Revision of Lecture Six

- We have discussed phase shift keying, in particular, one bit per symbol BPSK and two bits per symbol QPSK
- Carrier recovery operation and clock recovery operation, in particular,
  - Time-2 carrier recovery: suitable for binary modulation scheme
  - Time-2, early-late, zero crossing clock recovery schemes: suitable for binary signalling, and synchroniser clock recovery: generally applicable

  Carrier recovery and clock recovery are important, as each transceiver has a pair
- This lecture looks into bandwidth much more efficient modulation scheme called **quadrature amplitude modulation**, with emphasis on general **design considerations** of digital modulation scheme
- From channel capacity, one can trade off between bandwidth and SNR
  - When one gains bandwidth efficiency, one has to pay penalty in power efficiency

Linear and Nonlinear Modulation

- PSK belongs to the class of **linear modulation**, where the RF signal amplitude varies linearly with the modulating digital signal
  - This can be seen from the PSK RF signal:

$$s_i(t) = A\left(\cos(\phi_i(t))\cos(\omega_c t) - \sin(\phi_i(t))\sin(\omega_c t)\right)$$

  - Another example is amplitude shift keying

- Linear modulation techniques are **bandwidth** more efficient but **power** less efficient, requiring expensive linear amplifiers

- A class of **nonlinear modulation** are constant envelope modulation techniques
  - An example is frequency shift keying

- Nonlinear modulation techniques are **bandwidth** less efficient but **power** more efficient, no need for expensive linear amplifiers

Frequency Shift Keying

- FSK: a **constant envelope** modulation, f_i carries symbol information

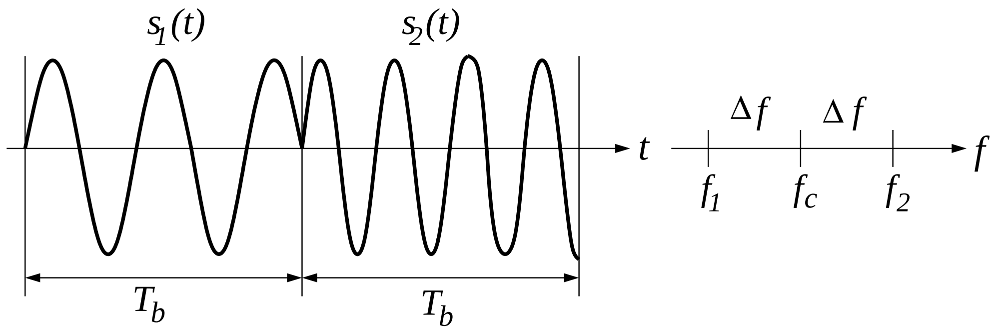
$$s_i(t) = A\cos(2\pi f_i t + \theta),\ 1 \le i \le M,\ 0 \le t \le T_s$$

  - BFSK, QFSK, etc. **Coherent** and **non-coherent** detection can be used

- BFSK: $M = 2$, bit 0: $f_1 = f_c - \Delta f$, bit 1: $f_2 = f_c + \Delta f$

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi(f_c - \Delta f)t)$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi(f_c + \Delta f)t)$$



- BFSK RF bandwidth (assume raised cosine): $B_p = 2\Delta f + 2B = 2\Delta f + (1+\gamma)R_b$, where B is the baseband digital signal bandwidth

  - By comparison, BPSK RF bandwidth $B_p = (1+\gamma)R_b$

Continuous Phase FSK

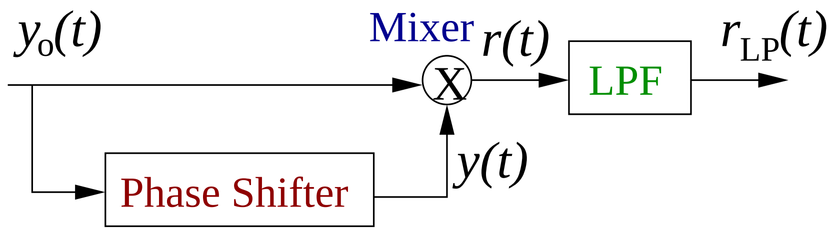
- Continuous phase FSK received signal can be expressed as

$$y_o(t) = A\cos\left(2\pi f_c t + 2\pi h \int_{-\infty}^{t} m(\tau)d\tau\right) + n_1(t)$$

