Revision of Lecture Five

Recall basic components of MODEM in slide 58

- A central point is associated with a digital modulation scheme, there is **channel capacity**, as we learn in Digital Coding and Transmission
  - This channel capacity is smaller than the channel capacity of ideal AWGN channel with Gaussian signal – recall capacity is maximised if signal PDF is Gaussian
  - Nevertheless, we may use the latter as upper limit for our practical digital modulated channel as first approximation

- As channel capacity is linked with bandwidth and signal to noise ratio, not surprisingly, performance measures of a digital modulation scheme are:
  - **Power efficiency** and **bandwidth efficiency**

- This lecture we continue on Modem, and look into **phase shift keying modulation**, in particular, BPSK and QPSK
  - with emphasis on operations of **carrier recovery** and **timing recovery**
  - and introduce concepts of coherent and non-coherent systems

Phase Shift Keying

- In PSK, the modulation signal set is:

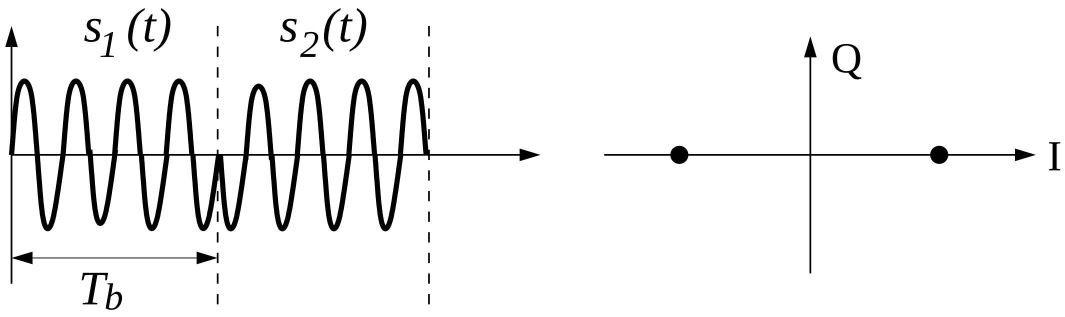
$$s_i(t) = A\cos(2\pi f_c t + \phi_i(t)), \; i = 1, \cdots, M, \; 0 \le t \le T_s$$

  - T_s is symbol period, A is carrier amplitude (constant), "**phase**" $\phi_i(t)$ carries symbol information, and $\log_2 M$ bits per symbol
  - BPSK, QPSK, 8-PSK, etc with 1 bit per symbol, 2 bits per symbol, 3 bits per symbol, etc, and minimum phase separation 180°, 90°, 45°, etc, respectively

- BPSK: $M = 2$. It is convention to use $m_1 = 1$ for bit 0, and $m_2 = -1$ for bit 1

$$s_i(t) = A\cos(2\pi f_c t + (i-1)\pi + \theta_c), \; 0 \le t \le T_b$$

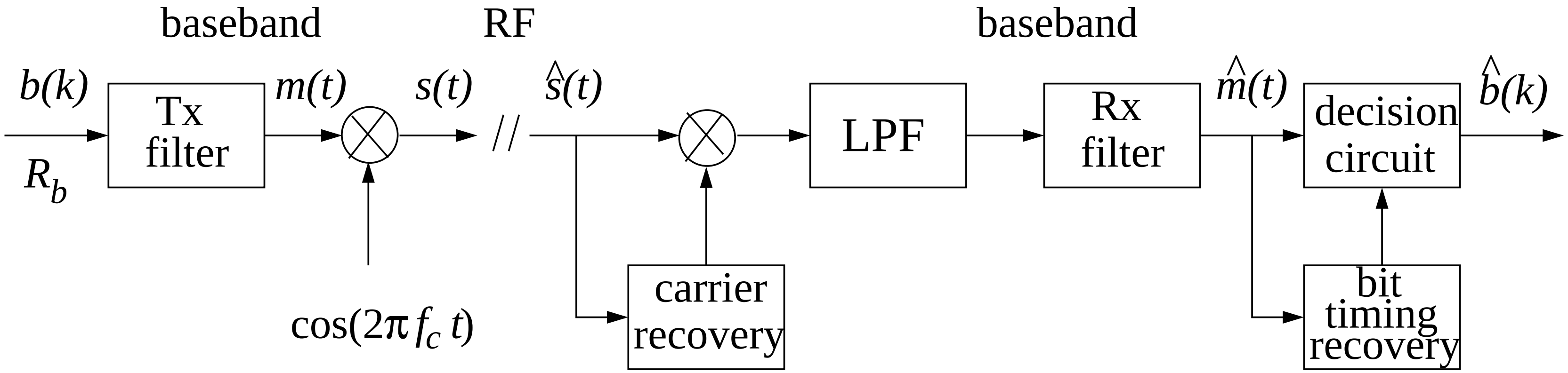
  - T_b: bit period
  - θ_c: an initial phase
  - Energy per bit for BPSK: $E_b = \frac{1}{2}A^2 T_b$ or $A = \sqrt{\frac{2E_b}{T_b}}$



