pre-equalisation at transmitter is advantageous, as no noise enhancement problem, e.g. system CIR is $1 + a \cdot z^{-1}$ with $|a| < 1$
  - Pre-coding or pre-equalisation is implemented as $\frac{1}{1+a \cdot z^{-1}}$, which codes data symbols $\{x(k)\}$ to $\{y(k) = x(k) - a \cdot y(k-1)\}$ for transmission

Practical Implementation

- In practice, a pulse shaping filter must be **causal** to be realisable
- Consider theoretical raised cosine pulse shaping filter, which is non-causal
  - Truncate the infinite pulse to a finite but sufficient length and delay it



  - Sampled values are obtained from the waveform of the truncated pulse and an FIR or transversal filter is used to realize the required Tx / Rx filters:



Summary

- Modulation overview: basic components of MODEM

1. Symbol set (constellation diagram) $\underleftrightarrow{\text{One} - \text{to} - \text{One}}$ modulation signal set
2. Modulation methods: use amplitude, phase or frequency of carrier
3. Performance measures: power efficiency and bandwidth efficiency, upper bound limit of bandwidth efficiency (channel capacity)

- Pulse shaping:

1. Nyquist pulse shaping: Nyquist criterion for zero ISI, minimum bandwidth for zero ISI, raised cosine (roll off factor) pulse shaping
2. Non-Nyquist pulse shaping: Gaussian (α parameter) pulse shaping, trade off between reducing RF bandwidth and increasing ISI
3. Practical implementation: causal realisation of pulse shaping filter, pre-coding for non-Nyquist pulse shaping