  - f_c: carrier frequency, $m(t)$: filtered baseband signal
  - $h = 2f_d T_s$: modulation index, with f_d frequency deviation and T_s symbol period
  - **Continuous-phase** $2\pi\Delta f_c \cdot t = 2\pi h \int_{-\infty}^{t} m(\tau)d\tau$ carries data information

- **Differential** FSK demodulator
  - **Phase shifter** frequency response

$$\varphi(f) = -\frac{\pi}{2} + 2\pi K(f - f_c), \; K = 1, 2, \cdots$$



  - Phase shifter output

$$y(t) = A\cos\left(2\pi f_c t + 2\pi h \int_{-\infty}^{t} m(\tau)d\tau - \frac{\pi}{2} + 2\pi K h m(t) + n_1(t)\right) + n_2(t)$$

  - Under condition that $2\pi K h m(t)$ is small

$$r_{LP}(t) = \text{LPF}\left(y_o(t) \cdot y(t)\right) \approx \frac{A^2}{2} 2\pi K h \cdot m(t)$$

which contains a scaled transmitted baseband signal $m(t)$

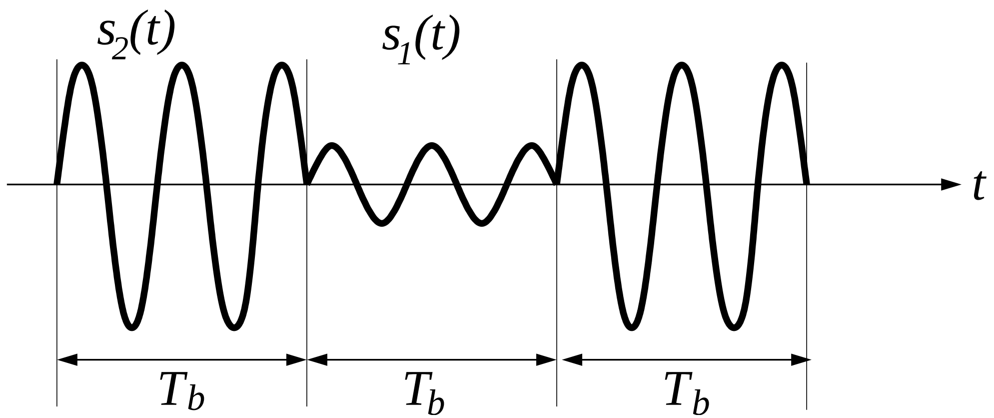
Amplitude Shift Keying

- ASK: A_i carries symbol information

$$s_i(t) = A_i \cos(\omega_c t + \theta), \ 1 \le i \le M, \ 0 \le t \le T_s$$

- BASK: $M = 2$, bit 0: $A_1 = A$, bit 1: $A_2 = 3A$

$$s_1(t) = A \cos(\omega_c t + \theta)$$
$$s_2(t) = 3A \cos(\omega_c t + \theta)$$



- For ASK, **linear amplifier is essential**. A nonlinear channel can seriously distort ASK signals, and ASK is rarely used on it own

- Typically, ASK and PS are combined $\Rightarrow$ quadrature amplitude modulation