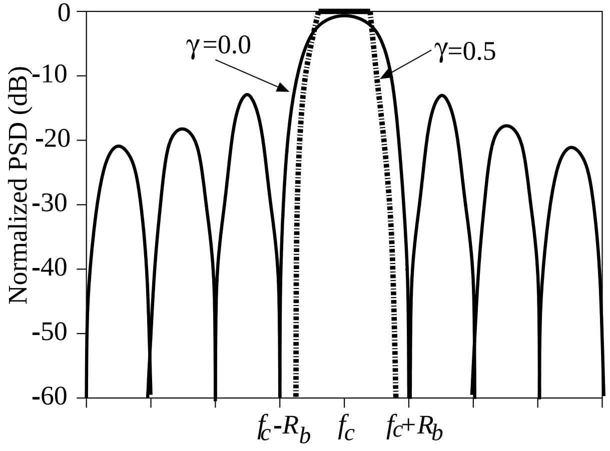
Note on $s_i(t)$ waveform: as carrier frequency f_c is very large, e.g. GHz, in each symbol period, there are millions of carrier waveform periods

BPSK MODEM

- Simplified BPSK **MODEM** diagram



- **Transmitter**: Data bit stream with bit rate R_b are filtered by a lowpass filter (square root of raised cosine pulse shaping filter) to generate baseband signal $m(t)$, which is then modulated by carrier
  - PSD of BPSK RF signal with raised cosine pulse shaping:
  - Baseband complex envelope signal