Quadrature Amplitude Modulation

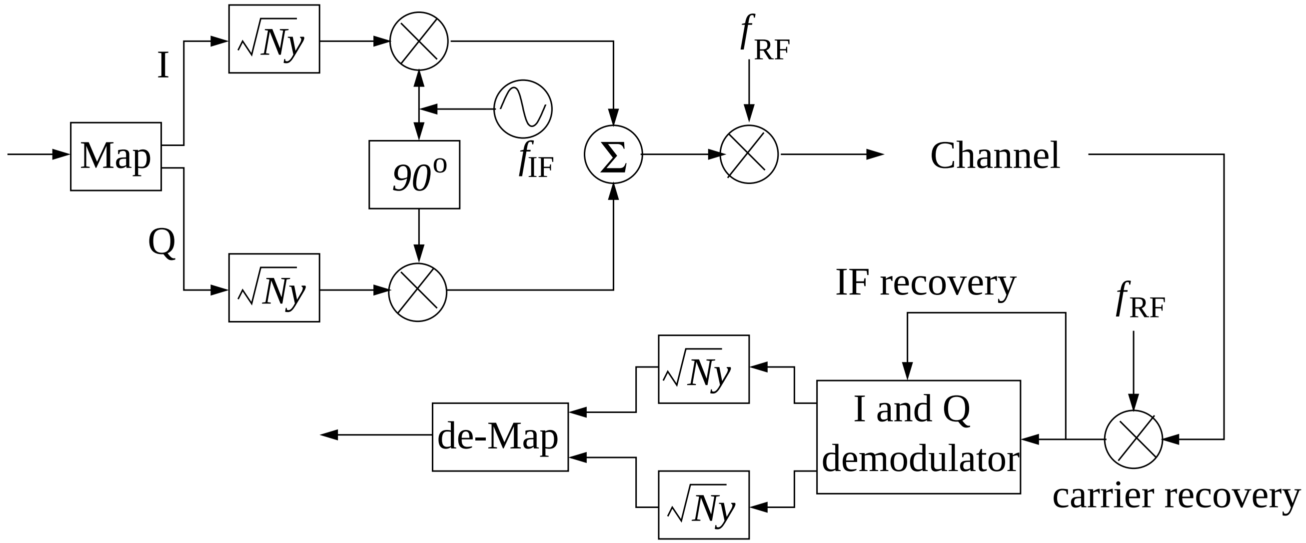
- QAM: **combined amplitude/phase keying**

$$s_i(t) = A_i \cos(\omega_c t + \phi_i(t)), \; 0 \le t \le T_s, \; 1 \le i \le M$$

  - T_s being symbol period, as both amplitude and phase are used to carry symbol information, it is very bandwidth efficient
  - symbol set size M: $2^1 \times 2^1 = 4$, $2^2 \times 2^2 = 16$, $2^3 \times 2^3 = 64$, etc $\longrightarrow$ 4QAM, 16QAM, 64QAM with 2, 4, 6 BPS etc
  - The larger M is, the better bandwidth efficiency but lower robustness against noise and fading

- Simplified QAM modem:

  Pulse shaping filter is split into two $\sqrt{Ny}$ parts for transmitter and receiver