$$g(t) = m(t)A\exp(j\theta_c)$$

$m(t)$ being pulse shaped symbol m_1 or m_2

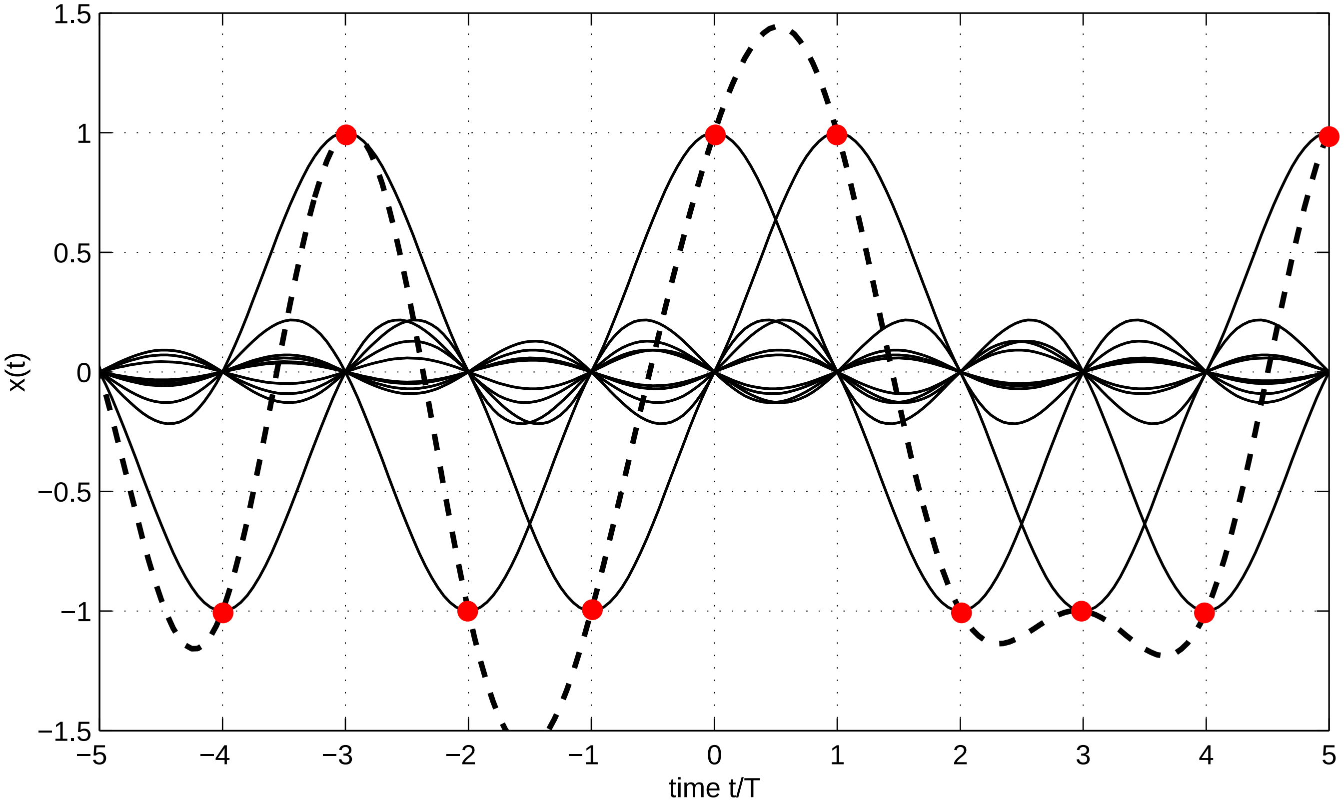
  - Transmitted BPSK signal

$$s(t) = \text{Re}[g(t)\exp(j2\pi f_c t)] = m(t)A\cos(2\pi f_c t + \theta_c)$$

- BPSK symbol information ± 1 are carried in baseband signal $m(t)$

BPSK Baseband Signal

- Recall in Digital Coding and Transmission, we learn BPSK **baseband signal** $m(t)$ is the dashed curve, which carries BPSK symbol information



- Transmitted RF signal $s(t)$ is obtained by modulating **carrier** $A\cos(2\pi f_c t + \theta_c)$ by **modulating signal** $m(t)$

BPSK MODEM: Receiver

- **Receiver**: received RF signal is given by $\widehat{s}(t) = \alpha \cdot m(t) A \cos(2\pi f_c t + \theta) + n(t)$
  - α: channel gain or attenuation, θ: random phase including phase shift due to channel delay, $n(t)$: channel AWGN
- For illustrating basic concept of demodulation, assume no channel distortion, omit noise and drop amplitude A, then received RF signal is simplified to

$$\widehat{s}(t) = m(t) \cos(2\pi f_c t + \theta)$$

  - Carrier recovery must obtain **carrier** $\cos(2\pi f_c t + \theta)$ in order to demodulate $\widehat{s}(t)$:

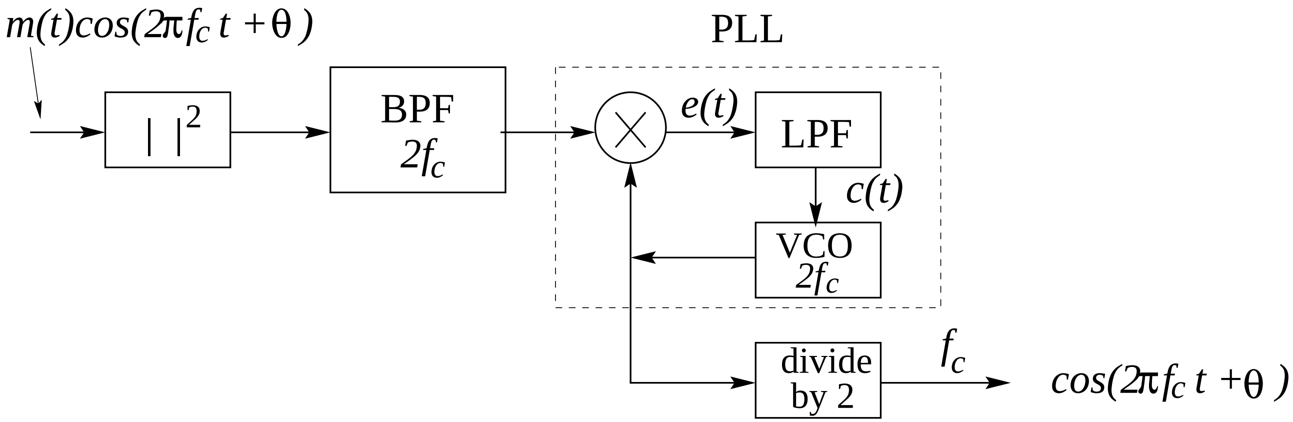
$$\widehat{s}(t) \cdot \cos(\omega_c t + \theta) = \frac{m(t)}{2} \cdot (1 + \cos(2\omega_c t + 2\theta))$$

  - The LPF at receiver then filters this to obtain baseband signal $m(t)$
  - Bit (symbol) timing recovery recovers clock pulses to obtain samples at appropriate instances for decision circuit, which detects transmitted bits (symbols)
- Carrier recovery: operate at RF to try to align receiver local oscillator with transmitted carrier frequency (and phase), which is only required for coherent or synchronous demodulation
- Clock recovery: operate at baseband to try to synchronise receiver clock with baseband symbol rate transmitter clock, which is needed for any receiver (coherent or non-coherent demodulation)
- Coherent receiver has better performance but higher complexity than non-coherent receiver

Carrier Recovery for BPSK

- Let received RF signal be $\widehat{s}(t) = m(t) \cdot \cos(\omega_c t + \theta)$
  - If receiver knows carrier $\cos(\omega_c t + \theta)$, it can use this information to demodulate $\widehat{s}(t)$ so as to obtain baseband signal $m(t)$
  - Recover the carrier (phase): **time-2 carrier recovery** scheme, which works well for BPSK signals, but not for quadrature signals with equal average power in each quadrature branch

- **Time-2 carrier recovery**:
  - Square device, BPF, PLL which produces $\cos(2\omega_c t + 2\widehat{\theta})$, and
  - frequency divider which generates $\cos(\omega_c t + \widehat{\theta})$



- Nonlinear square device generates $m^2(t)\cos^2(\omega_c t+\theta) = \frac{1}{2}m^2(t)\big(1+\cos(2\omega_c t+2\theta)\big)$. BPF centred at $2f_c$ gets $\cos(2\omega_c t + 2\theta)$ and uses it to drive a phase locked loop

Time-2 Carrier Recovery: PLL

- **Phase locked loop** consists of a lowpass filter, a multiplier and a **voltage controlled oscillator**
  - VCO oscillates at $2f_c$ with an initial phase $\widehat{\theta}$, and its output, $\sin(2(\omega_c t + \widehat{\theta}))$, is multiplied by $\cos(2(\omega_c t + \theta))$ to obtain

$$e(t) = \frac{1}{2}\sin(4\omega_c t + 2(\theta + \widehat{\theta})) + \frac{1}{2}\sin(2(\theta - \widehat{\theta}))$$

  - The first term is removed by the LPF, while the second term

$$c(t) = \frac{1}{2}\sin(2(\theta - \widehat{\theta})) \approx \Delta\theta \quad \text{if} \quad \Delta\theta \ll 1$$

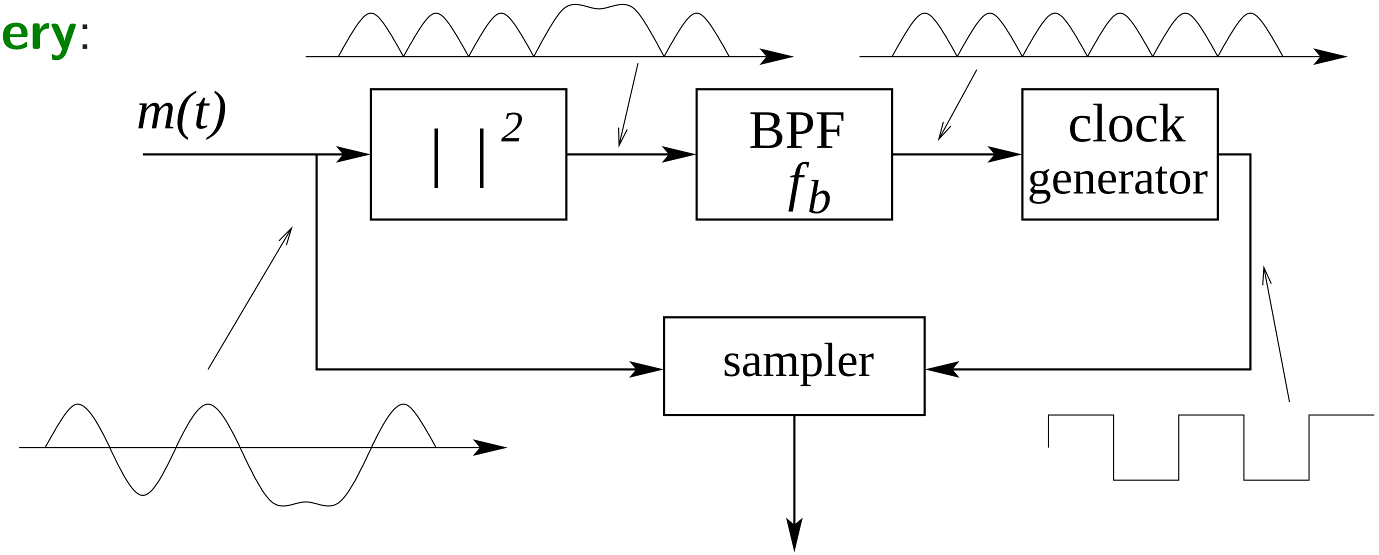
    is used to drive the VCO, so that its phase $\widehat{\theta}$ locks to θ, i.e. $\widehat{\theta} \to \theta$
  - In order for $c(t) \to 0$, initial phase $\widehat{\theta}$ of VCO should not be far away from true carrier phase θ