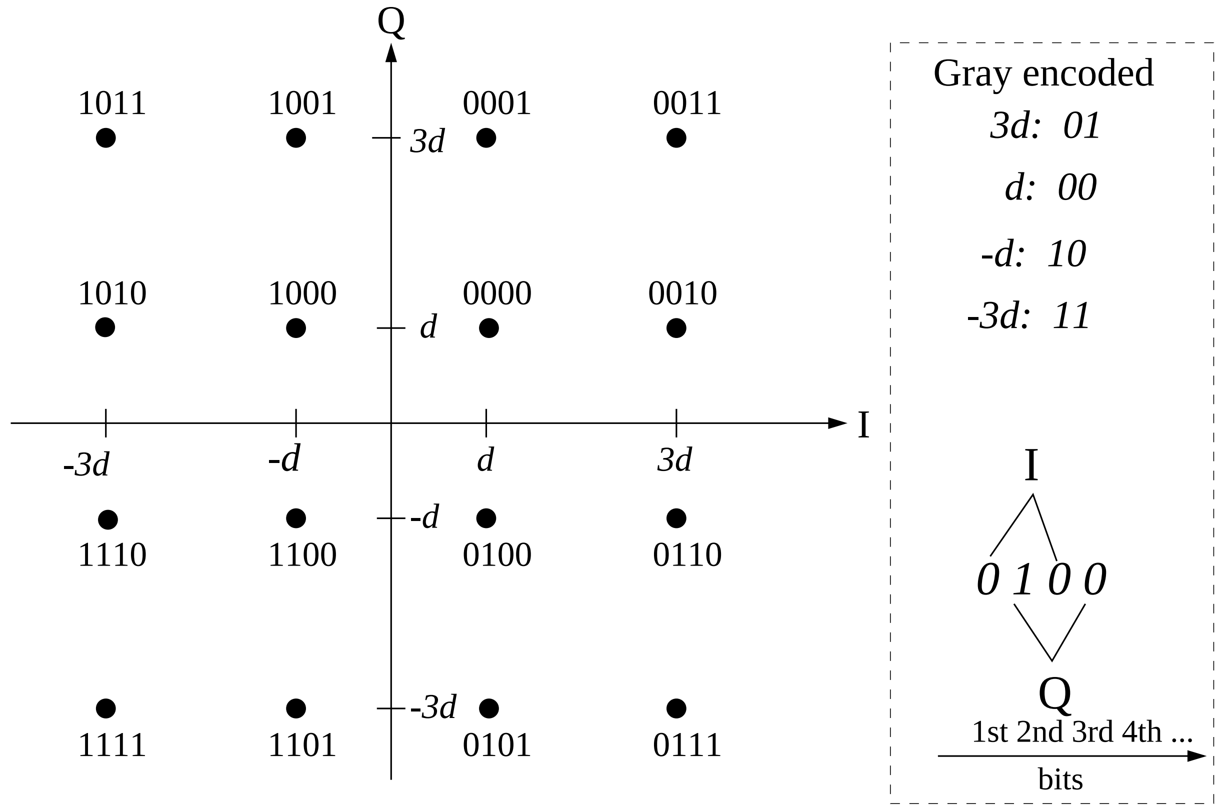


Mapping Bits to Symbols

- Example: squared 16-QAM

  Shortest distance among the neighbour points is $2d$

- Bit stream is split into I and Q streams. I and Q are Gray encoded (neighbour points only differ in 1 bit position) to produce symbol points in constellation