- Frequency divider then divides $\cos(2(\omega_c t + \theta))$ to produce **the carrier** $\cos(\omega_c t + \theta)$

Alternative PLL with its VCO operates at f_c – Work out its circuit diagram

Time-2 Clock Recovery

- Properly sample baseband signal $m(t)$ is vital to recover transmitted data, and for BPSK signal, we have
- **Time-2 clock recovery**:



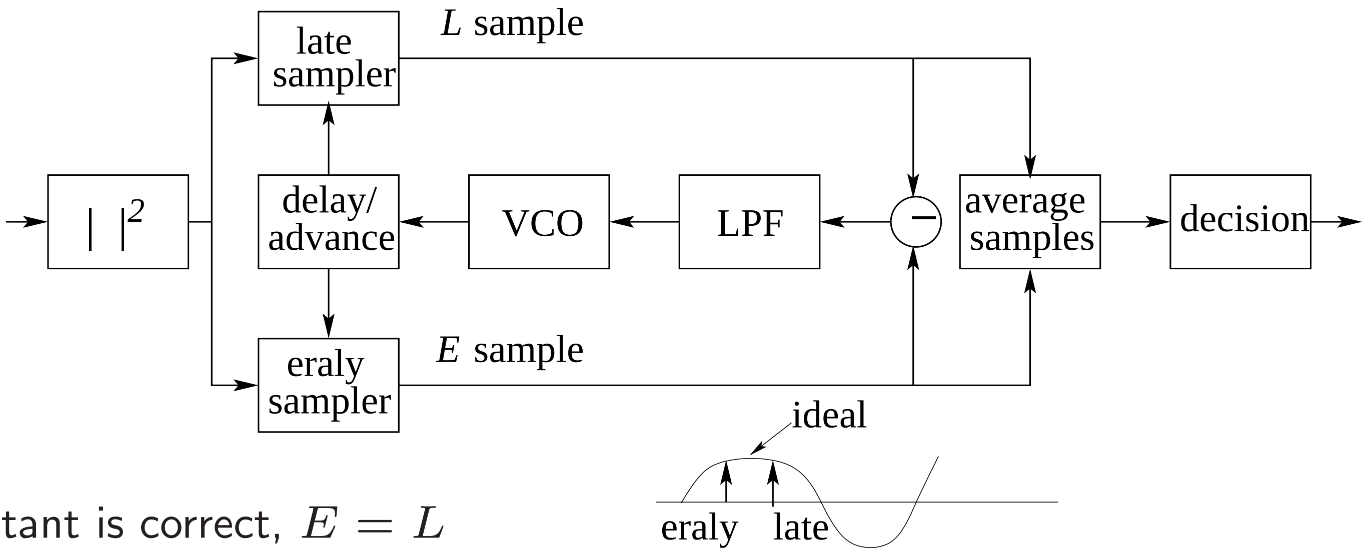
  - When the received demodulated signal is squared, it possesses a periodic frequency domain component at the symbol rate. A BPF tuned close to the symbol rate extracts this periodic signal. A clock pulse regenerator (a saturating amplifier) produces a rectangular pulse shape with the required timing information

- It works well for binary modulation schemes, but not so for multilevel signalling schemes, as the symbol rate component is less clear in the squared signal

Early-Late Clock Recovery

- **Early-late clock recovery**: It takes two samples E, L, both equi-spaced around predicted sampling instant

  Assumption is peaks in squared waveform are correct sampling points



  - If predicted sampling instant is correct, $E = L$
  - If $E > L$, recovered clock is sampling too late
  - If $E < L$, recovered clock is sampling too early
  - With LPF used to reduce noise, filtered difference signal adjusts frequency of VCO to delay or advance the arrival of next clock impulse, and results are averaged over several samples

- It works well for binary modulation schemes but less so with multilevel ones, as there are fewer distinctive peaks for a (squared) multilevel baseband signal

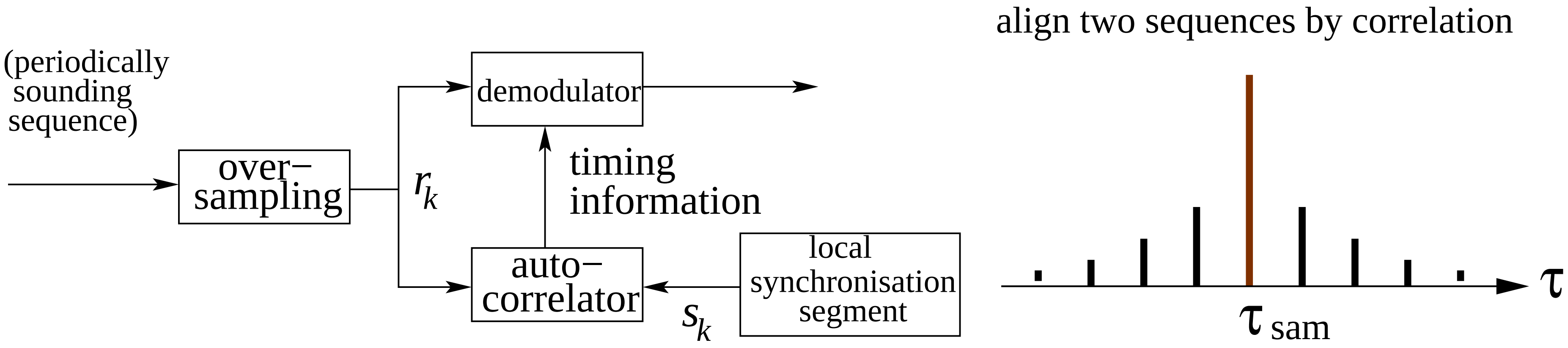
Zero Crossing Clock Recovery

- **Zero crossing clock recovery**: it looks for zero crossings at the incoming waveform, instead of peaks of incoming squared waveform as in the early-late scheme
  - For binary signalling, the circuit is similar to that of the early-late
  - This works as for symmetrical signalling the received waveform will pass through zero midway between the sampling points

- The time-2, early-late, and zero crossing clock recovery schemes all work well for binary signalling but less successful for multilevel signalling
  - The time-2 clock recovery performs poorly for multilevel signalling
  - As zero crossing is not always at middle of sampling period for multilevel signalling, a control logic block is needed for zero crossing scheme to enable or disable adjustments,
    - ∗ If a transition occurs between two symbols of equal magnitude but opposite polarity, the zero crossing associated with this transition is in the middle of a symbol period, and only these zero crossings are used to update the timing
  - Later we will discuss modified early-late clock recovery for multilevel signalling

Synchroniser Clock Recovery

- **Synchroniser** clock recovery:



  - Transmitter periodically sends a **sounding** or training sequence
  - Receiver aligns local training sequence with received sounding sequence by correlation
- Let r_k be the oversampled (typically 4 times of symbol rate) received sounding signal and s_k the locally generated sounding signal
  - Align the two sequences by shifting correction lag τ to find peak in correlation

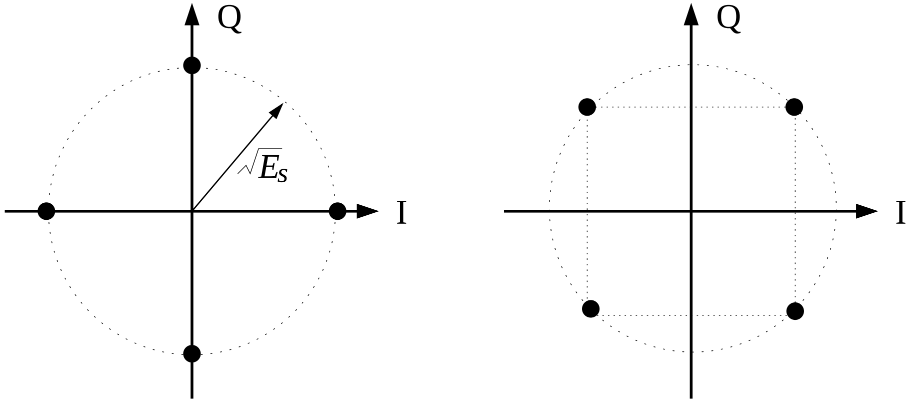
$$\tau_{\text{sam}} = \arg\max_{\tau} \sum_k r_{k+\tau} \cdot s_k$$

  - τ_{sam} at which the **maximum correlation** occurs is the correct sampling point
- This method works well for both binary and multilevel modulation schemes but requires extra bandwidth overhead for sounding sequence