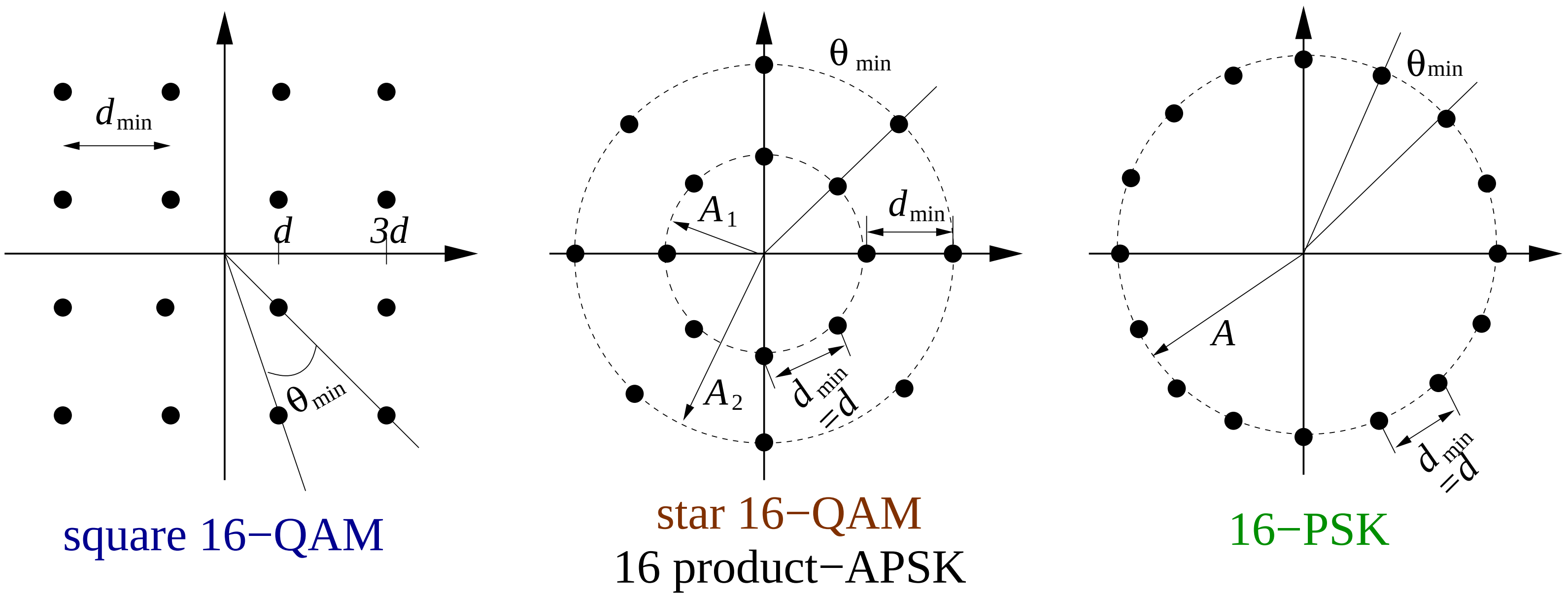


- For squared 16QAM, I and Q components are Gray encoded by assigning the bits 01, 00, 10 and 11 to the levels $3d$, d, $-d$ and $-3d$

  For 4-QAM, I and Q are BPSK; for 16QAM, I and Q are 4-ary; For M-QAM, I and Q are $\sqrt{M}$-ary

Constellation Design

- Consider different (4 bits per symbol) constellation schemes with same average symbol energy E_s



- Design considerations:
  1. The **minimum distance** $d_{\min}$ among phasors constellation points, which is the characteristic of the noise immunity of the scheme
  2. The **minimum phase rotation** $\theta_{\min}$ among constellation points, determining the phase jitter immunity and hence resilience against clock recovery errors and channel phase rotation
  3. The **ratio of peak-to-average power** r, which is a measure of robustness against nonlinear distortions introduced by the power amplifier

Constellations: Comparison

1. Square 16QAM: $22.5^o < \theta_{\min} < 45^o$, peak energy is $18d^2$, and average energy is $E_s = 10d^2$
   - Thus $d_{\min} = 2d = 2\sqrt{E_s/10} \approx 0.63\sqrt{E_s}$ and $r = \frac{18d^2}{10d^2} = 1.8$
2. Star 16QAM: $\theta_{\min} = 45^o$, $d = 2A_1 \cos(67.5^o)$ or $A_1 \approx 1.31d$, and $A_2 = A_1 + d \approx 2.31d$
   - Since average energy is $E_s = \frac{A_1^2 + A_2^2}{2} \approx 3.53d^2$, $d_{\min} \approx \sqrt{E_s/3.53} \approx 0.53\sqrt{E_s}$
   - Also peak energy is A_2^2, so $r \approx \frac{(2.31d)^2}{3.53d^2} \approx 1.5$
3. 16PSK: obviously $\theta_{\min} = 22.5^o$, $r = 1$, and $E_s = A^2$
   - $d = 2A\cos(78.75^o) \approx 0.39A$, hence $d_{\min} = 0.39\sqrt{E_s}$

Constellation	$\theta_{\min}$	$d_{\min}$	r
square 16QAM	$22.5^o < \theta_{\min} < 45^o$	$d_{\min} = 0.63\sqrt{E_s}$	$r = 1.8$
star 16QAM	$\theta_{\min} = 45^o$	$d_{\min} = 0.53\sqrt{E_s}$	$r = 1.5$
16PSK	$\theta_{\min} = 22.5^o$	$d_{\min} = 0.39\sqrt{E_s}$	$r = 1$

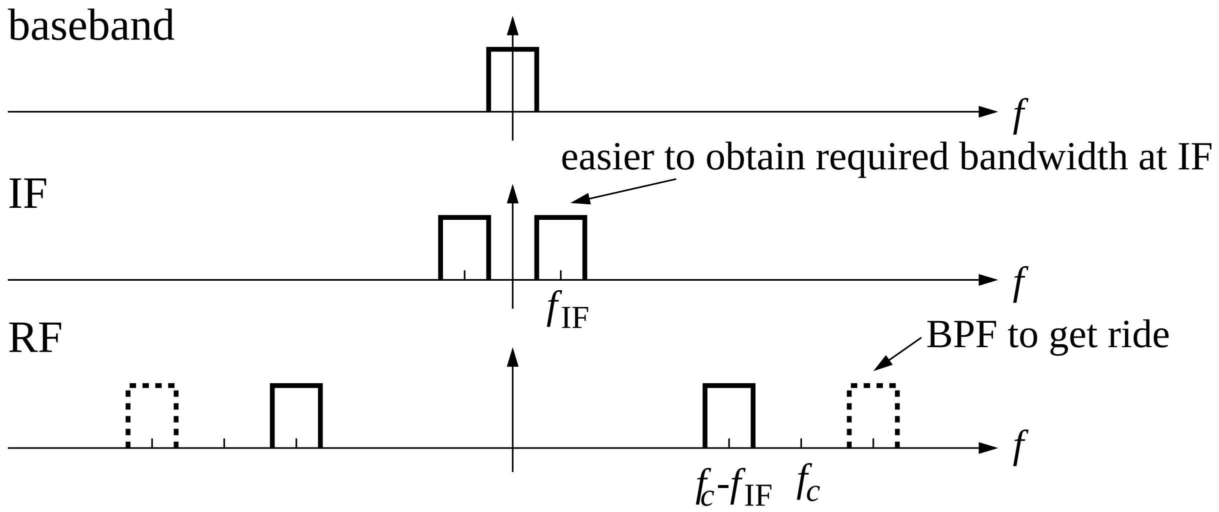
- **Star** 16QAM has higher jitter immunity and slightly lower peak-to-average energy ratio than square 16QAM, and it is more robust to fading and has better power efficiency
- **Square** 16QAM has almost 20% ((0.63-0.53)/0.53) higher minimum distance at the same average energy than star 16QAM, and it is optimum in terms of error probability for AWGN channels
- 16PSK measures worst in terms of minimum distance and minimum phase rotation, but has **ideal peak-to-average power ratio** of 1

Intermediate Frequency Modulation

- In theory, the modulation is done by multiplying baseband signal with **carrier**

$$s(t) = m_I(t)\cos(2\pi f_c t) + m_Q(t)\sin(2\pi f_c t)$$

  – That is, modulation is mixing modulating signal $m(t)$ with carrier $\cos(2\pi f_c t)$
  – which will introduce noise and spread power into adjacent bands
  – BPF (not shown in slide 92) must be used to obtain required bandwidth
  – This is easier at lower IF frequency than at RF



- In practice, therefore, it is often seen that modulation is first via an intermediate frequency modulation and then RF modulation by f_c, as in slide 92 of QAM modem

Example

The Shannon-Hartley law is given by:

$$\frac{C}{B_p} = \log_2(1 + \text{SNR}) \quad \frac{\text{bps}}{\text{Hz}}$$

where B_p is the RF channel bandwidth in Hz and C is the channel capacity in bps.

1. Assuming the roll-off factor $\gamma = 0$, state the minimum required channel SNR for supporting 1, 2 and 4 bit/symbol (BPS) BPSK, QPSK and 16QAM, respectively, signalling over Gaussian channels.