Quadrature Phase Shift Keying

- QPSK: $M = 4$, 2 BPS, symbol period $T_s = 2T_b$, energy per symbol $E_s = 2E_b$
- QPSK signal constellation:

  Minimum phase separation is 90°



- QPSK signal set: $i = 1, 2, 3, 4$,

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + (i-1)\frac{\pi}{2}\right) \quad 0 \le t \le T_s$$

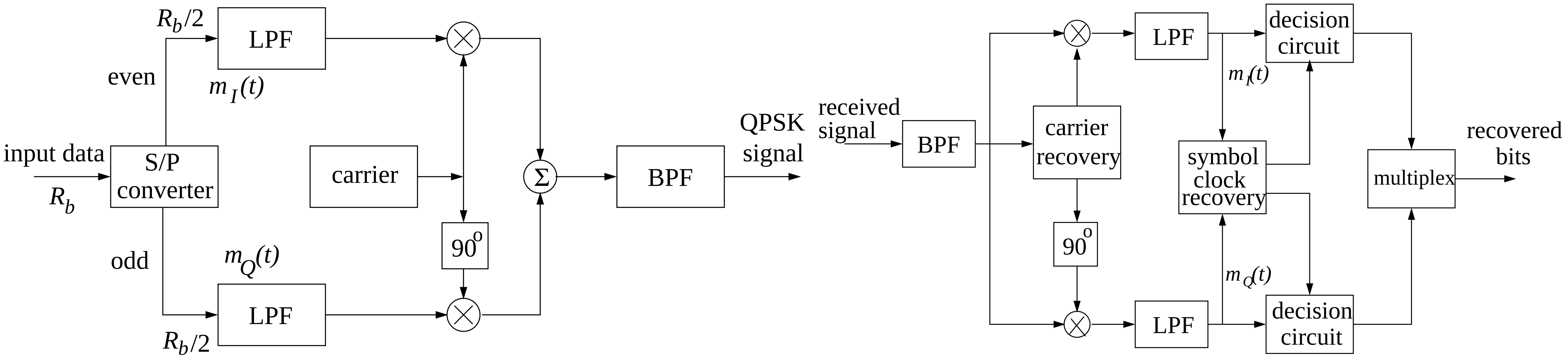
- The transmitted QPSK RF signal can also be written as:

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left((i-1)\frac{\pi}{2}\right) \cos(\omega_c t) - \sqrt{\frac{2E_s}{T_s}} \sin\left((i-1)\frac{\pi}{2}\right) \sin(\omega_c t)$$

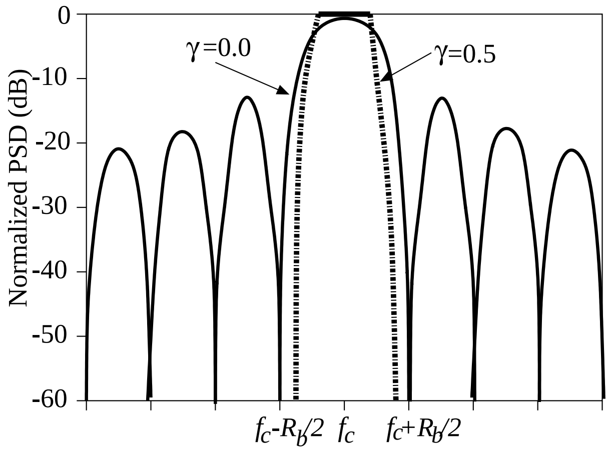
Note both inphase and quadrature branches (two orthogonal carriers) are utilised

QPSK Modem

- QPSK Modem: S/P splits bit stream into inphase and quadrature streams, Tx/Rx LPF pair are for pulse shaping, two orthogonal carriers are used for inphase and quadrature carrier modulations



- QPSK RF signal PSD (raised cosine):
- BPF at transmitter limits signal power spectrum within allocated band, as raised cosine pulse shaping is truncated
- BPF at receiver filters out out of band noise
- Time-2 carrier recovery **does not work** for QPSK
- As I and Q are BPSK, all clock recovery schemes work
- P/S after decision circuits recovers original data streams