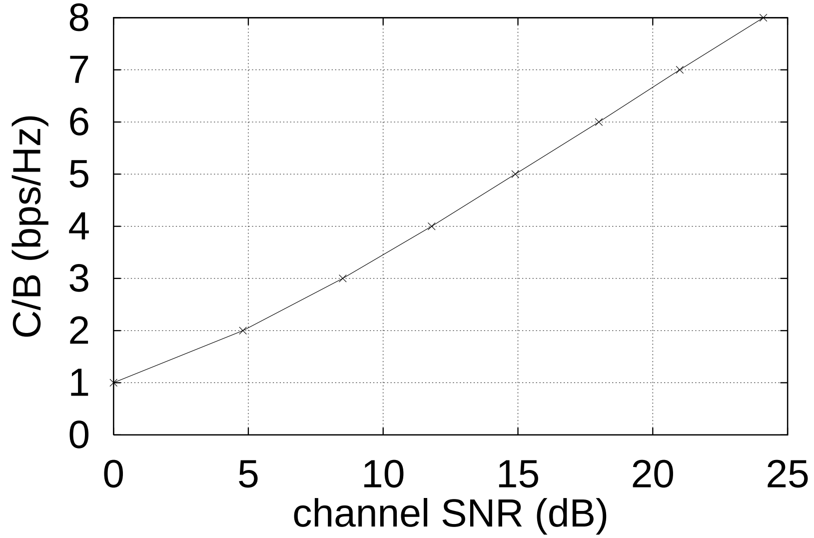
2. Assuming $\gamma = 0.5$ and a channel bandwidth of 600 kHz, calculate the achievable bit rate of BPSK, QPSK and 16QAM.

Note: Shannon-Hartley channel capacity is for Gaussian signal. BPSK, QPSK and 16QAM are not Gaussian signal, although PDF of 16QAM signal is much closer to a Gaussian PDF than PDF of BPSK signal. Nevertheless, we use this ideal case channel capacity as though it were the channel capacity of BPSK, QPSK and 16QAM channels, i.e. we use the upper limit.

Solution

From the Shannon-Hartley law, we have the following table and figure:

SNR		$\frac{C}{B_p}$
ratio	dB	bps/Hz
1	0	1
3	4.8	2
7	8.5	3
15	11.8	4
31	14.9	5
63	18.0	6
127	21.0	7
255	24.1	8



1. Noting $\gamma = 0$, the symbol or transmission rate $f_s = B_p$, that is, C/B_p is the same as BPS. Thus, the minimum required channel SNRs for supporting 1, 2 and 4 BPS BPSK, QPSK and 16QAM are 0, 4.8 and 11.8 dB, respectively.

2. For $\gamma = 0.5$, $B_p = (1 + \gamma)f_s = 1.5f_s$, leading to $f_s = 400$ kHz. The data rate (bps) $R = BPS \times f_s$. Thus, $R_{\text{BPSK}} = 400$ kbps, $R_{\text{QPSK}} = 800$ kbps, and $R_{\text{16QAM}} = 1600$ kbps.

Summary

- We have covered all basic modulation schemes, including PSK, ASK and FSK
- We have emphasized QAM, a combination of ASK and PSK, bandwidth very efficient linear modulation scheme
  - QAM modem diagram, mapping bits to symbols
  - Bandwidth efficiency is obtained at cost of power efficiency
- We exam **design considerations** or criteria of constellation schemes
  1. Minimum distance among constellation points, which is the characteristic of the noise immunity of the scheme
  2. Minimum phase rotation among constellation points, determining phase jitter immunity, i.e. resilience against clock recovery errors and channel phase rotation
  3. Peak-to-average power ratio r, which measures robustness against nonlinear distortions introduced by power amplifier
- We **have not yet** considered carrier recovery and clock recovery for QAM