Differential Binary Phase Shift Keying

- **Coherent receiver** requires to know the carrier phase, which is a difficult task
- **Non-coherent receiver** with no carrier recovery is easy and cheap to build, but the problem:
  - Received signal $\widehat{s}(t) = A\cos\left(\omega_c t + \varphi\right) + n(t)$, and receiver local carrier $\cos\left(\omega_c t + \widetilde{\varphi}\right)$
  - No carrier recovery, hence $\phi = \widetilde{\varphi} - \varphi \neq 0$, and demodulated baseband signal

$$y(t) = x(t+\tau)e^{\mathrm{j}\phi} + n(t)$$

  - Timing recovery to sample $y(t)$ results in $y_k = x_k e^{\mathrm{j}\phi} + n_k$
  - There is a random **unknown** channel state information $e^{\mathrm{j}\phi}$, could not recover transmitted symbols $\{x_k\}$ properly from $\{y_k\}$
  - Other means must be adopted to resolve this problem, including **differential encoding**
- At DBPSK transmitter, bit sequence $\{b_k\}$ with $b_k \in \{0, 1\}$ are differentially encoded by

$$d_k = \overline{b_k \oplus d_{k-1}}$$

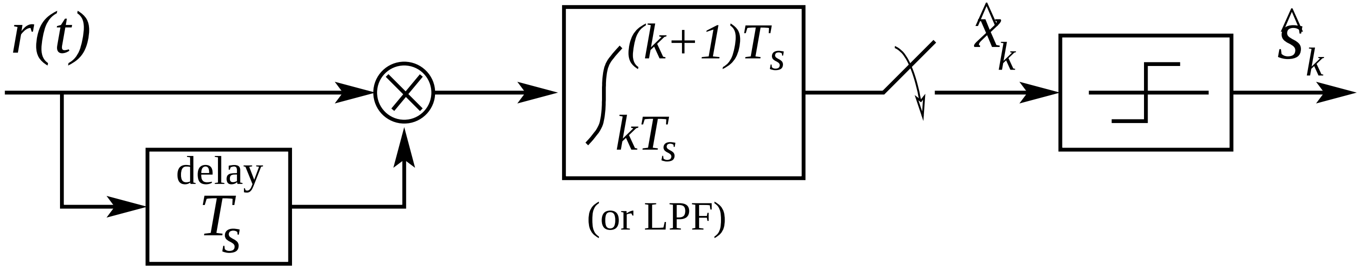
  - Initial condition d_0 is given and known to both transmitter and receiver, e.g. $d_0 = 1$
  - Encoded bit sequence $\{d_k\}$ is then BPSK modulated into BPSK symbol sequence $\{s_k\}$
  - and transmitted DBPSK signal

$$x(t) = \sqrt{\frac{2E_s}{T_s}}\cos\left(2\pi f_c t + \frac{1}{2}(1 - s_k)\pi\right), \quad kT_s \leq t < (k+1)T_s$$

where f_c is carrier frequency, E_s symbol energy, and T_s symbol duration

DBPSK: Non-Coherent Receiver

- For **noncoherent demodulator**, no local carrier is needed



  - Let $A^2 = \frac{2E_s}{T_s}$ and $\frac{1}{2}(1 - s_k)\pi = \varphi_k$, then received RF signal (minus noise)

$$r(t) = A\cos\left(2\pi f_c t + \varphi_k + \vartheta\right), \; kT_s \le t < (k+1)T_s$$

  - Noting $\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$, input to integrator

$$A^2\cos\left(4\pi f_c t + 2\vartheta + \varphi_k + \varphi_{k-1}\right) + A^2\cos\left(\varphi_k - \varphi_{k-1}\right)$$

  - First term average over one period is zero, and $\cos\left(\varphi_k - \varphi_{k-1}\right) = 1$ if $s_k = s_{k-1}$; $\cos\left(\varphi_k - \varphi_{k-1}\right) = -1$ if $s_k = -s_{k-1}$
  - Hence, after timing recovery, from sampled signal $\{\hat{x}_k\}$, we know:

$$\hat{x}_k = \begin{cases} E_s, & s_k = s_{k-1} \\ -E_s, & s_k = -s_{k-1} \end{cases}$$

  - Detection is achieved by $\{\hat{x}_k\} \rightarrow \{\hat{s}_k\} \rightarrow \{\hat{d}_k\} \rightarrow \{\hat{b}_k\}$

Summary

- Phase shift keying: general form, phase carries symbol information
  - Bits per symbol: define system throughput or bandwidth efficiency
  - Coherent receiver: require carrier phase reference
  - BPSK: signal waveform and constellation, 1 BPS, transmitter/receiver
  - QPSK: signal waveform and constellation, 2 BPS, transmitter/receiver
  - DBPSK: for non-coherent receiver with differential encoding
- Time-2 carrier recovery: suitable for binary modulation scheme
- Clock recovery
  - Time-2, early-late, zero crossing clock recovery schemes: suitable for binary signalling
  - Synchroniser clock recovery: suitable for any signalling schemes at cost of a reduced bandwidth efficiency
- Carrier recovery and clock recovery are important components of transceiver